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**Time-frequency analysis techniques for long range
sediment tomography**

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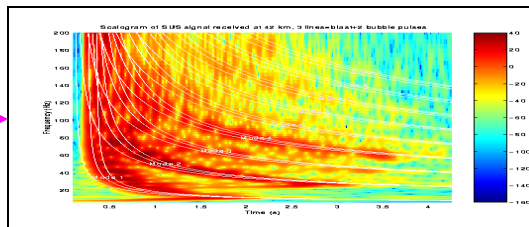
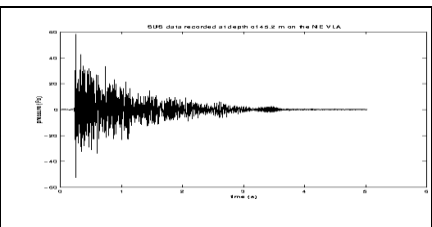
Outline

- Sediment tomography
- CSS data analysis using Dispersion Based STFT (D-STFT)
- Empirical Mode Decomposition – results
- Summary and Future work

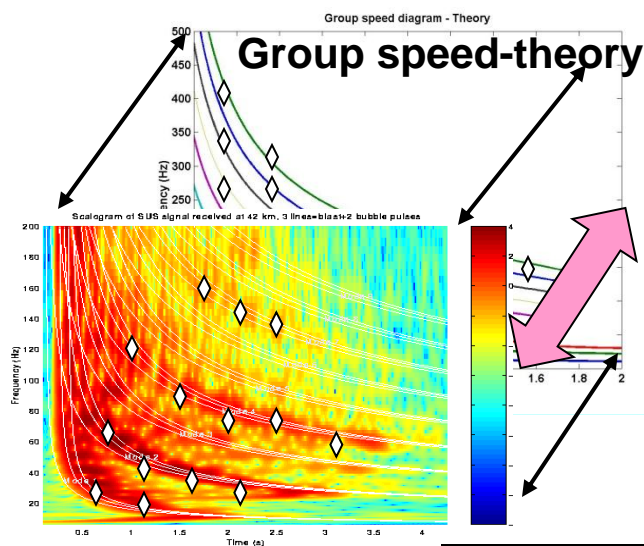
Inversion Scheme for Compressional Speeds

Group speed dispersion by Time-frequency analysis

Genetic Algorithm Optimization



Broadband data



***A Posteriori* analysis
mean, standard deviation**

Parameters for GA search

- EOF coefficients
- Sediment compressional speeds
- Bathymetry
- Source-receiver range

Objective Function for m^{th} parameter set

$$E(m) = ||w (d^{\text{obs}} - d^{\text{pred}})||_2$$

w – diagonal weight matrix

d^{obs} – observed data

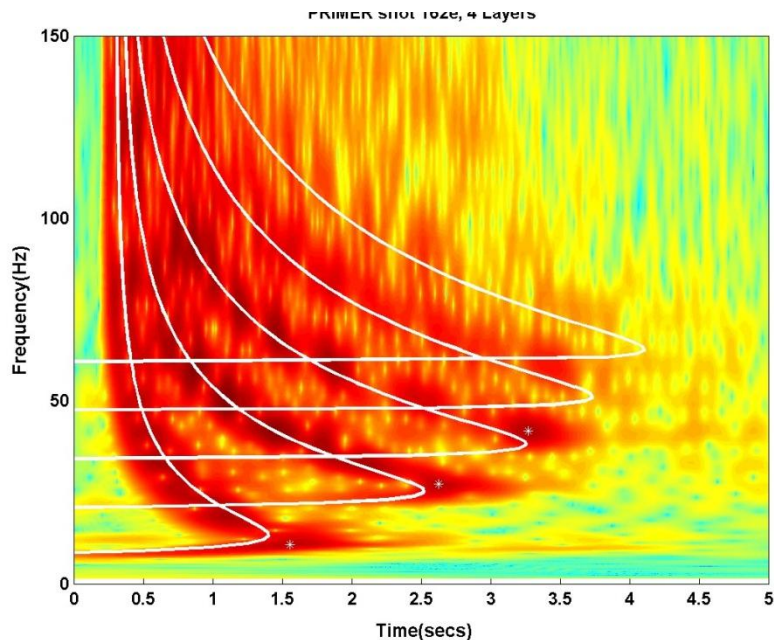
d^{pred} – predictions of forward model

Levenberg-Marquardt
non-linear least squares method

Range: 40 km
Water depth \cong 100 m
Charge Weight: 0.8 kg
Source depth: 18 m

Arrival spread 4 s and 10- 150 Hz.

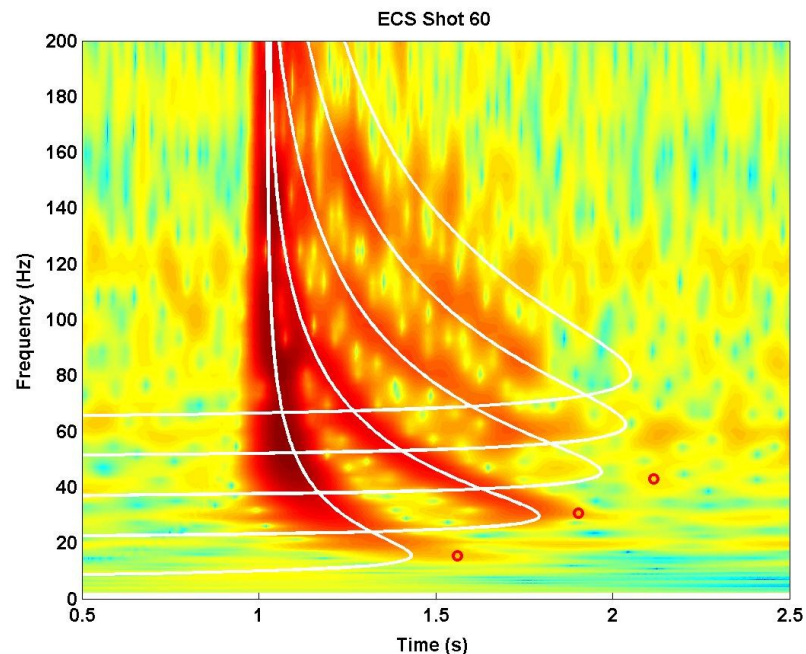
PRIMER (1996)



