A model for four-dimensional coastal internal waves with applications to acoustics

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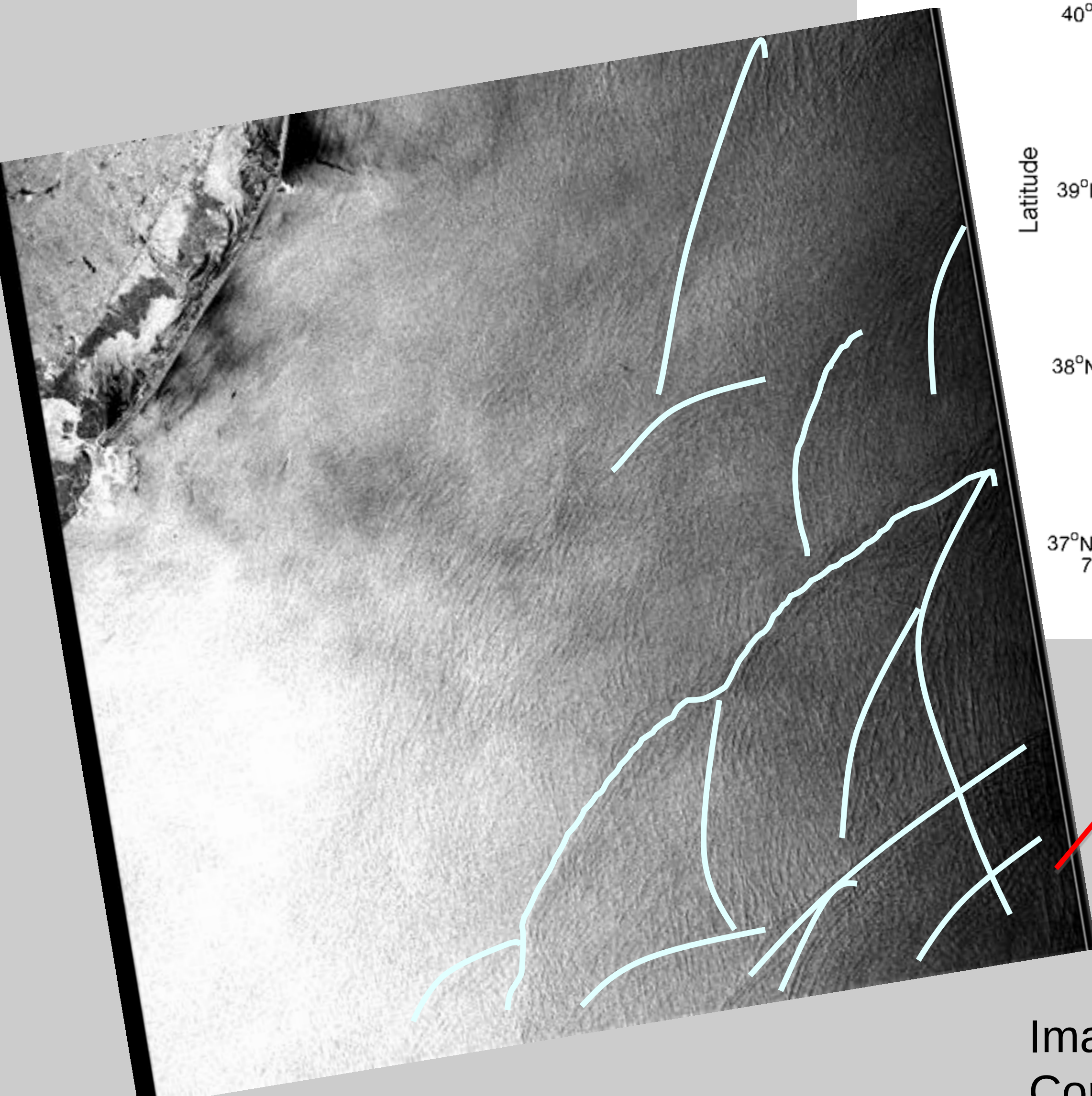

