

**Acoustical Society of America  
San Antonio– October 26-30, 2009**

**Time-frequency analysis techniques for long range  
sediment tomography**

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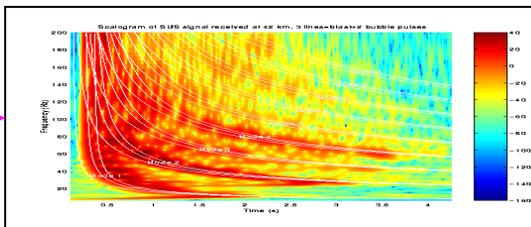
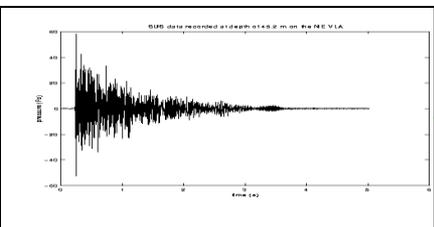
**Work supported by Office of Naval Research code 3210A**

# Outline

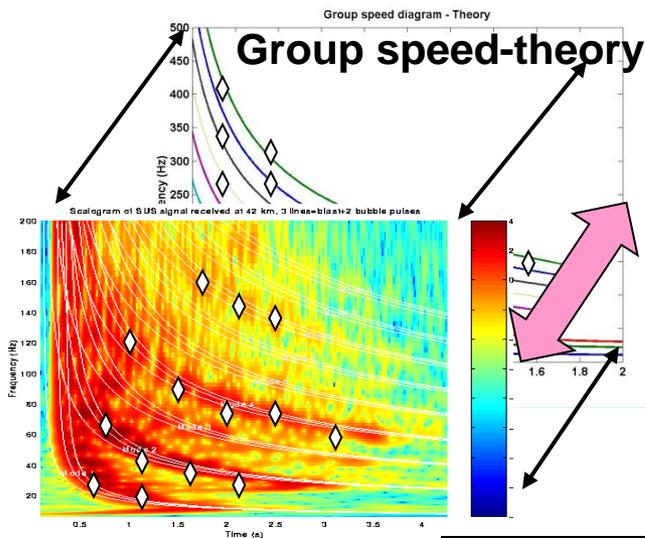
- Sediment tomography
- CSS data analysis using Dispersion Based STFT (D-STFT)
- Empirical Mode Decomposition – results
- Summary and Future work

# Inversion Scheme for Compressional Speeds

## Group speed dispersion by Time-frequency analysis



### Broadband data



**A Posteriori analysis  
mean, standard deviation**

## Genetic Algorithm Optimization

### Parameters for GA search

- EOF coefficients
- Sediment compressional speeds
- Bathymetry
- Source-receiver range

### Objective Function for m<sup>th</sup> parameter set

$$E(m) = ||w (d^{obs} - d^{pred})||_2$$

w – diagonal weight matrix

d<sup>obs</sup> – observed data

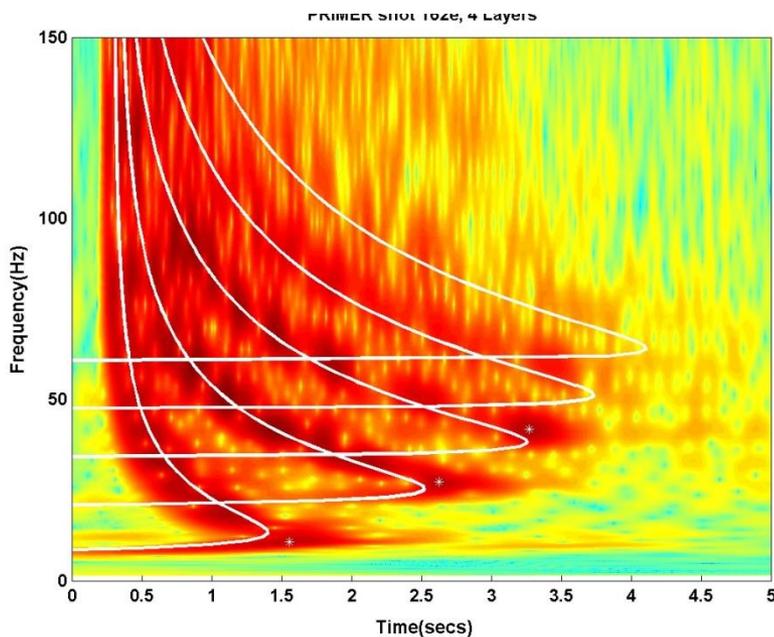
d<sup>pred</sup> – predictions of forward model

Levenberg-Marquardt  
non-linear least squares method

**Range:** 40 km  
**Water depth**  $\cong$  100 m  
**Charge Weight:** 0.8 kg  
**Source depth:** 18 m

Arrival spread 4 s and 10- 150 Hz.