Range: 30km
Water depth \cong 100 m
Charge Weight: 38 g;
Source depth: 50 m

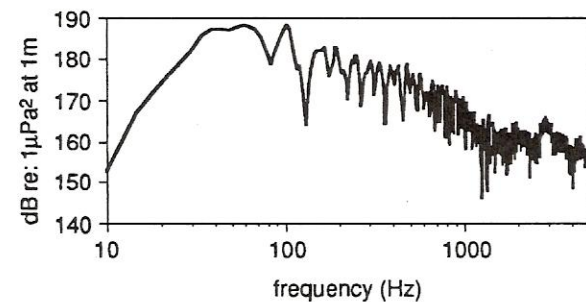
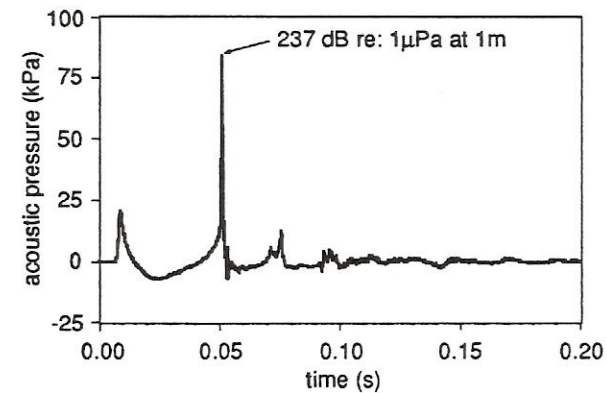
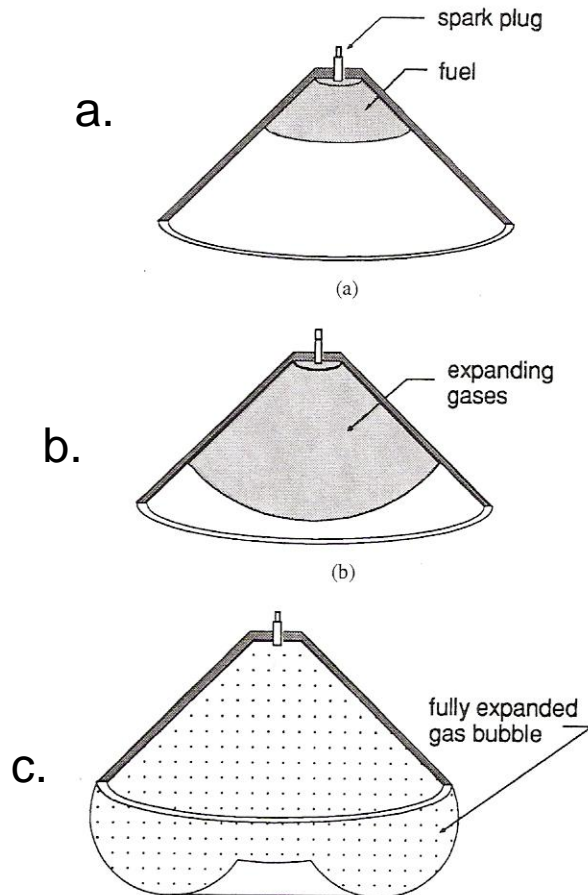
Arrival spread 1 s and 10- 200 Hz.

ASIAEX-ECS Shot 60



Combustive Sound Source (CSS) SW 06 (2006)

From: Wilson, P. S, Ellzey, J. L., and Muir, T. G., "Experimental Investigation of the Combustive Sound Source," IEEE J. Oceanic. Eng., 20(4), 1995.



**A typical CSS pressure signature
(produced by the combustion of 5.0 l
stoichiometric hydrogen and oxygen
and the power spectrum**

Cross section of CSS combustion Chamber

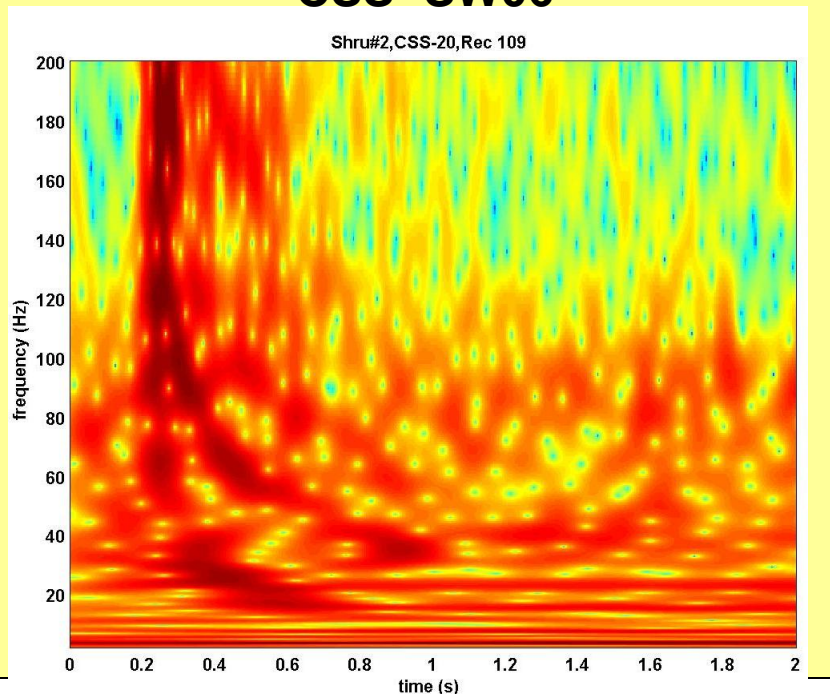
- a. Unburnt gaseous fuel/oxygen mixture
- b. Gases expand during combustion
- c. Bubble assumes a toroidal shape upon full expansion

Combustive Sound Source (CSS) during SW-06

Range: 21.24 km
Water depth \cong 90 m
Source depth: 26 m

Arrival spread 0.8 s and 10- 200 Hz.

CSS- SW06



The chamber we used in SW06 was a cylinder with a hemispherical cap. The bubble motion is not the same for the cylinder and the cone, although the radiated acoustic pulse is similar.

Time- Frequency Analysis Techniques

- Over the years the source levels have become lower resulting in shorter ranges
- Less separation between mode arrivals and lower SNR
- CSS used in SW06 gave two to three modes; will provide properties of deeper sediments; lower depth resolution
- Need for high resolution time-frequency techniques
- Hong et al. developed an adaptive time-frequency analysis method, whose time-frequency tiling depends on the dispersion characteristics of the wave signal to be analyzed

Jin-Chul Hong, Kyung Ho Sun, and Yoon Young Kim, "Dispersion-based short-time Fourier transform applied to dispersive wave analysis," J. Acoust. Soc. Am. **117** (5), May 2005

Short time Fourier Transform

$$\begin{aligned} Sf(u, \xi) &= \int_{-\infty}^{\infty} f(t) \bar{g}_{(s,u,\xi)} dt \\ &= \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{-i\xi t} dt \\ g_{(s,u,\xi)}(t) &= \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{-i\xi t} \end{aligned}$$

Window function $g(t)$ is a Gaussian

$$g(t) = \pi^{-1/4} e^{-\frac{t^2}{2}}$$

$\left\{ \begin{array}{l} \bar{g} \text{ denotes the complex conjugate of } g \\ s \text{ determines the size of the window} \end{array} \right\}$

Dispersion based Short time Fourier transforms

D-STFT is defined using a basis function that include a new parameter d

$$Df(u, \xi) = \int_{-\infty}^{\infty} f(t) \overline{g}_{(s,u,\xi,d)}(t) dt$$

$$= \int_{-\infty}^{\infty} f(t) \left[\frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) \otimes (id)^{-1/2} e^{-i\left(\frac{t^2}{2d}\right)} \right] e^{-i\xi t} dt$$

$$g_{(s,u,\xi)}(t) = \left[\frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) \otimes (id)^{-1/2} e^{-i\left(\frac{t^2}{2d}\right)} \right] e^{-i\xi t}$$

Window function $g(t)$ is a Gaussian

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Group delay

The Fourier transform of $g(t)$ is :

$$G(s, u, \xi, d) = \sqrt{s} G[s(\omega - \xi)] e^{-i \left[u(\omega - \xi) + \left(\frac{d}{2} \right) (\omega - \xi)^2 \right]}$$