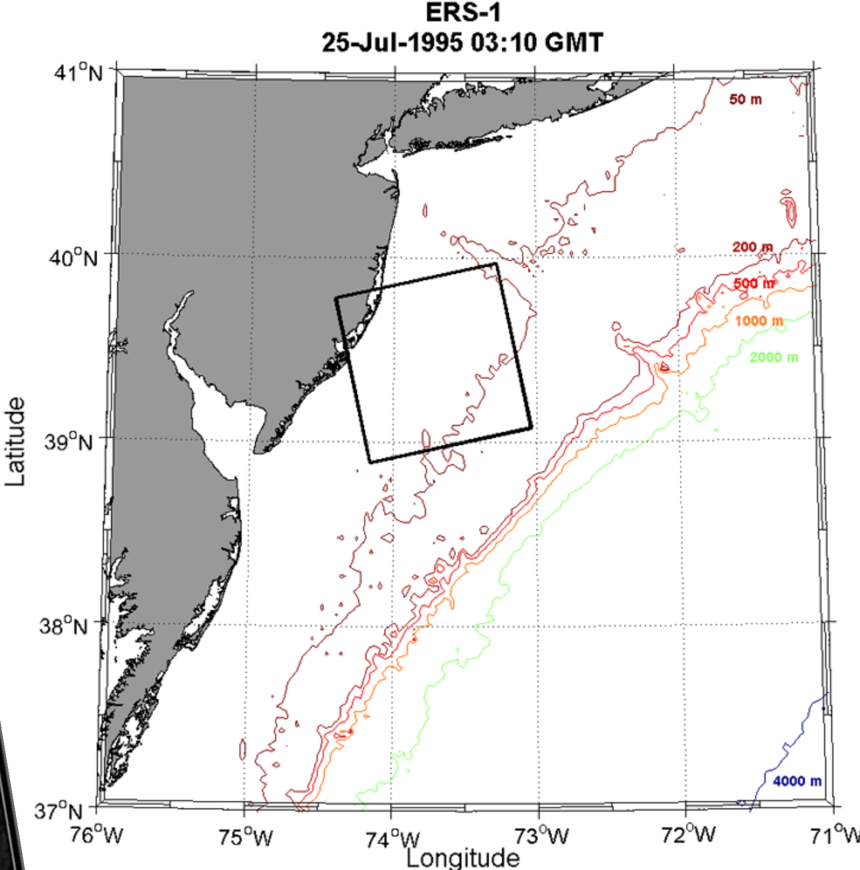

Image from Global Ocean Associates
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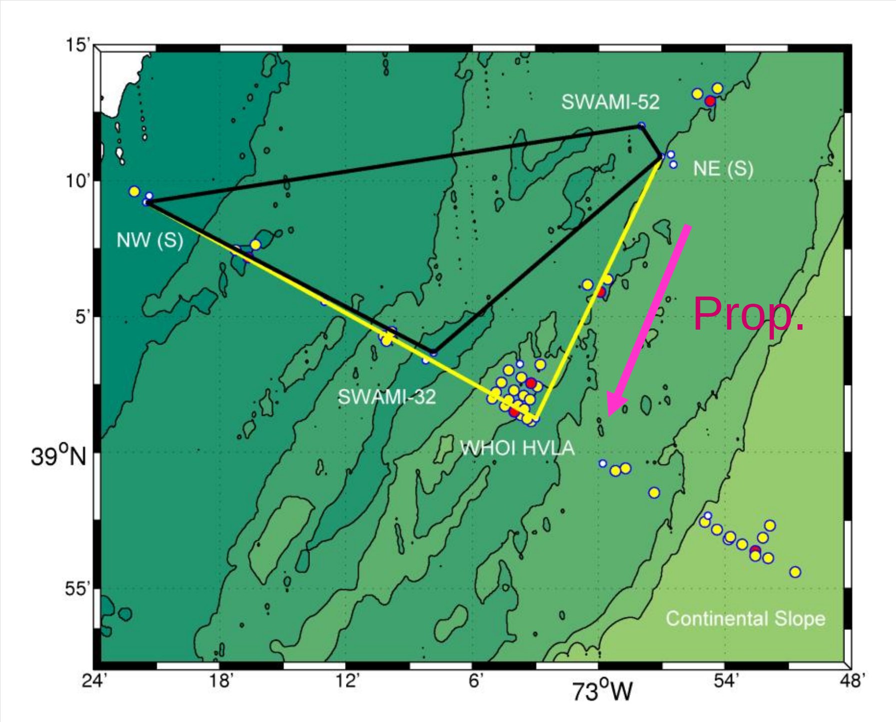
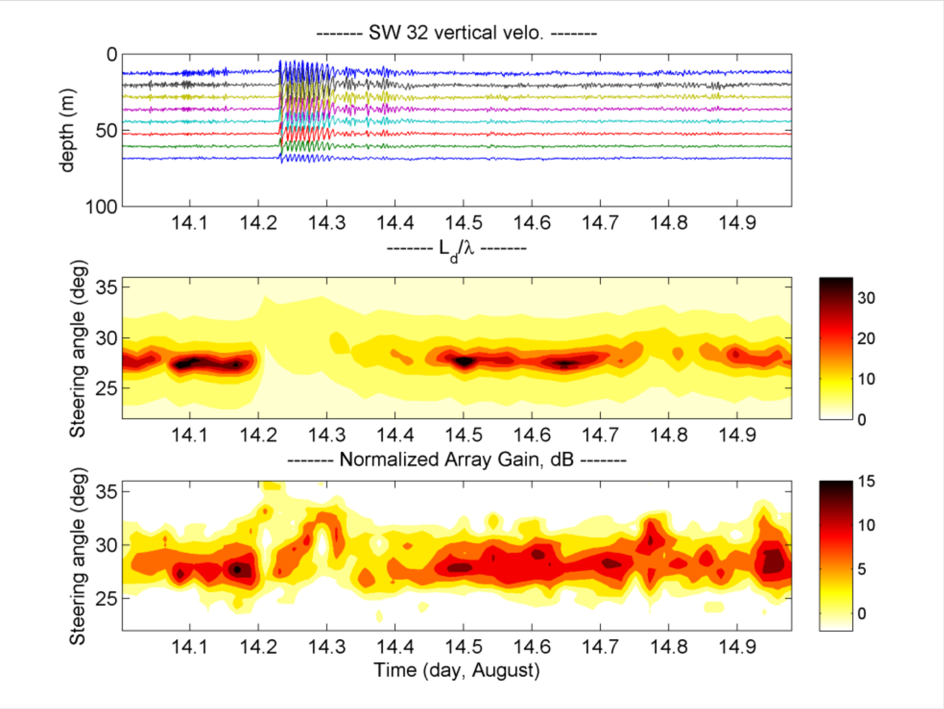
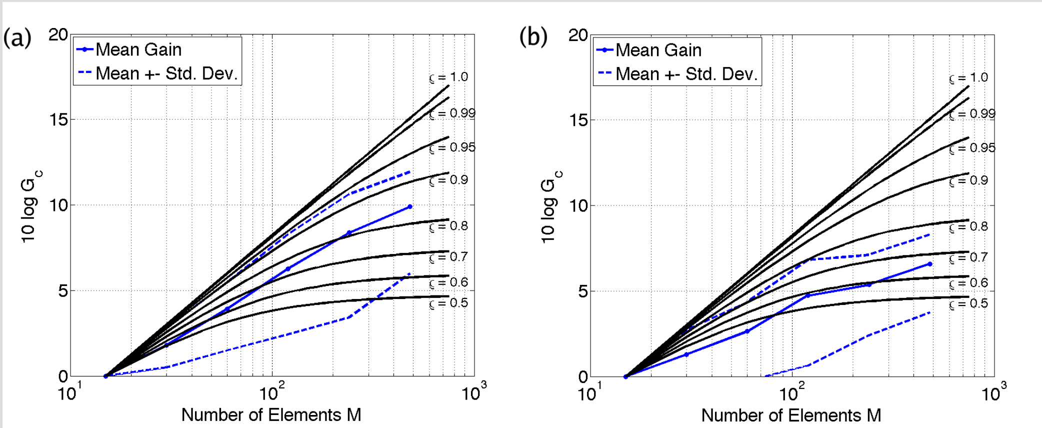
Internal-wave effects

- **Non-linear internal gravity waves (NLIWs) create strong anomalies of sound speed at the thermocline. Major effects:**
  - Shadowing
  - Focusing
  - Mode content alteration
  - Mode multipath
  - Mode ‘splitting’ (y-modes of z-modes)
  - Apparent bearing shifts
- **Advantages of localizing (predicting) the wave locations and parameters:**
  - Predict event arrivals at fixed locations
  - Make evolving maps of fluctuation statistics
  - Predict major/minor axes of anisotropic conditions
  - Predict strength of fluctuation effects
- **Nonlinear wave details are sensitive and by nature not predictable**