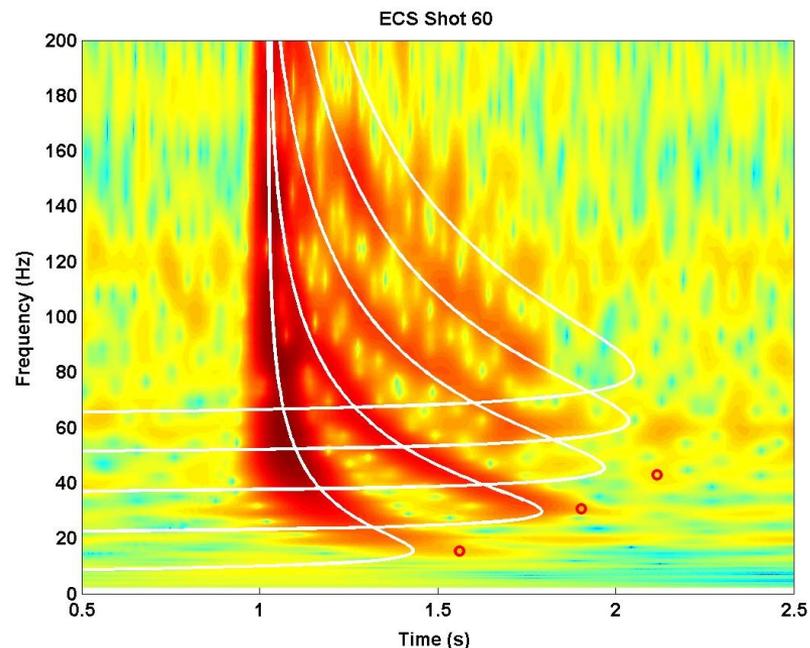
## PRIMER (1996)



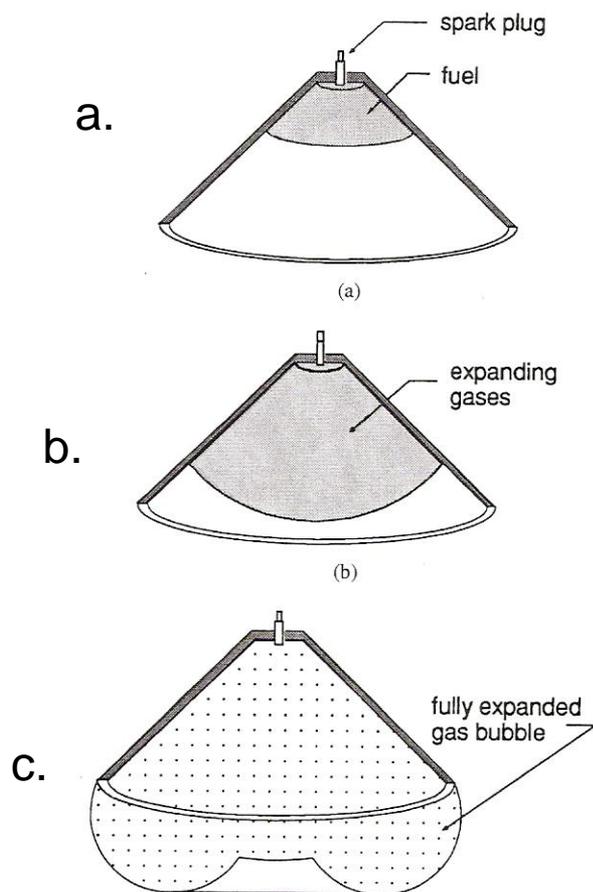
**Range:** 30km  
**Water depth**  $\cong$  100 m  
**Charge Weight:** 38 g;  
**Source depth:** 50 m

Arrival spread 1 s and 10- 200 Hz.

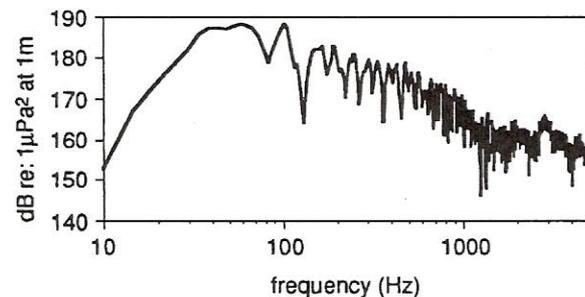
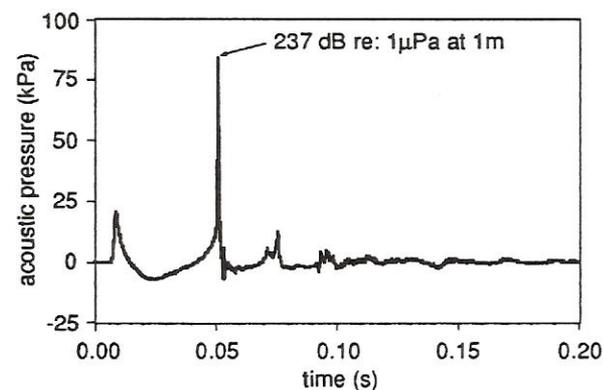
## ASIAEX-ECS Shot 60



# Combustive Sound Source (CSS) SW 06 (2006)



From: Wilson, P. S, Ellzey, J. L., and Muir, T. G., "Experimental Investigation of the Combustive Sound Source," *IEEE J. Oceanic. Eng.*, 20(4), 1995.