The group delay of the basis function is :

$$\tau(\omega) = \frac{d}{d\omega} \left[u(\omega - \xi) + \frac{d}{2} (\omega - \xi)^2 \right]$$

$$\tau(\omega) = u + d(\omega - \xi)$$

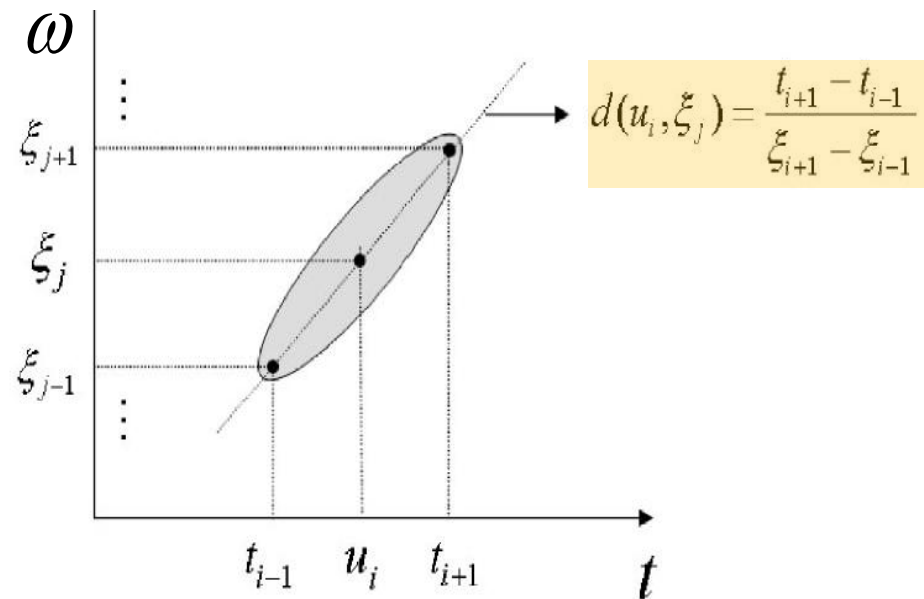
Dispersion based Short time Fourier transforms

d determines the amount of rotation
of the time - frequency box in (u, ξ)

$$d = d(u, \xi) = \frac{\Delta u}{\Delta \xi}$$

The group delay is :

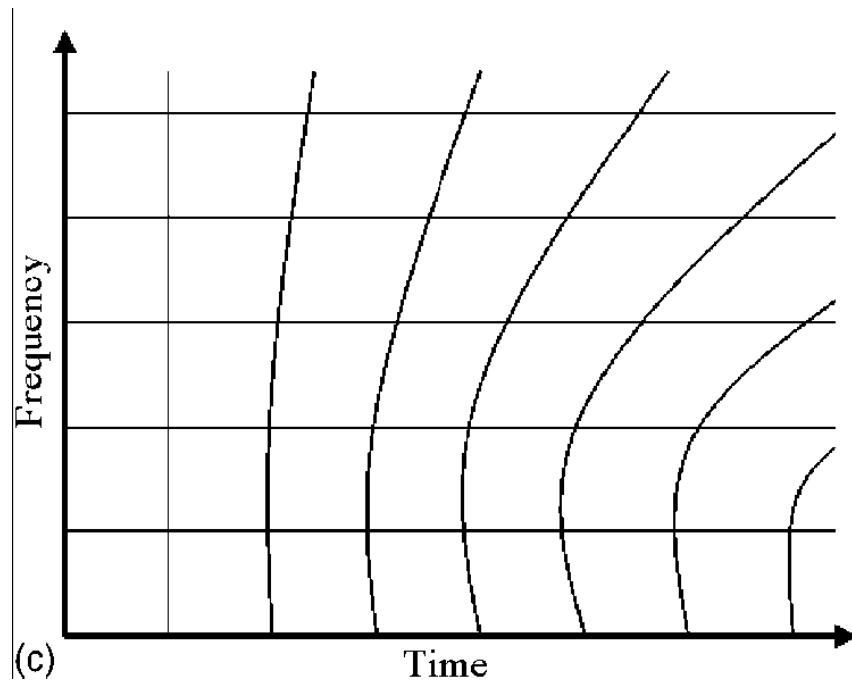
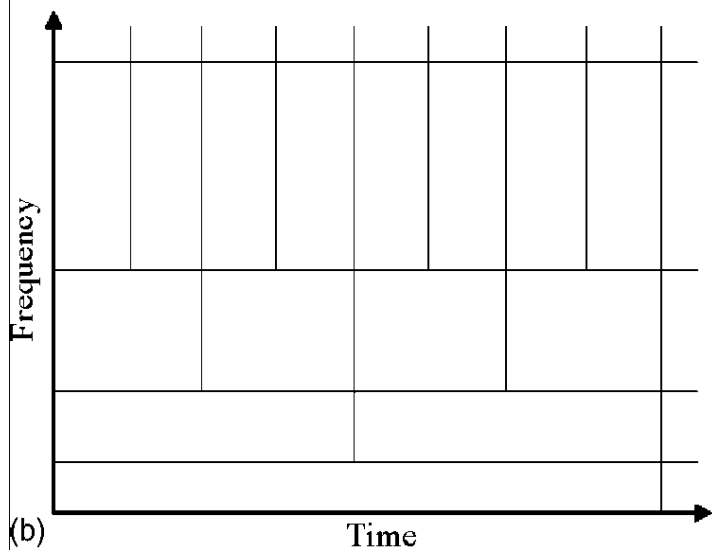
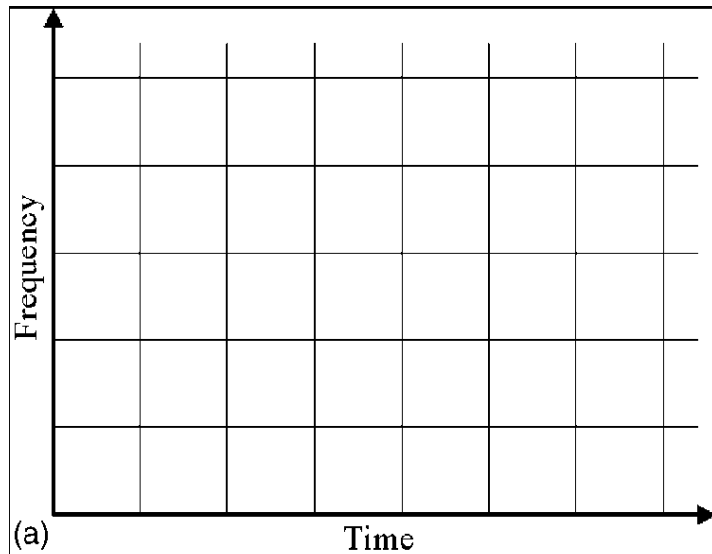
$$\tau(\omega) = u + d(\omega - \xi)$$



This implies that the time-frequency box in (u, ξ) can be obtained by rotating or shearing the time frequency box of standard STFT using the parameter $d(u, \xi)$

If $d(u, \xi)$ is chosen based on the local wave dispersion, then the resulting time frequency tiling will correspond to the entire wave dispersion behavior.

Time-frequency tiling in D-STFT is performed by adaptively rotating each of the analysis atoms with respect to the dispersion relationship



A comparison of time-frequency tilings.