SW06 Along-shelf Acoustic Observations: Transiting NLIW Groups



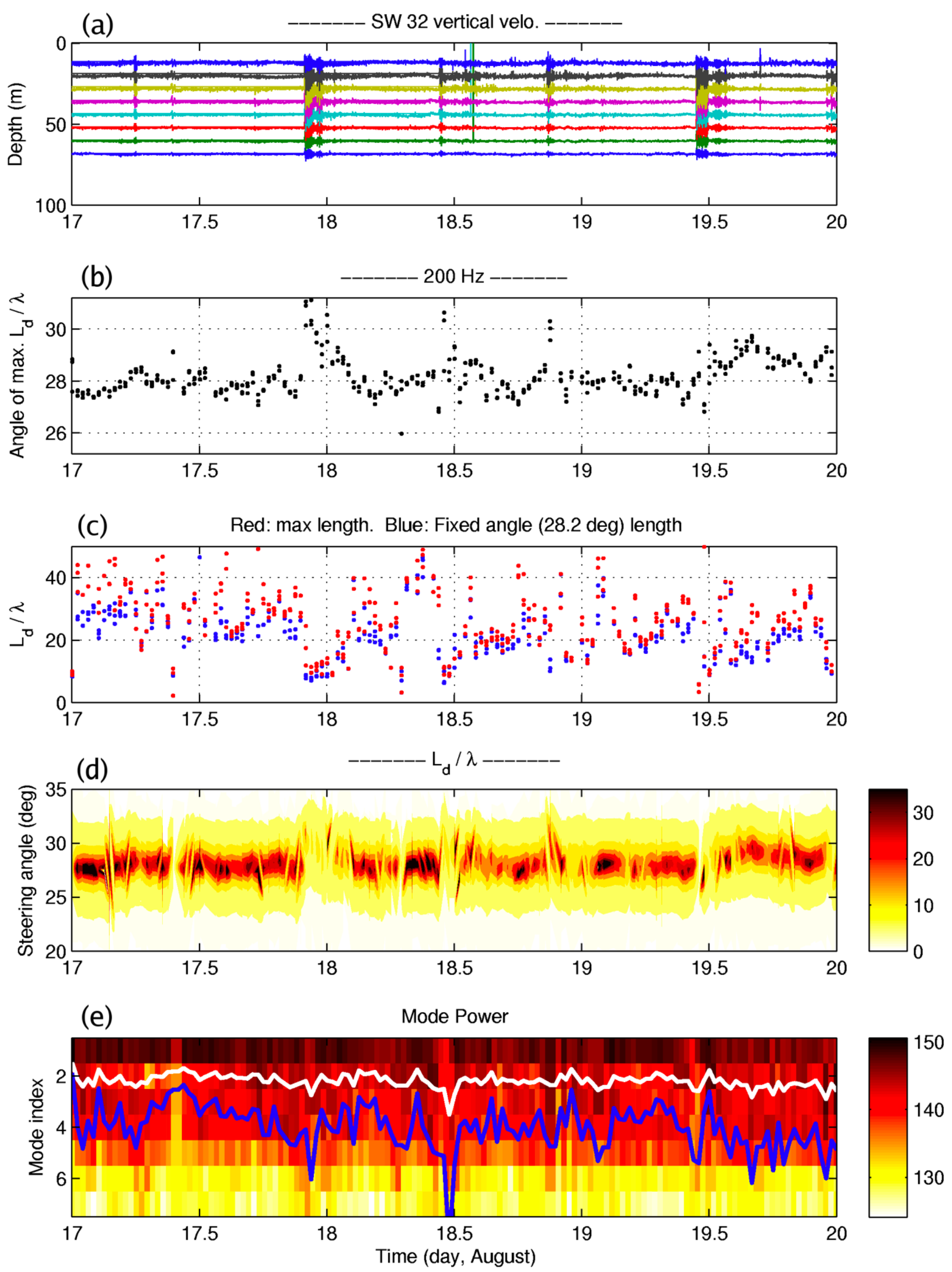




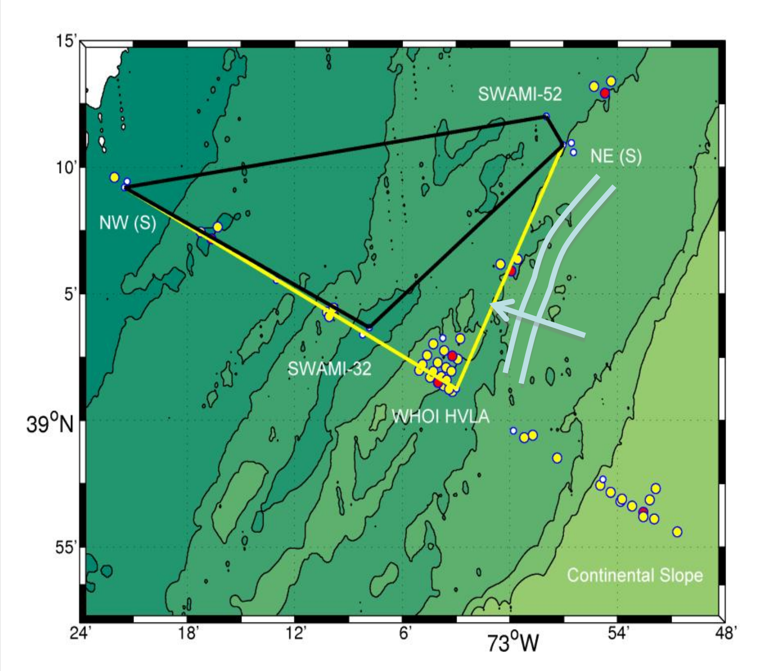
Above: Beam deflection and shortening of horizontal field coherence length (30 to 5 wave-length) at horiz. array as IW's pass. Array gain changes also.

Left: Typical max and min array gain curves vs aperture size. (w/o and with NLIW)

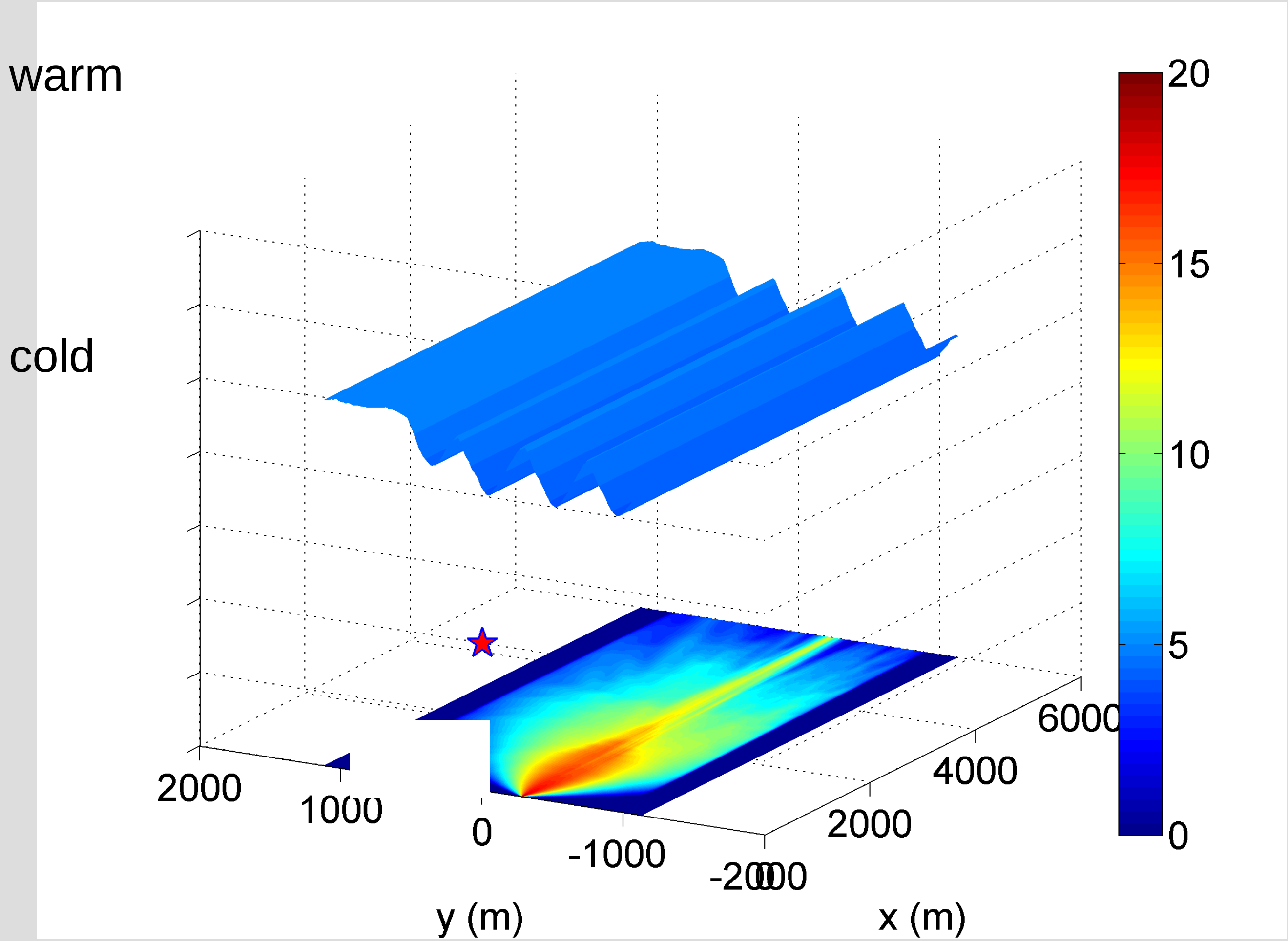
Collis, Duda, Lynch, DeFerrari, 9/2009 JASA EL.



