Range-independent inversion results from the Modal Inverse Methods Experiment (MIME)

Megan S. Ballard and Kyle M. Becker

Applied Research Laboratory and Graduate Program in Acoustics
The Pennsylvania State University
PO Box 30, State College, PA 16804
The New Jersey Shelf

chirp seismic reflection data

Chirp data provided by John Goff
Perturbative Inversion using QR

Relate perturbation in wave number to perturbation in sound speed of the sediment.

\[ \Delta k_n = \frac{1}{2k_n} \int_0^D \frac{\Delta q(z) Z_n^2}{\rho(z)} \, dz \quad \rightarrow \quad y = Ax \]

The parameter estimate is given by:

\[ \hat{x} = (A^T A + \lambda^2 L_q^T L_q)^{-1} A^T y \]

\( L_q \) is given by:

\[ L_q = L(I - \sum_{i=1}^r q_i q_i^T) \]

the set \( \{q_i\}_{i=1}^r \) is on orthogonal basis for \( Q \).

\( L \) is a discrete version of the differential operator \( \frac{d^n}{dx^n} \)
Along and Across Shelf Tracks

Ship tracks oriented along and across the shelf. The flattest 3 km segment of each track was used for wave number estimation and inversion and is shown by the dashed portion.
Water Column Sound Speed
Auto regressive (AR) techniques were used to estimate wave numbers from pressure field data. Mode shapes aided in identifying modes. Across shelf data shown here.

Window length of the AR estimator is 3000m.
Monte Carlo Error Estimates

The complete solution of the inverse problem requires not only the estimates of the model parameter values, but also a measure of the uncertainty of the estimates.

Monte Carlo Error Propagation:

\[ y + n_i = Ax_i \quad i = 1, 2, \ldots, N \]

The empirical estimate of the covariance matrix

\[ \text{Cov} = \frac{D^T D}{N} \quad \text{where} \quad D = x_i^T - \bar{x}_i^T \]

Assuming the model parameters are normally distributed, the 95\% confidence interval can be computed by

\[ C.I. = 1.96 \sqrt{\text{diag}(\text{Cov})} \]

where the factor 1.96 comes from

\[
\int_{-1.96\sigma}^{1.96\sigma} e^{-\frac{x^2}{2\sigma^2}} dx \approx .95
\]
Inversion Results

\[ R = \frac{\sum_{n=1}^{N} \sqrt{(k_n^{\text{meas}} - k_n^{\text{calc}})^2}}{N} \]

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Along Shelf</th>
<th>Across Shelf</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.0006</td>
<td>0.0010</td>
</tr>
<tr>
<td>75</td>
<td>0.0022</td>
<td>0.0023</td>
</tr>
<tr>
<td>125</td>
<td>0.0015</td>
<td>0.0015</td>
</tr>
<tr>
<td>175</td>
<td>0.0016</td>
<td>0.0016</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.0059</strong></td>
<td><strong>0.0064</strong></td>
</tr>
</tbody>
</table>
Mode Shapes: 50 Hz

Validity of the inversion results: comparison of measured and calculated mode shapes.
Mode Shapes: 75 Hz

Validity of the inversion results: comparison of measured and calculated mode shapes.
Mode Shapes: 125 Hz

Validity of the inversion results: comparison of measured and calculated mode shapes.
Mode Shapes: 175 Hz

Validity of the inversion results: comparison of measured and calculated mode shapes.
Current Work:
Range Dependent Inversion
Making the 3D Model

Across Shelf

Depth [m]

Range [m]
Simulation Results
50 Hz Pressure Field

Pressure Field in dB

Along Shelf Track

Oblique Shelf Track

Across Shelf Track
Simulation Results
75 Hz Pressure Field

Pressure Field in dB

Along Shelf Track

Oblique Shelf Track

Across Shelf Track
Simulation Results
125 Hz Pressure Field
Simulation Results
175 Hz Pressure Field

Pressure Field in dB

Along Shelf Track

Oblique Shelf Track

Across Shelf Track
Conclusions and Future Work

• Completed range independent inversion results
  – Produced reasonable inversion results with confidence intervals
  – Mode shapes of the first five modes calculated from the inversion result are a good match to the data
  – There is still work to be done: pressure fields do not make a good match across all four frequencies

• The next step: range dependent inversion results
  – Range dependant wavenumber estimates along each of the three tracks
  – Perform perturbation inversion for each range step
  – Interpolate between the tracks to obtain sound speed estimates for the entire grid
  – Match the pressure fields for each of the three tracks