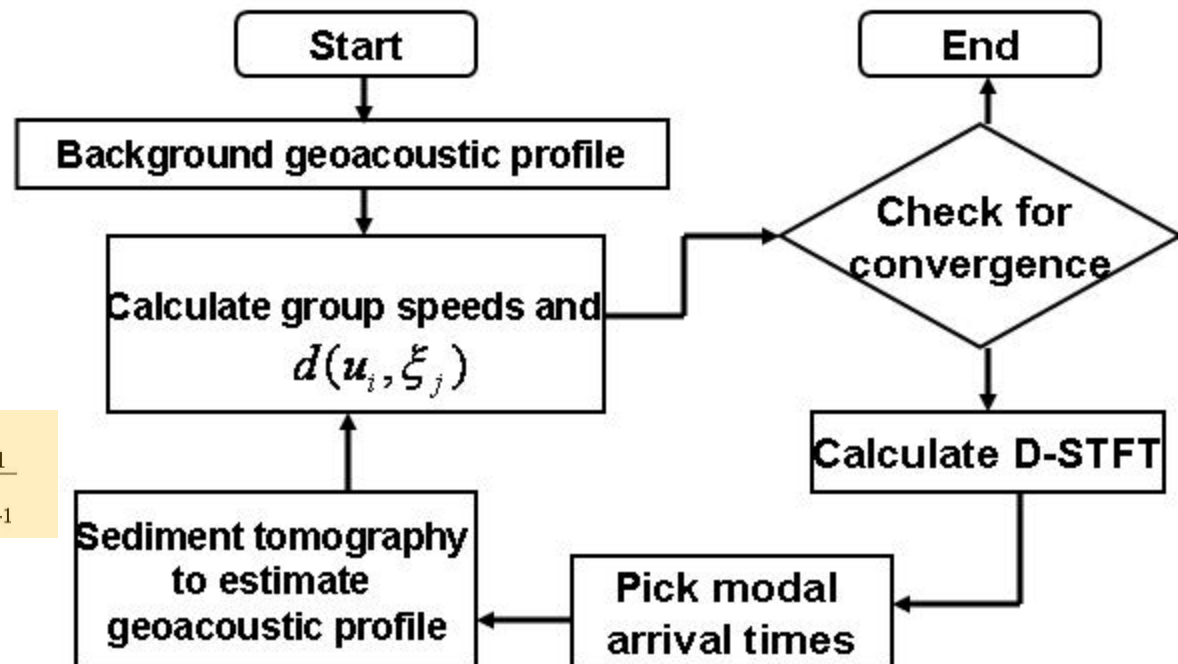
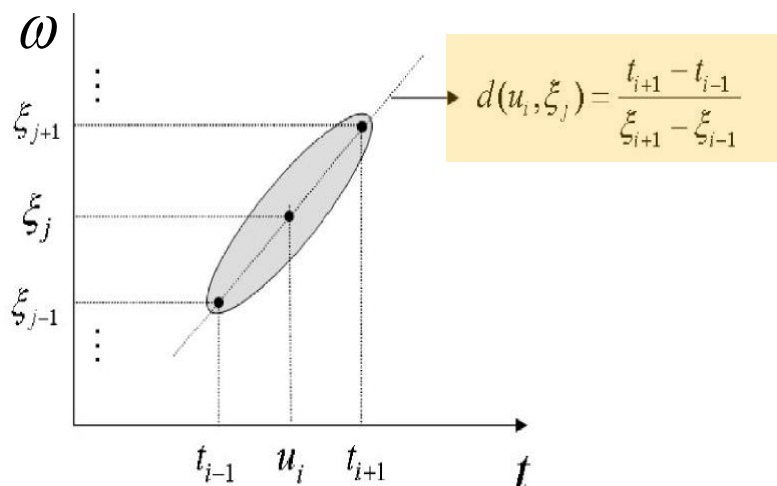
a. Short-time Fourier transform

b. continuous wavelet transform

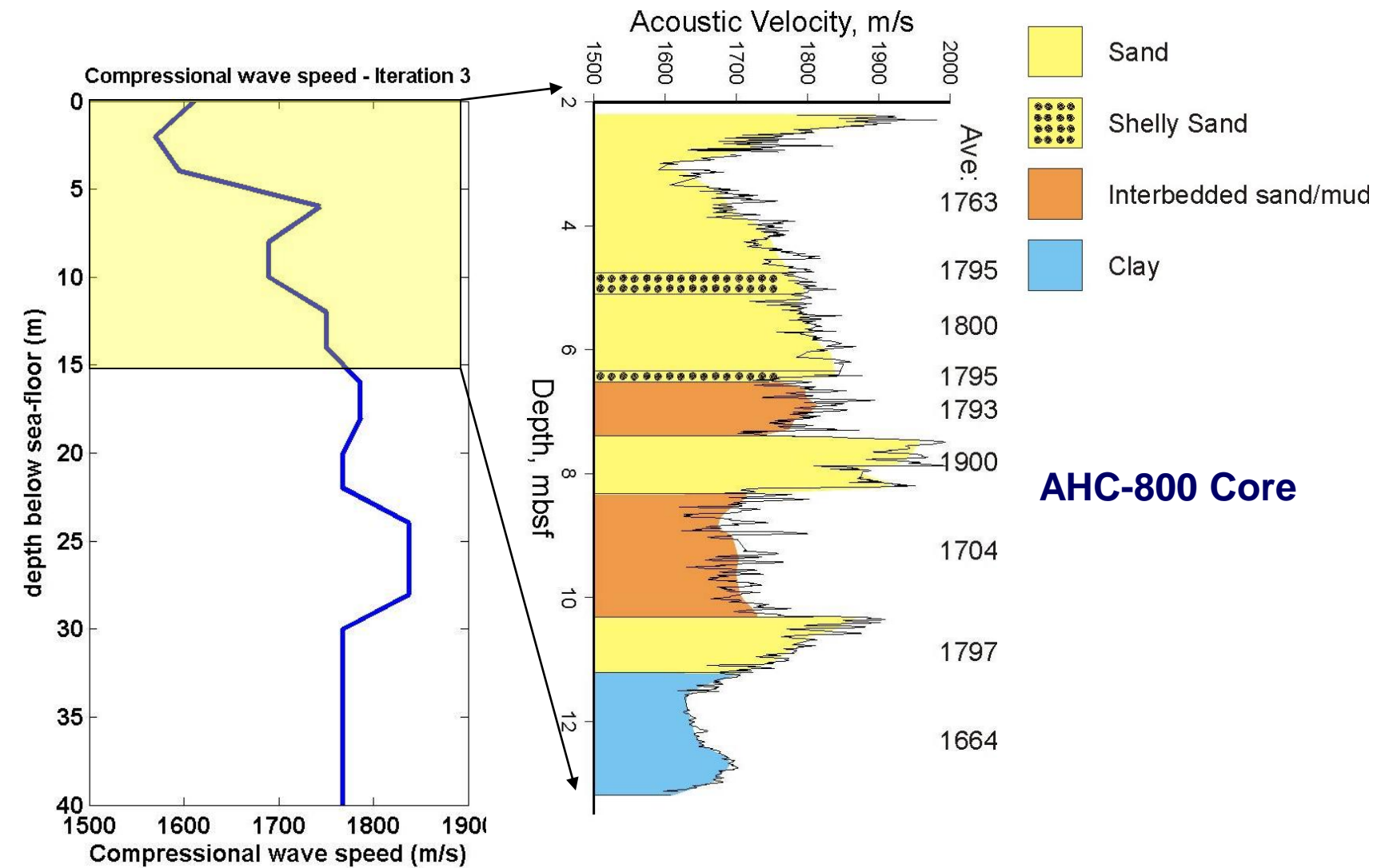
c. dispersion-based short-time Fourier transform.