(a)
------- SW 32 vertical velo. -------
Depth (m)
(b)
------- 200 Hz -------
Angle of max. L_d / λ
(c)
Red: max length. Blue: Fixed angle (28.2 deg) length
L_d / λ
(d)
------- L_d / λ -------
Steering angle (deg)
(e)
Mode Power
Mode index
Time (day, August)



SWAMI-52
NE (S)
NW (S)
SWAMI-32
WHOI HVLA
Continental Slope
39°N
73°W

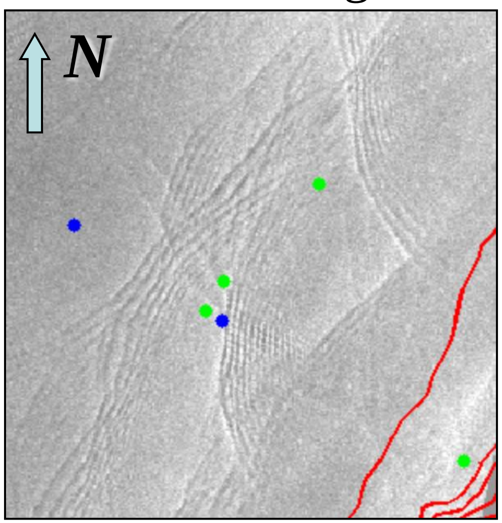


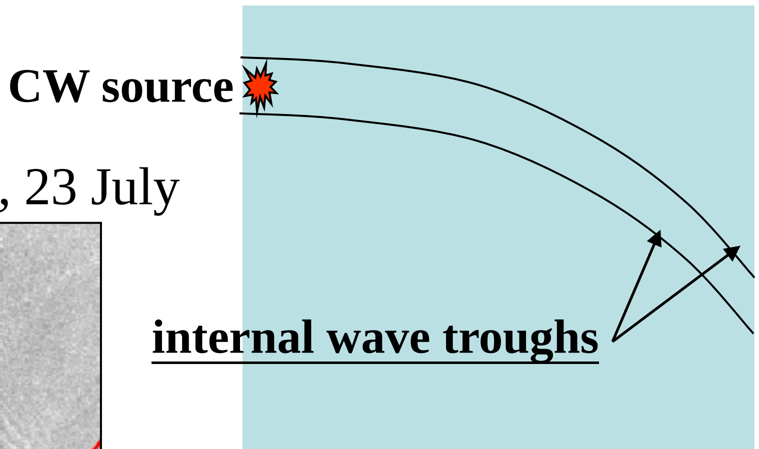
Idealized duct (South China Sea parameters)

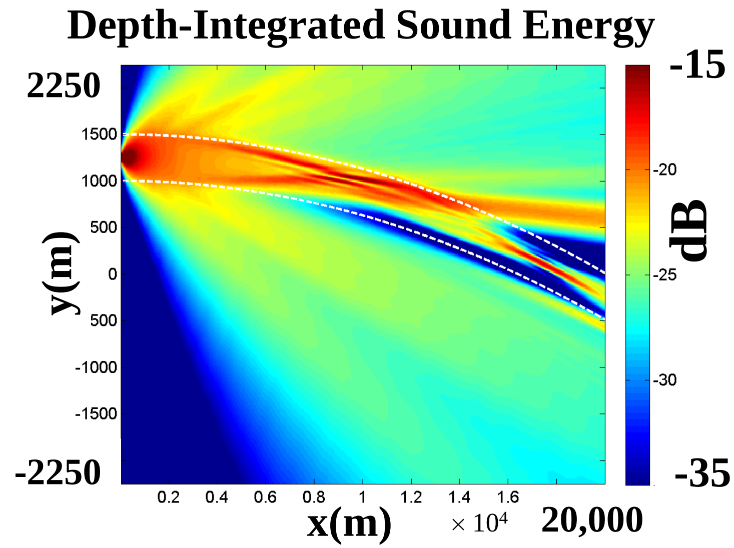
Acoustic mode focusing in a curved internal wave duct

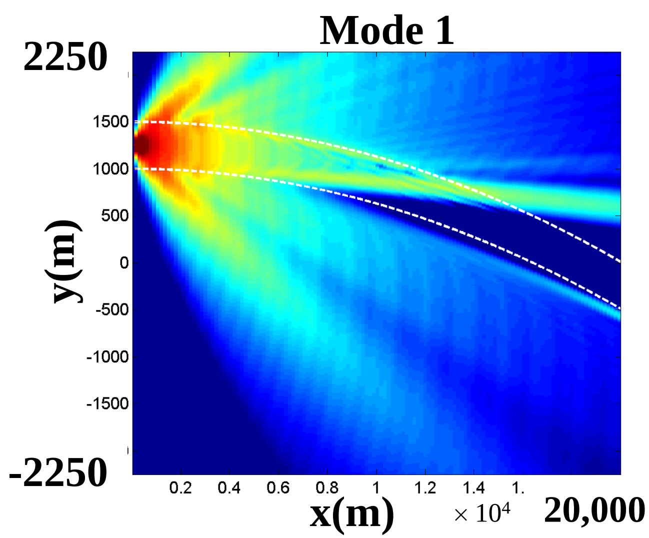
- Radius of curvature=135km, frequency = 100Hz
- Mode 1 penetrates through internal wave duct, but mode 2 focuses in the duct.

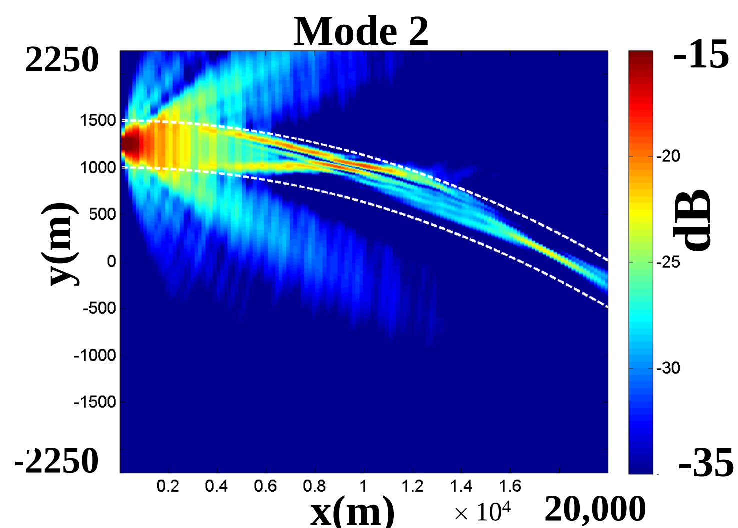
SW06 SAR image, 23 July

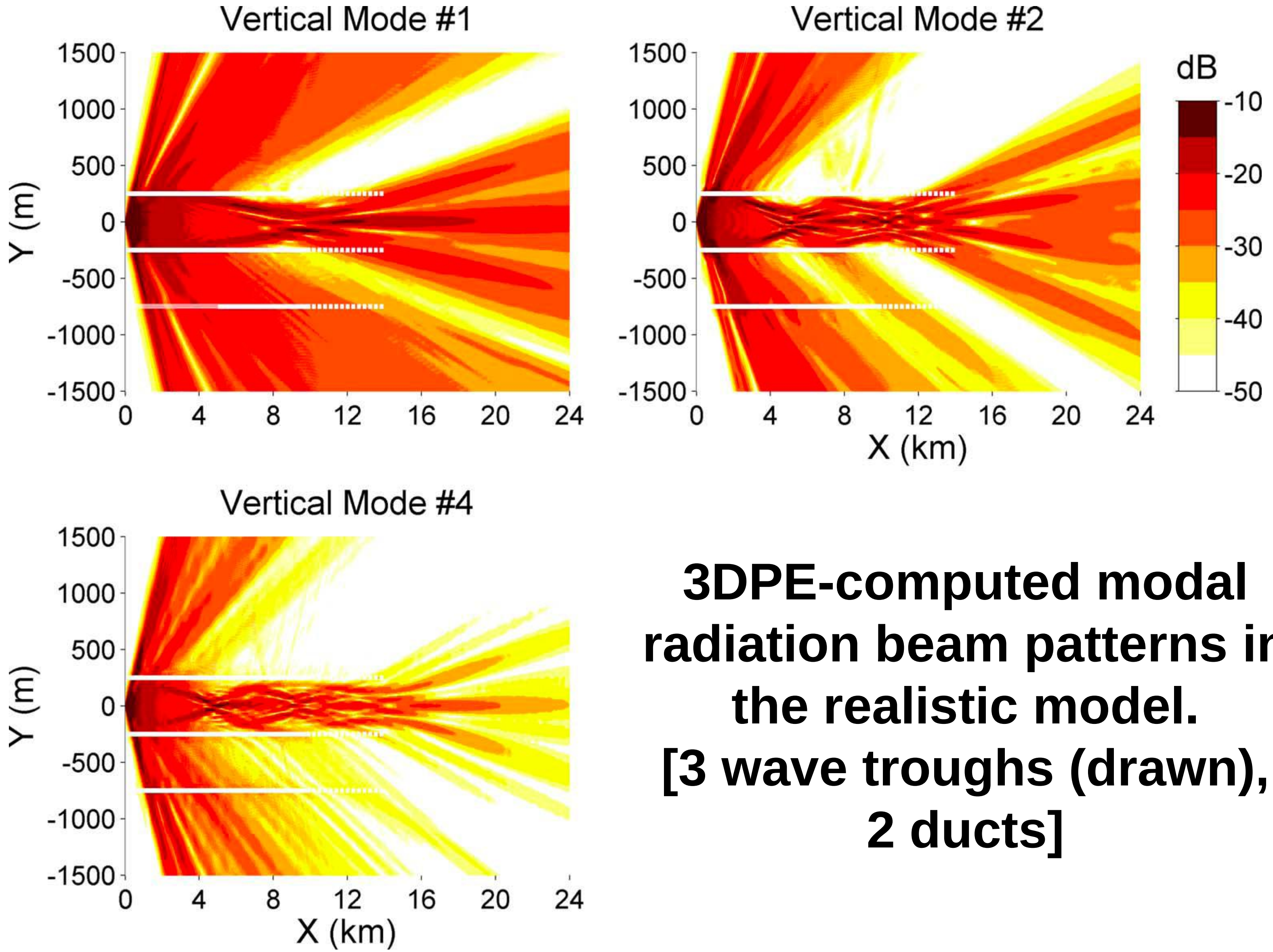












3DPE-computed modal radiation beam patterns in the realistic model.
[3 wave troughs (drawn), 2 ducts]

Mapping internal wave effects: Regional models

- **Type 1: With hydrostatic approximation (HA):**
  - Fast; pervasive in the community
- Can solve for:
  - Mesoscale: eddies and fronts
  - Internal tide (IT) source regions
  - Internal tide strength
  - Initial propag. of long-wavelength IT
- However, IT response may suffer when deep stratification is incorrect, when ITs enter open boundaries, and when boundary layer and mixing parameterizations are not optimal.

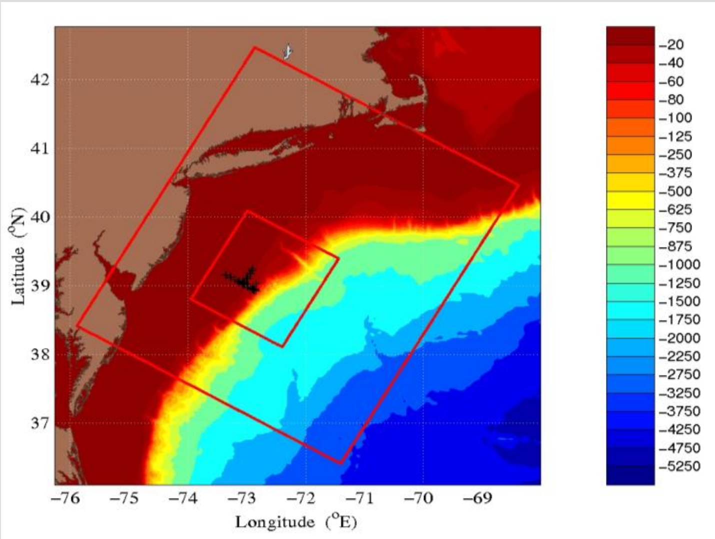
- **Type 2: Full physics, without Hydrostatic approximation: ("Non-hydrostatic"):**
  - Not fast; specialized.
  - Can solve for mesoscale and IT *and* actual internal wave packets with perfect initialization
  - Perfect initialization impossible

(two strikes)

A practical procedure: ***(1)*** *Use a combination of models to ascertain IT generation behavior (source locations, energy density, phase of orbital tide).*

(2) *Use HA model to directly map IT, or initialize using step-1 information and use robustly modeled HA-model eddy field to map internal tide speed and trace rays (make wavefronts).*

(3) *Add nonlinear waves to the wavefronts.*

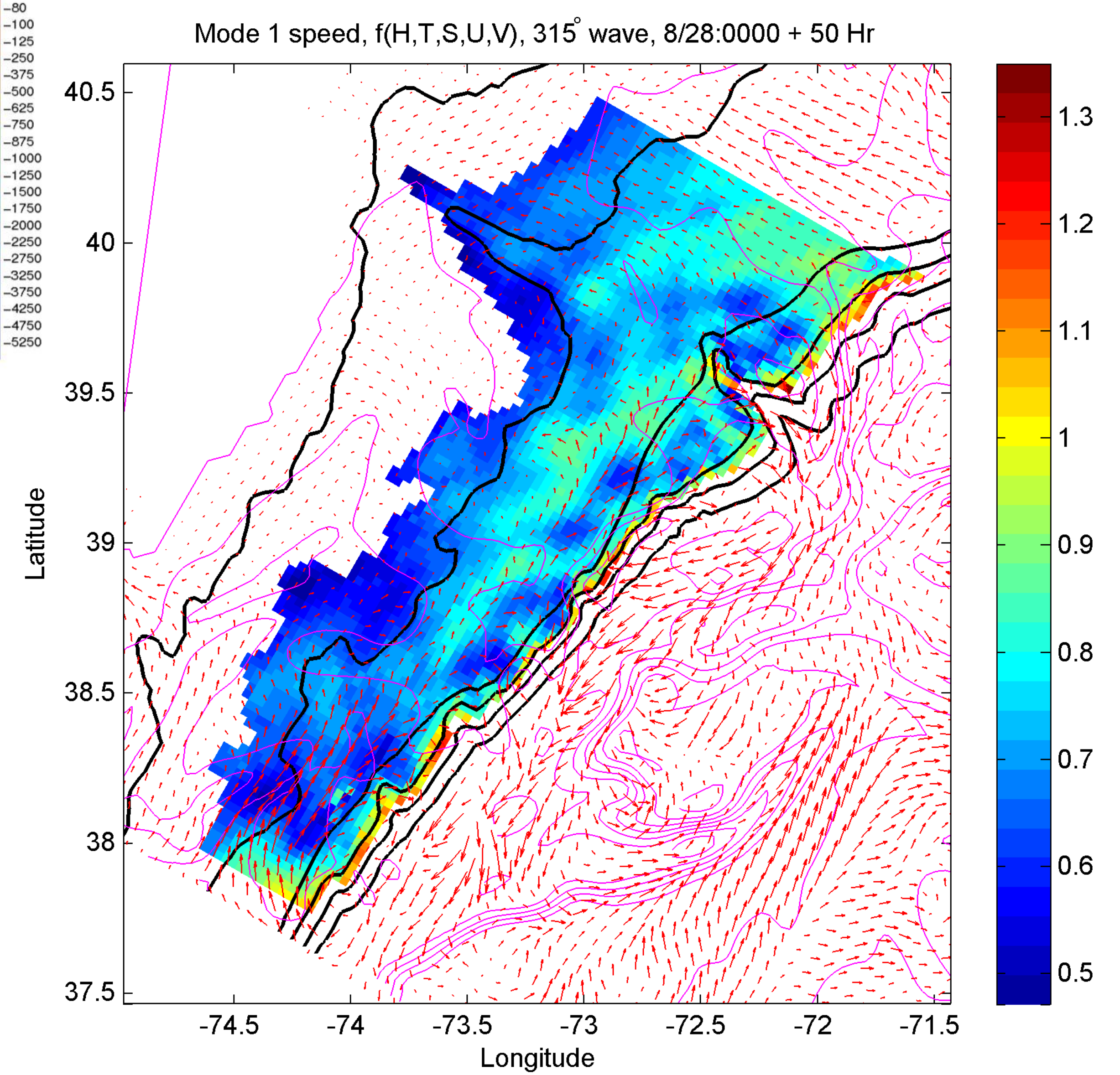


HOPS two-way nested modeling domains. 3km and 1km grids. Tides included.