Iterative Scheme for estimating modal group speeds

The key step in the algorithm is to connect each of the rotation parameters $d(u, \xi)$ to the actual dispersion relationship



D-STFT- Iteration: 3



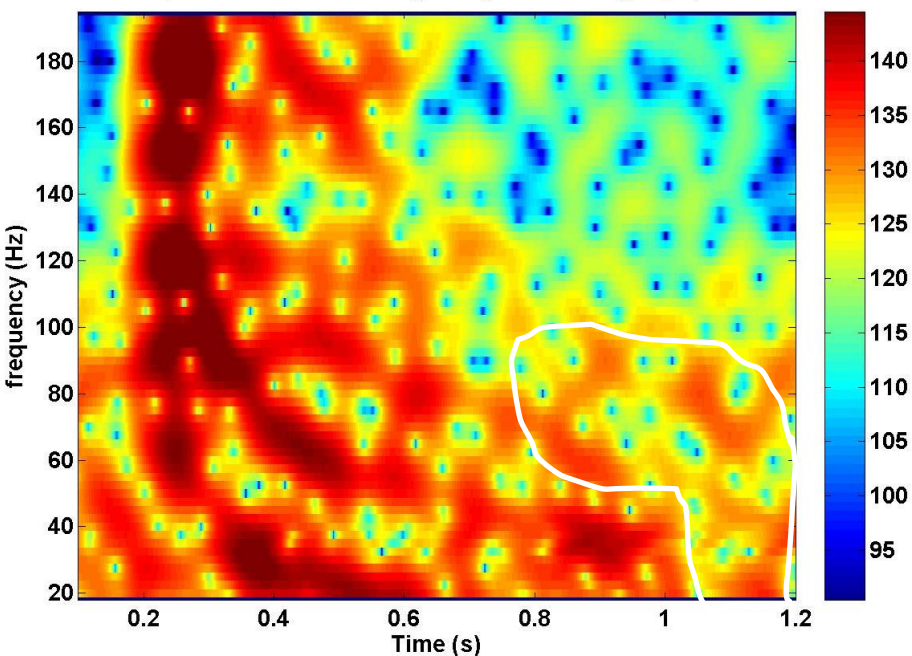
Comparison – D-STFT Vs Wavelet Scalogram

Modes 1, 2 and 3 D-STFT produces similar information

Mode 4 – possibly on a null

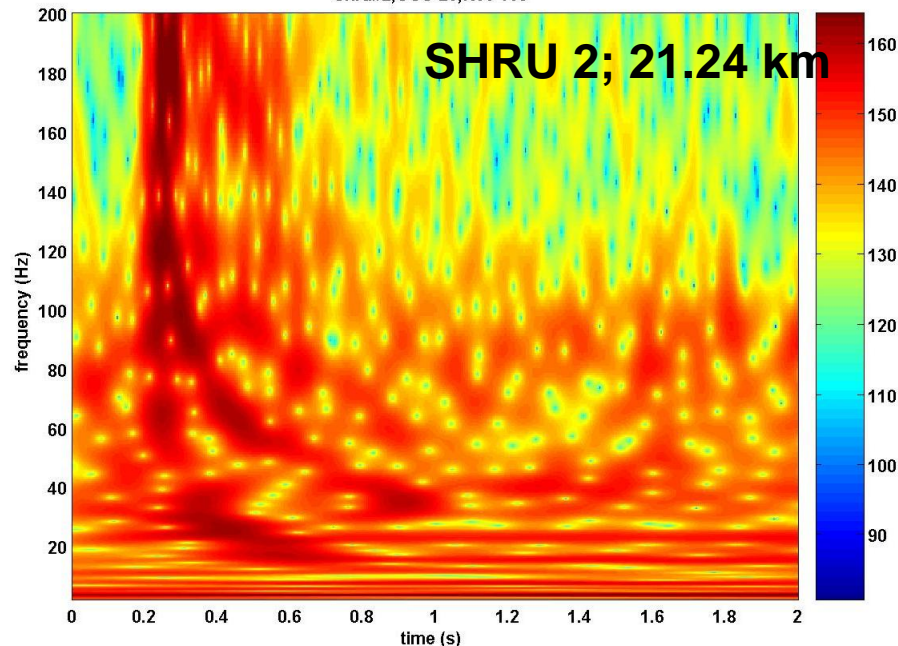
Mode 5 – D- STFT offers some promise as opposed to Scalogram

Dispersion based STFT - using a rough estimate of group speed



D-STFT

Shru#2,CSS-20,Rec 109



Wavelet Scalogram

Empirical Mode Decomposition

Empirical mode decomposition (EMD), is used to generate a set of intrinsic mode functions (IMF). EMD is a method of breaking down a signal without leaving the time domain.

The objective of the EMD is to empirically separate a signal into several subsignals of varying, and possibly overlapping, frequency content.

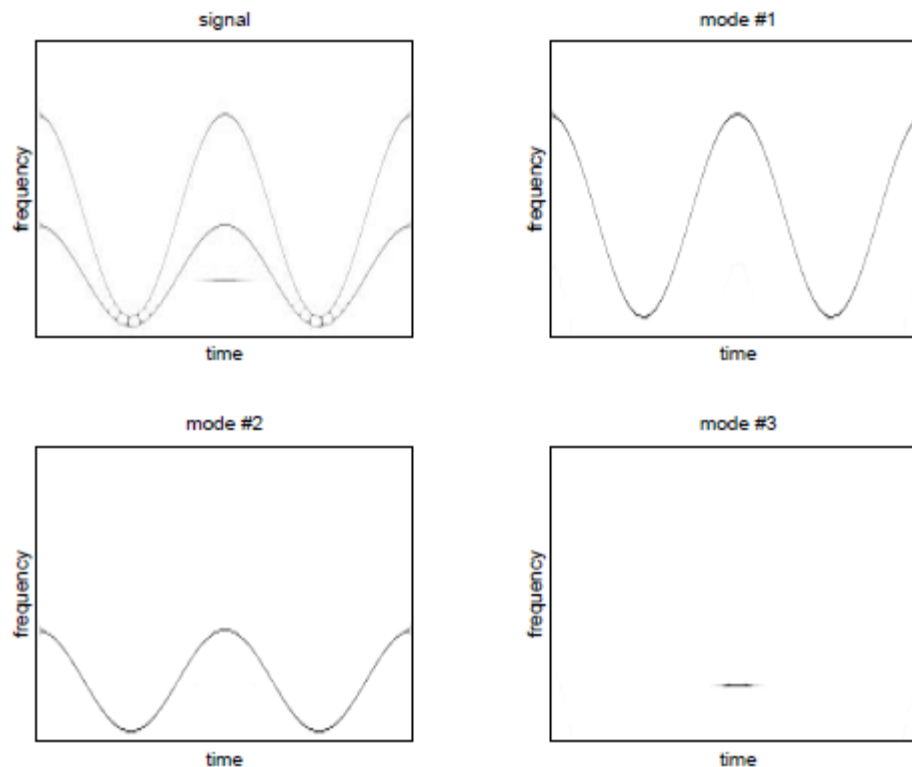
Each of the sub-signals is referred to as an intrinsic mode function because it is empirically derived from the data i.e., there are no user-specified filters.