Mode-1 internal tide speed. Anisotropic in variable current field [u(x,y,z), v(x,y,z)]

Speed at one heading plotted.

Explanation forthcoming.



Anisotropic wave speed: Taylor-Goldstein equation

Used often for stability analysis of viscid and inviscid plane-parallel shear flow. (Find ‘fastest growing modes’, high imaginary speed component, i.e. exponential growth, with $c = \omega/k$)

Earth’s rotation neglected. (Need to fix that and re-derive.)

We analyze the eigenmode with fastest real speed, having (essentially) zero imaginary speed component.

$$w_{zz} + \left[\frac{N^2}{(c-U)^2} + \frac{U_{zz}}{c-U} - k^2 \right] w = 0$$

$$\mathbf{u}' = (u', 0, w')$$

$$w' = (x, z, t) = w(z) e^{i(kx - \omega t)}$$

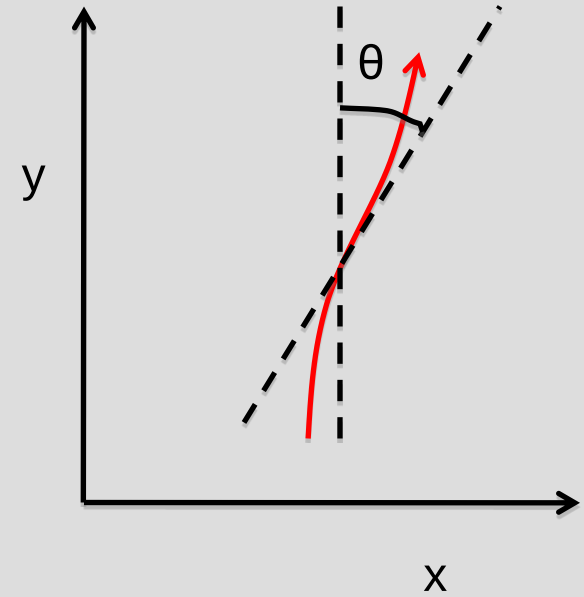
$$U(z) = \text{background current; project [U,V]}$$

Non-canonical form of raytrace equations

$$\frac{d\theta}{dy} = \frac{1}{c}\frac{\partial c}{\partial y}\tan\theta - \frac{1}{c}\frac{\partial c}{\partial x}$$

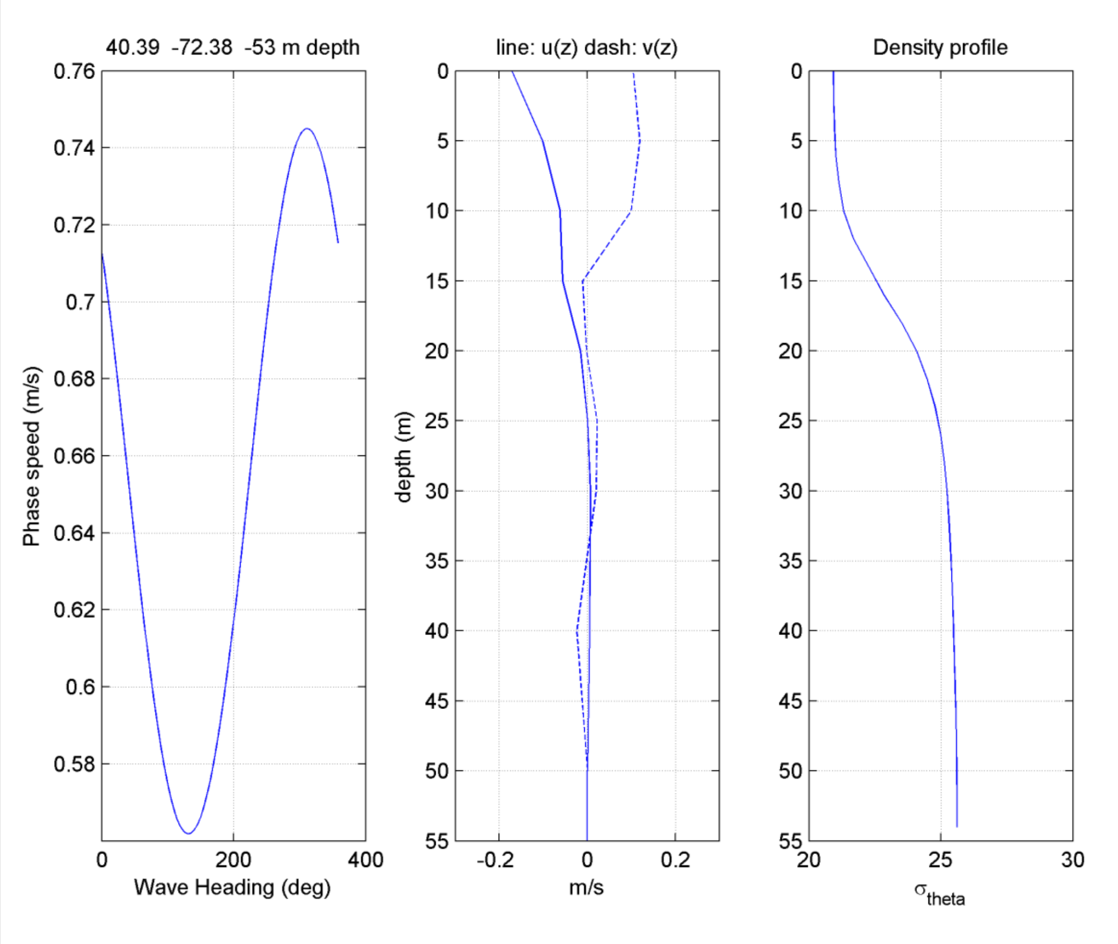
$$\frac{dx}{dy} = \tan\theta$$

$$\frac{dt}{dy} = \frac{\sec\theta}{c}$$



$c_I = c(x, y)$ Isotropic wave speed

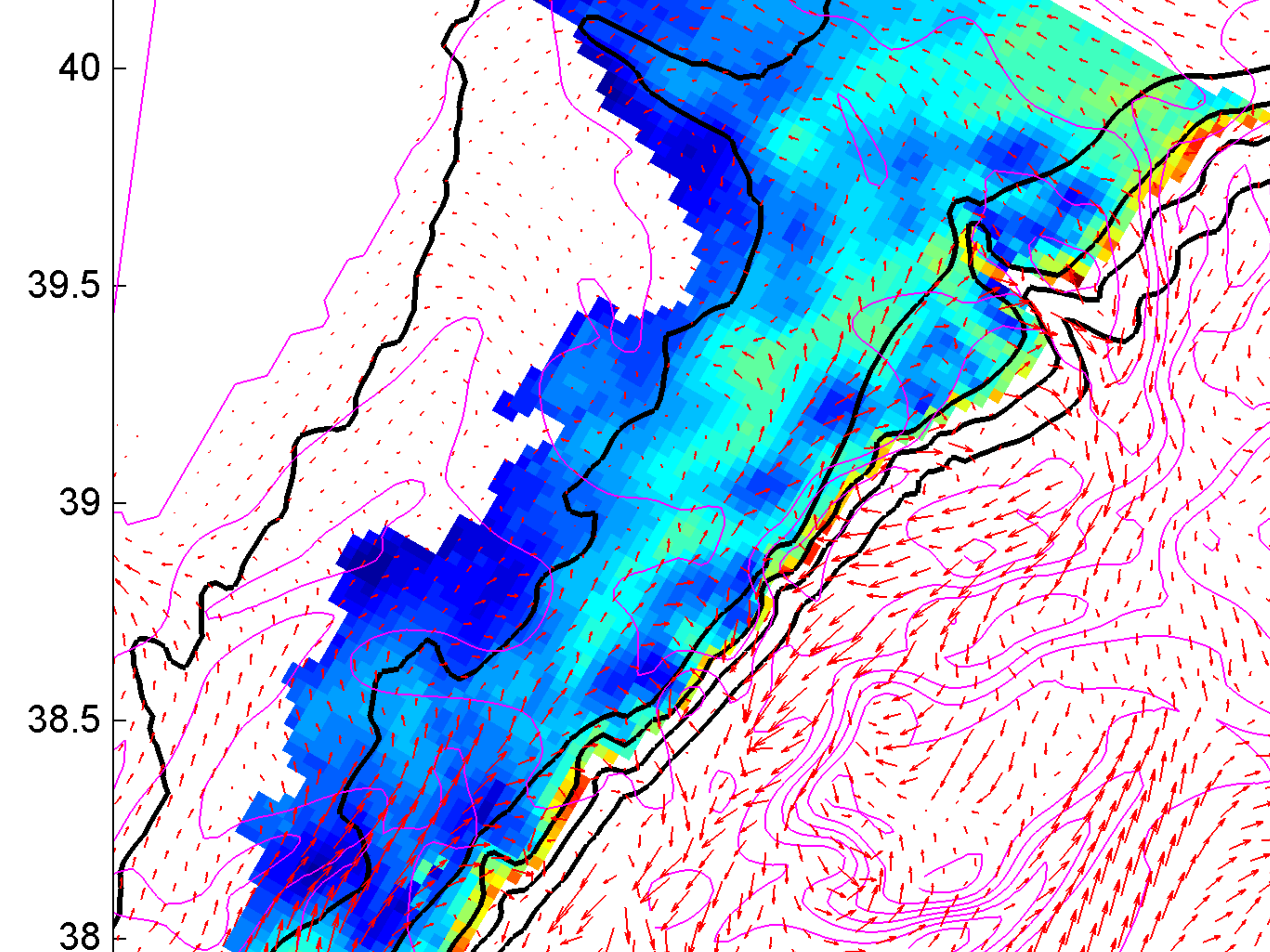
$c_A = c(x, y, \theta)$ Anisotropic wave speed



$$w_{zz} + \left[\frac{N^2}{(c-U)^2} + \frac{U_{zz}}{c-U} - k^2 \right] w = 0$$

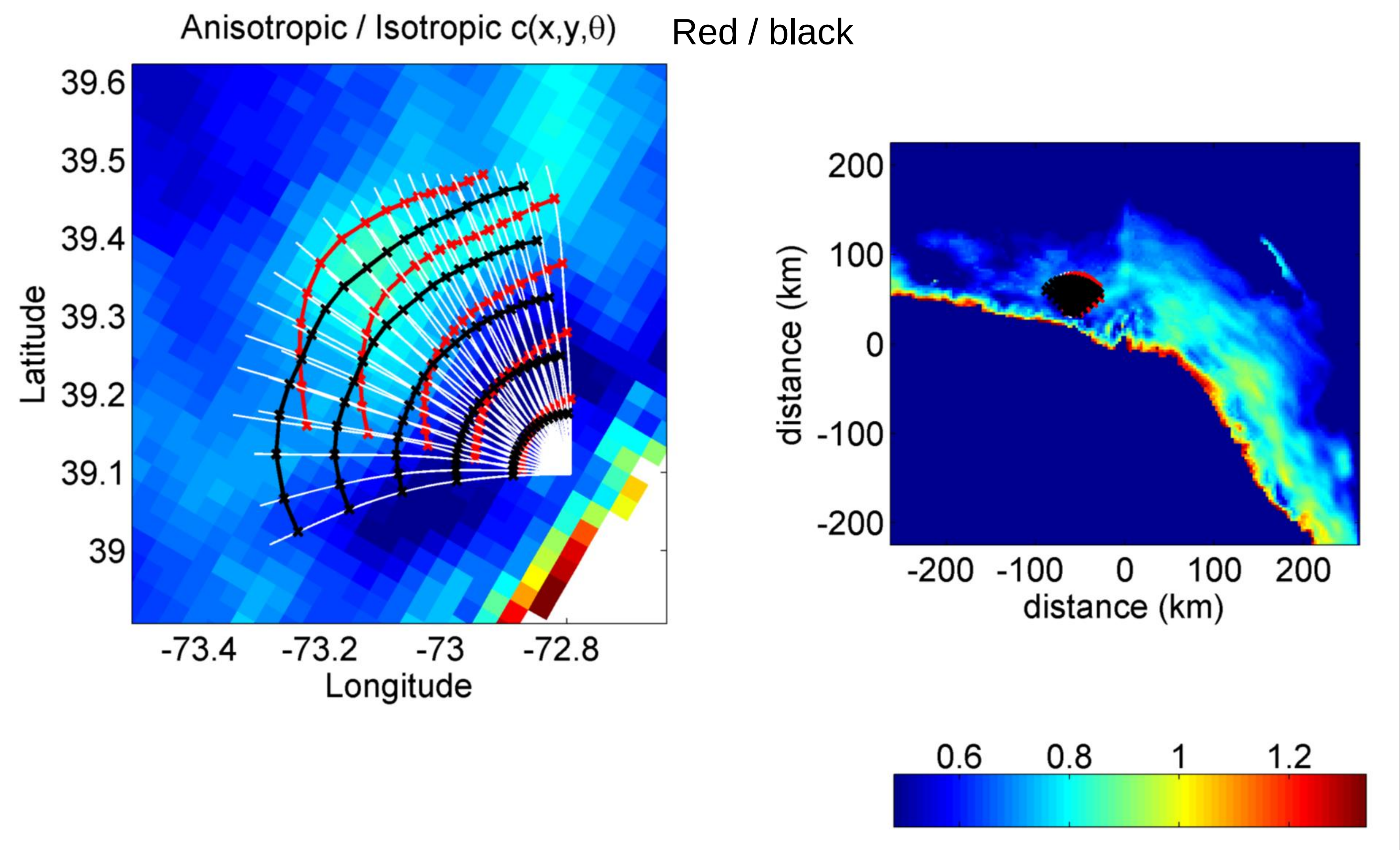
$U(z) =$ background current; project [U,V]