The EMD produces a bank of IMFs whose sum yields the original signal.

The first IMFs produced contain the highest frequency components of a signal while the latter contain the lowest frequency components.

N.E. Huang, Z. Shen, S.R. Long, M.L. Wu, H.H. Shih, Q. Zheng, N.C. Yen, C.C. Tung and H.H. Liu, "The empirical mode decomposition and Hilbert spectrum for nonlinear and nonstationary time series analysis," *Proc. Roy. Soc. London A*, Vol. 454, pp. 903–995, 1998.

EMD – Example (Rilling et al.)



EMD of a 3-component signal. *The analyzed signal is the sum of 2 sinusoidal FM components and 1 Gaussian wavepacket. The time frequency analysis of the total signal (top left) reveals 3 time-frequency signatures which overlap in both time and frequency. The time-frequency signatures of the first 3 IMF's extracted by EMD evidence that these modes efficiently capture the 3-component structure of the analyzed signal.*

Hilbert – Huang spectrum

X_n : bank of IMFs

Y_n : bank of their Hilbert transforms

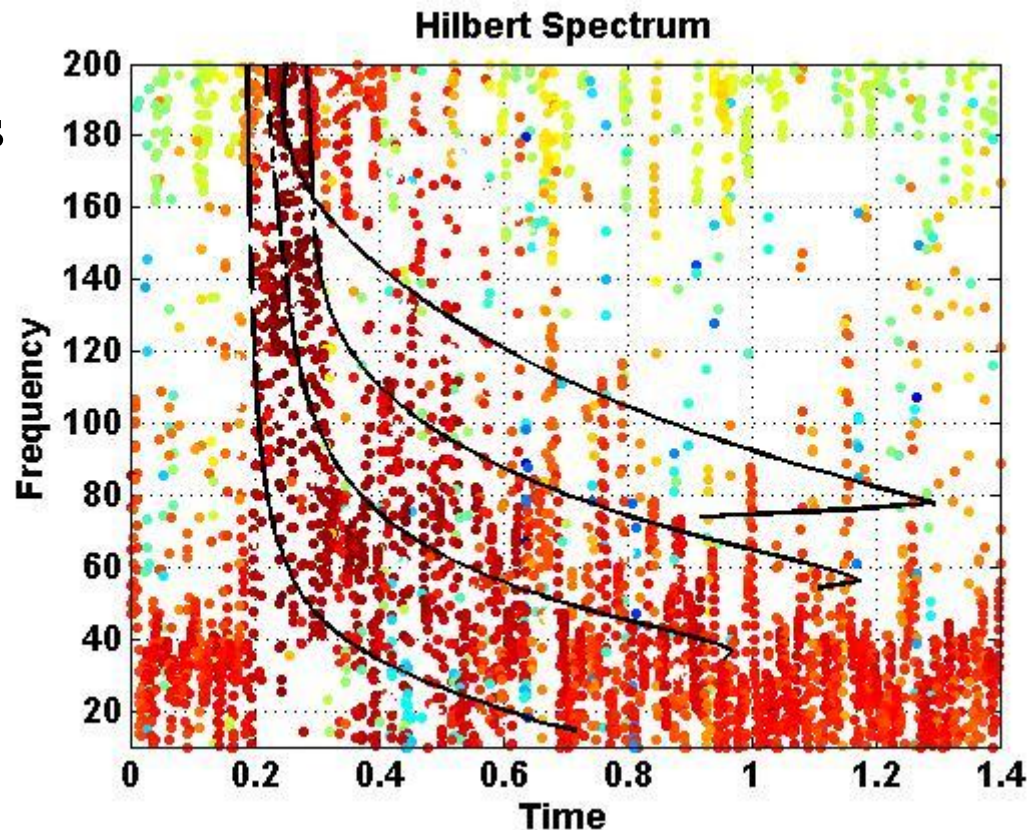
Amplitude $a_n(t) = \sqrt{X_n(t)^2 + Y_n(t)^2}$

Phase $\phi_n(t) = \tan^{-1} \left(\frac{Y_n(t)}{X_n(t)} \right)$

Instantaneous frequency :

$$f_n(t) = \frac{1}{2\pi} \frac{d\phi_n}{dt}$$

Amplitude, phase and frequency can be time-sorted and displayed in a time-frequency fashion.



Time – frequency structure not clear !!!!!

Intrinsic Mode Function (IMF) # 8

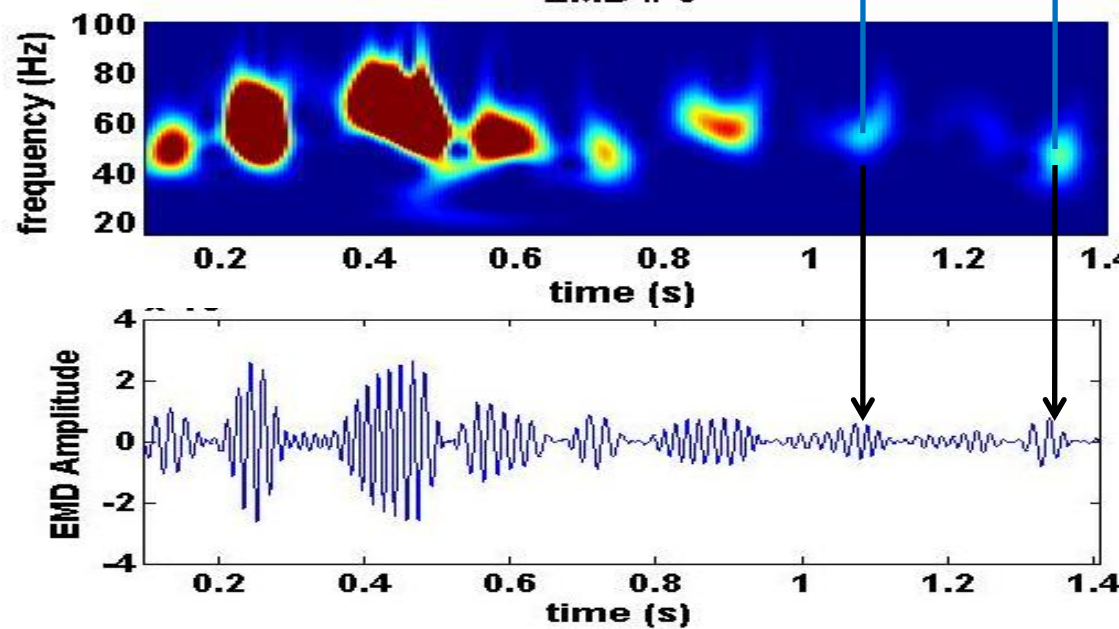
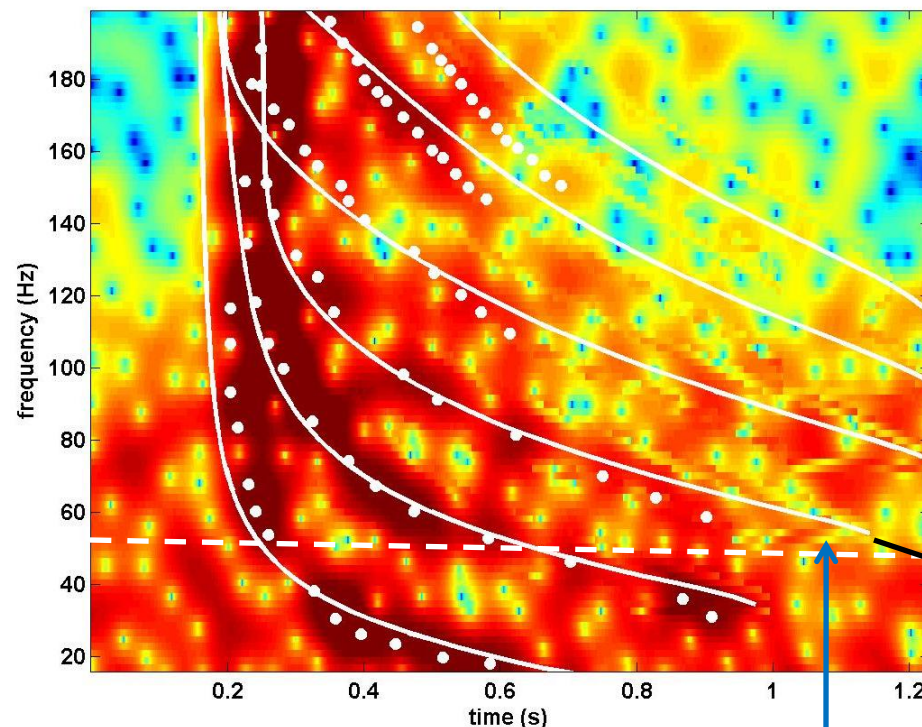
IMF # 8 – 40 to 60 Hz

Mode 1 and 2 dominant (0.4
and 0.6 sec respectively)

Mode 3 energy at 1.1 sec
(~50 Hz) and 1.35 sec (~40
Hz)

Arrivals before mode 1 (50
Hz)

Dispersion based STFT - using Mode 1-6- Iteration 2



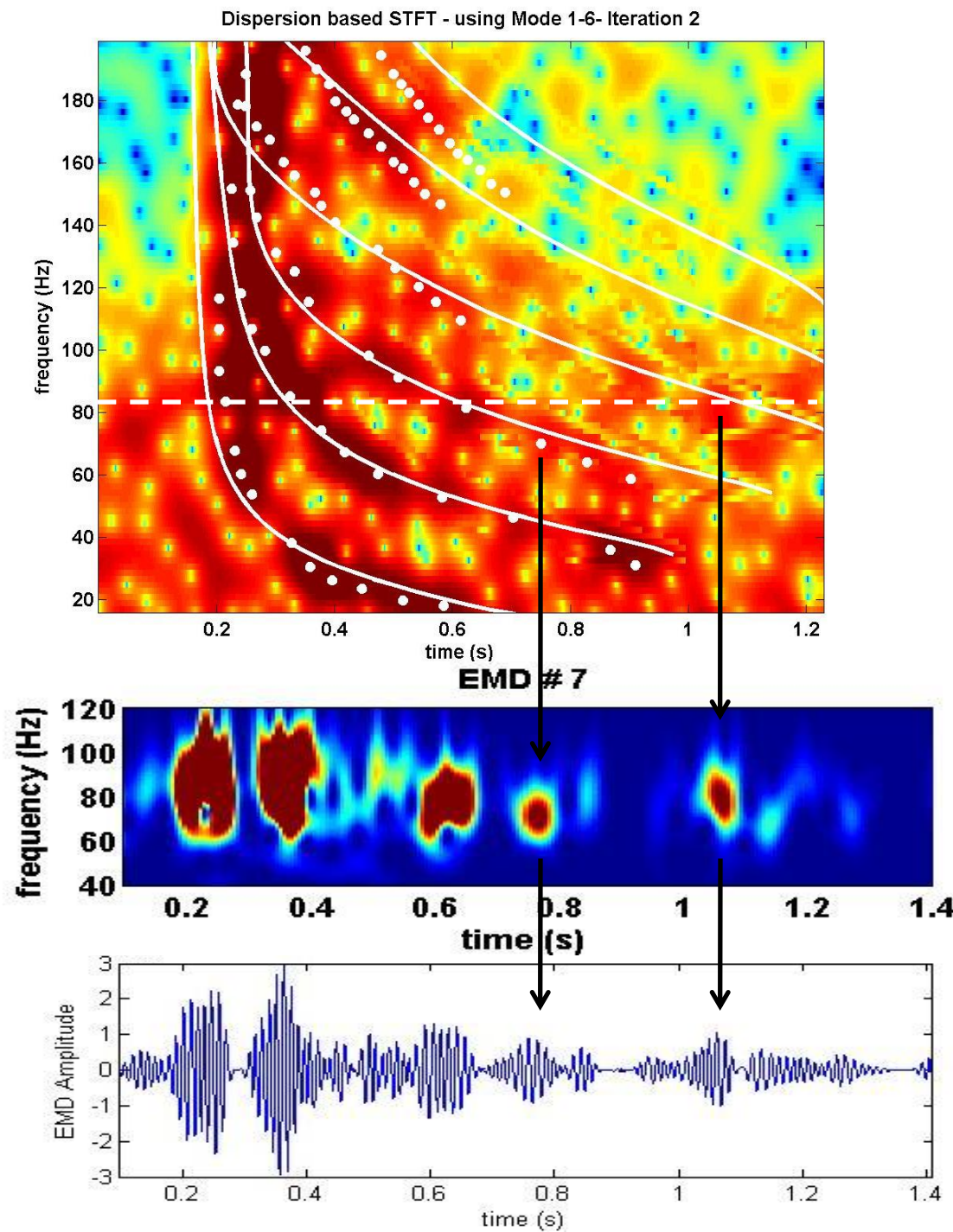
Intrinsic Mode Function (IMF) # 7

IMF # 7 – 60 to 100 Hz

Mode 1 and 2 dominant (0.2
and 0.4 sec respectively)

Mode 3 energy at 0.6 sec
(~80 Hz) and 0.8 sec (~ 65
Hz)

Mode 4 energy at 1.05 sec
(75 Hz)