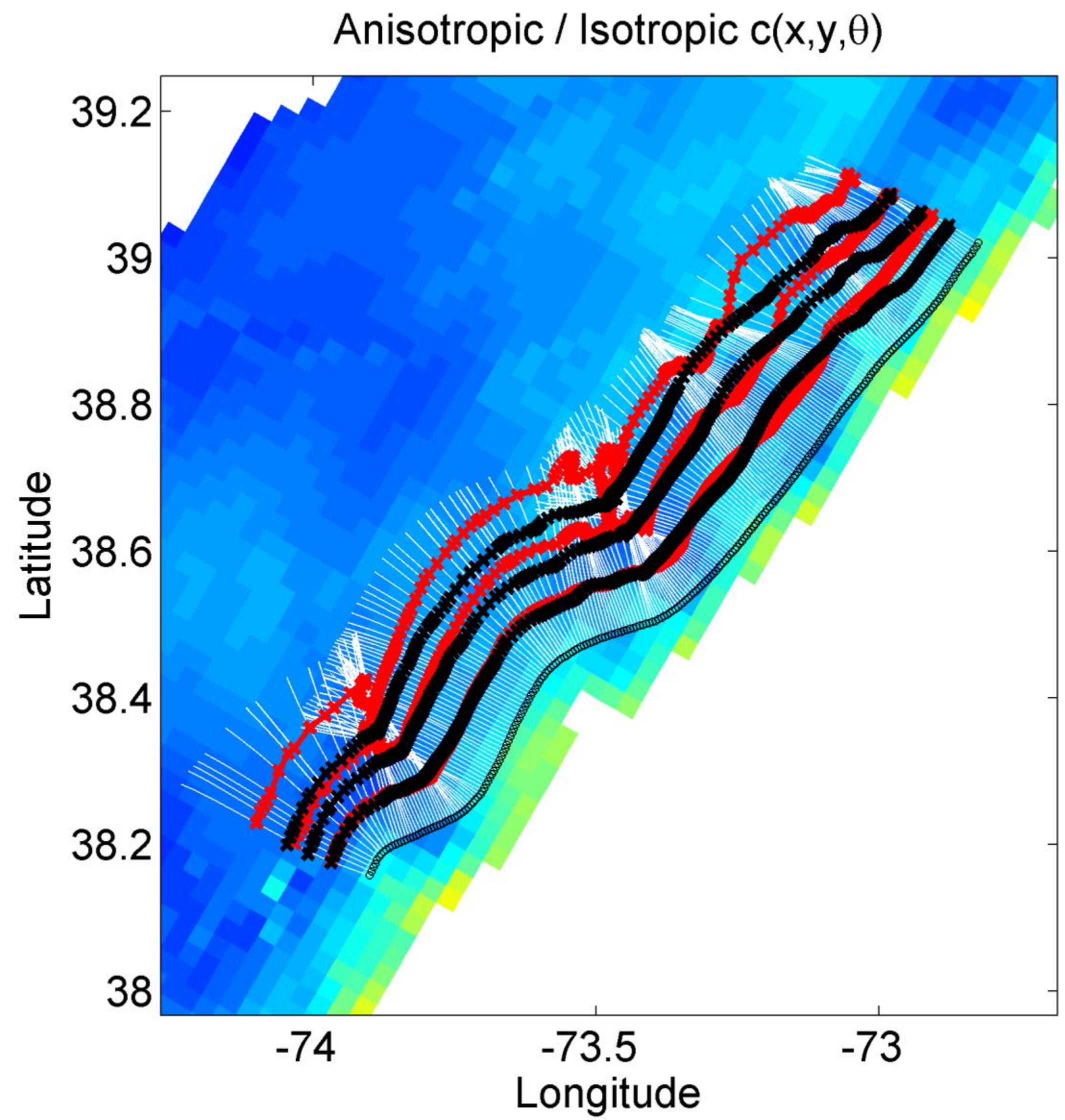
Project current onto all directions to solve for eigenmodes at all points of the compass.



40
39.5
39
38.5
38

Anisotropic / Isotropic wave speed comparison





Anisotropic / Isotropic c(x,y,θ)
39.2
39
38.8
38.6
38.4
38.2
38
Latitude
-74
-73.5
-73
Longitude

Add nonlinear internal waves to IT wave front

Using IT time series and model density conditions as input:

- Solve Korteweg-de Vries equation or a relative, full solution or KdV dnoidal variant.
  - or, solve Euler equations
  - or, solve 2D non-hydrostatic model
- Include energy density from ray convergence.

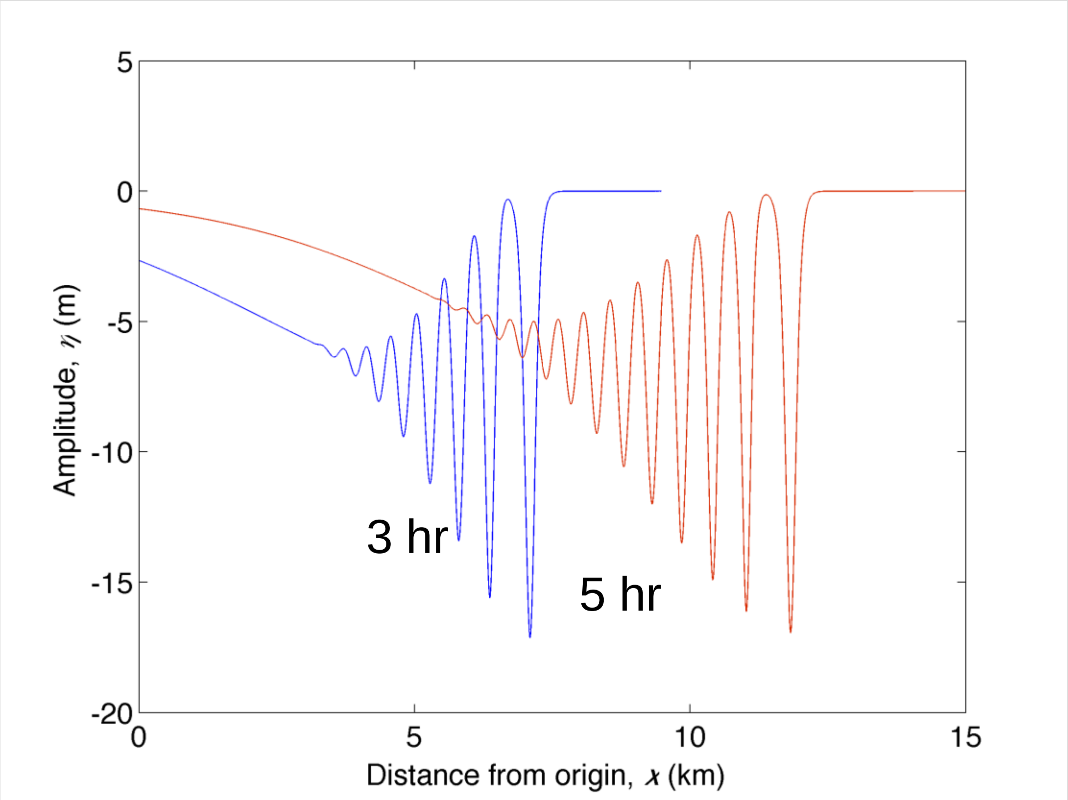
KdV equation

$$\frac{\partial \eta}{\partial t} + c\frac{\partial \eta}{\partial x} + \alpha\eta\frac{\partial \eta}{\partial x} + \beta\frac{\partial^3 \eta}{\partial x^3} = 0$$

α and β are found by depth integrating functions of the internal-wave normal mode shapes (cube of derivative, for example).

η is the amplitude of the first normal mode in our case.

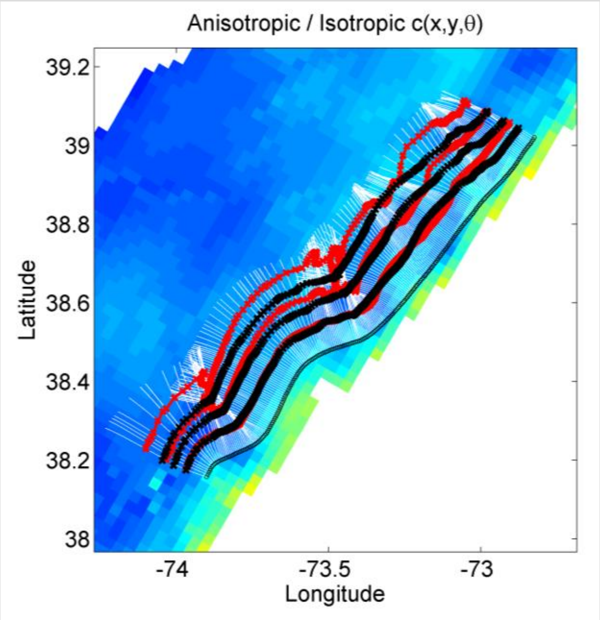
John Apel's"dnoidal" total solution



Conclusions

A nested scheme for mapping long-wavelength and nonlinear internal wave is challenging, interesting, and requires theoretical validation and perhaps some input data.

Stands a chance of providing useful predictions of wave patterns.



Conclusions

Under construction

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Under construction