Intrinsic Mode Function - (IMF) # 6

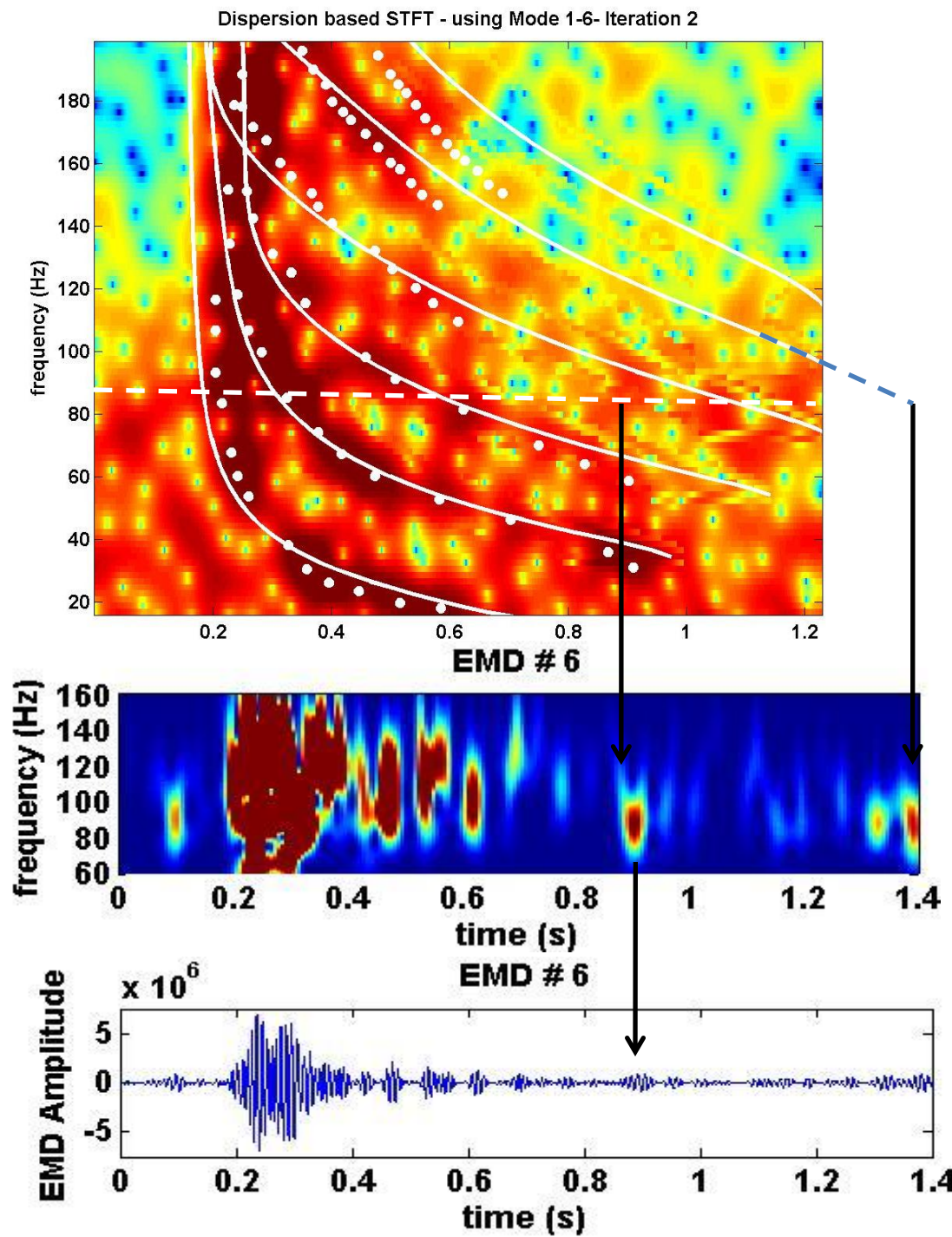
IMF # 7 – 60 to 160 Hz

Mode 1 and 2 dominant (0.2
and 0.3 sec respectively)

Mode 3 energy smeared
between 0.4 and 0.6 sec

Mode 4 energy at 0.9 sec
(80 Hz)

Mode 5 energy at 1.4 sec
(80 Hz)



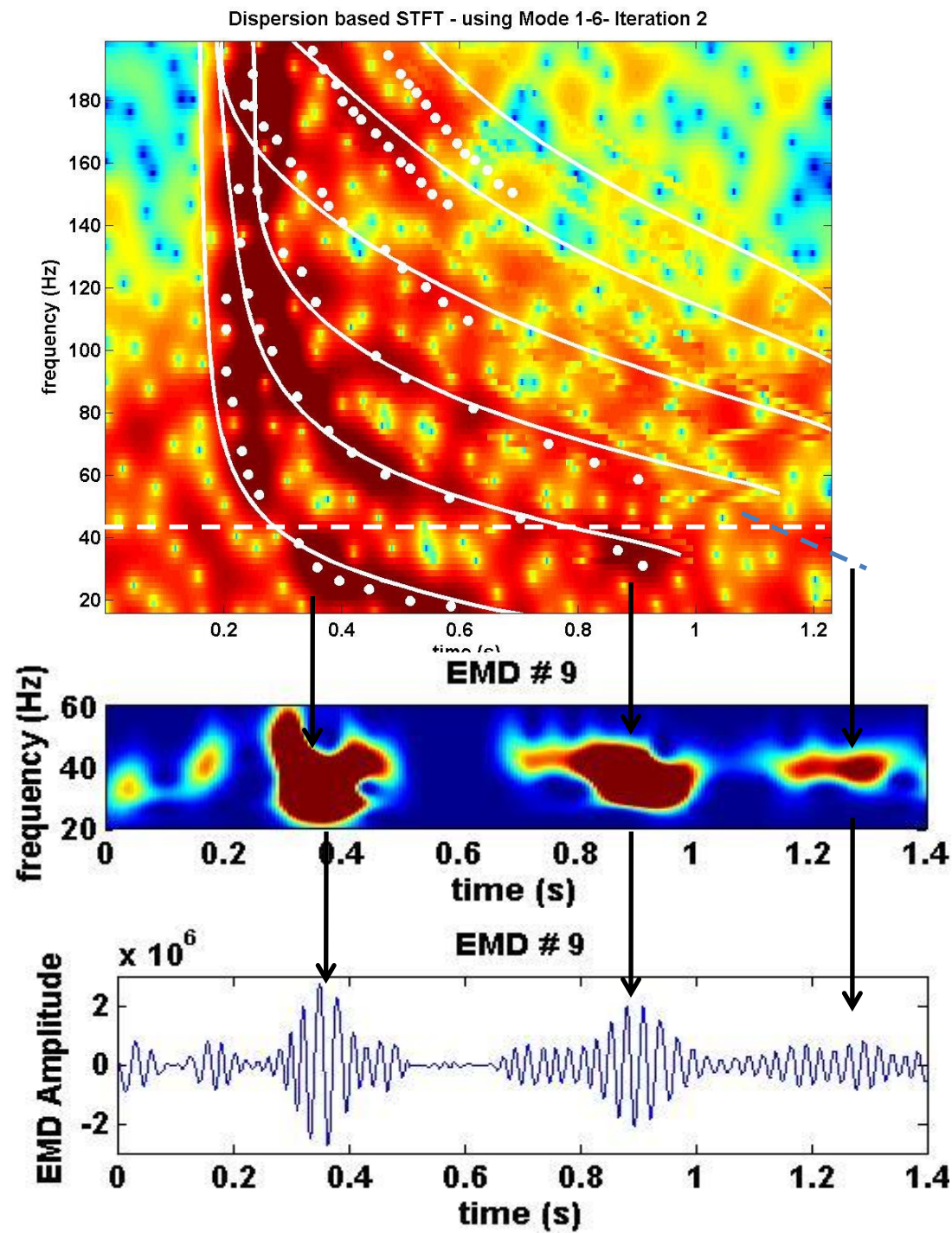
Intrinsic Mode Function (IMF) # 9

IMF # 9 – 60 to 160 Hz

Mode 1 energy(0.35 sec)

Mode 2 energy at 0.9 sec
(40 Hz)

Mode 3 energy at 1.3 sec
(40 Hz)



Summary and Future Work

- **D-STFT was applied to CSS data to improve the performance of time-frequency data.**
- **Individual EMFs provide insights into the modal arrivals at specific frequency bands.**
- **EMFs can improve the D-STFT (or wavelet) dispersion information by identifying or confirming mode arrival information especially at low frequency region.**

Questions ??