**A typical CSS pressure signature (produced by the combustion of 5.0 l stoichiometric hydrogen and oxygen and the power spectrum**

**Cross section of CSS combustion Chamber**

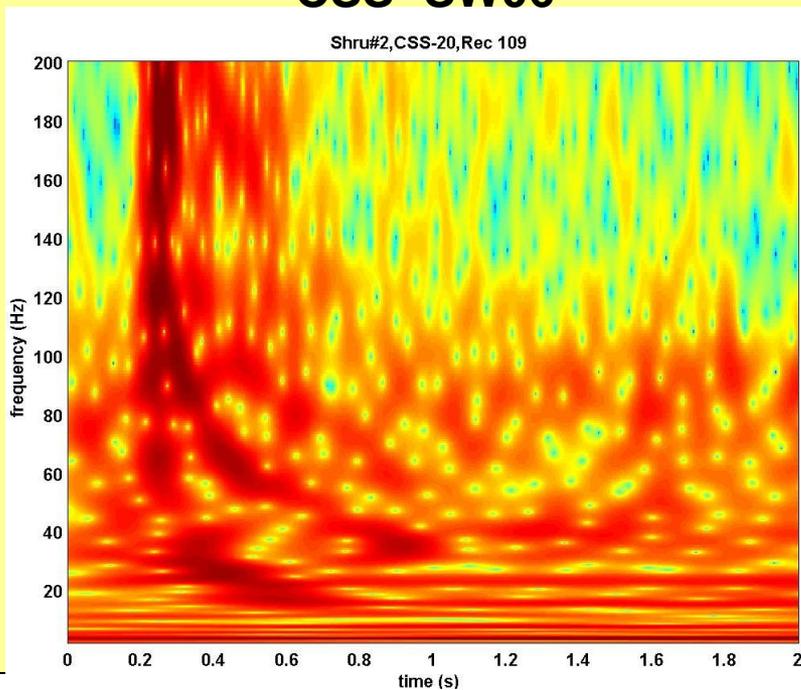
- Unburnt gaseous fuel/oxygen mixture
- Gases expand during combustion
- Bubble assumes a toroidal shape upon full expansion

# Combustive Sound Source (CSS) during SW-06

**Range:** 21.24 km  
**Water depth**  $\cong$  90 m  
**Source depth:** 26 m

**Arrival spread 0.8 s and 10- 200 Hz.**

## CSS- SW06



The chamber we used in SW06 was a cylinder with a hemispherical cap. The bubble motion is not the same for the cylinder and the cone, although the radiated acoustic pulse is similar.

# Time- Frequency Analysis Techniques



- **Over the years the source levels have become lower resulting in shorter ranges**
- **Less separation between mode arrivals and lower SNR**
- **CSS used in SW06 gave two to three modes; will provide properties of deeper sediments; lower depth resolution**
- **Need for high resolution time-frequency techniques**
- **Hong et al. developed an adaptive time-frequency analysis method, whose time-frequency tiling depends on the dispersion characteristics of the wave signal to be analyzed**

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Jin-Chul Hong, Kyung Ho Sun, and Yoon Young Kim, "Dispersion-based short-time Fourier transform applied to dispersive wave analysis," J. Acoust. Soc. Am. **117** (5), May 2005

# Short time Fourier Transform

$$\begin{aligned} Sf(u, \xi) &= \int_{-\infty}^{\infty} f(t) \bar{g}_{(s,u,\xi)} dt \\ &= \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{-i\xi t} dt \\ g_{(s,u,\xi)}(t) &= \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{-i\xi t} \end{aligned}$$

Window function  $g(t)$  is a Gaussian

$$g(t) = \pi^{-1/4} e^{-\frac{t^2}{2}}$$

$\left\{ \begin{array}{l} \bar{g} \text{ denotes the complex conjugate of } g \\ s \text{ determines the size of the window} \end{array} \right\}$

# Dispersion based Short time Fourier transforms

**D-STFT is defined using a basis function that include a new parameter  $d$**

$$Df(u, \xi) = \int_{-\infty}^{\infty} f(t) \bar{g}_{(s,u,\xi,d)}(t) dt$$

$$= \int_{-\infty}^{\infty} f(t) \left[ \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) \otimes (id)^{-1/2} e^{-i\left(\frac{t^2}{2d}\right)} \right] e^{-i\xi t} dt$$

$$g_{(s,u,\xi)}(t) = \left[ \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) \otimes (id)^{-1/2} e^{-i\left(\frac{t^2}{2d}\right)} \right] e^{-i\xi t}$$

Window function  $g(t)$  is a Gaussian

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# Group delay

The Fourier transform of  $g(t)$  is :

$$G(s, u, \xi, d) = \sqrt{s} G[s(\omega - \xi)] e^{-i \left[ u(\omega - \xi) + \left( \frac{d}{2} \right) (\omega - \xi)^2 \right]}$$

The group delay of the basis function is :

$$\tau(\omega) = \frac{d}{d\omega} \left[ u(\omega - \xi) + \frac{d}{2} (\omega - \xi)^2 \right]$$

$$\tau(\omega) = u + d(\omega - \xi)$$

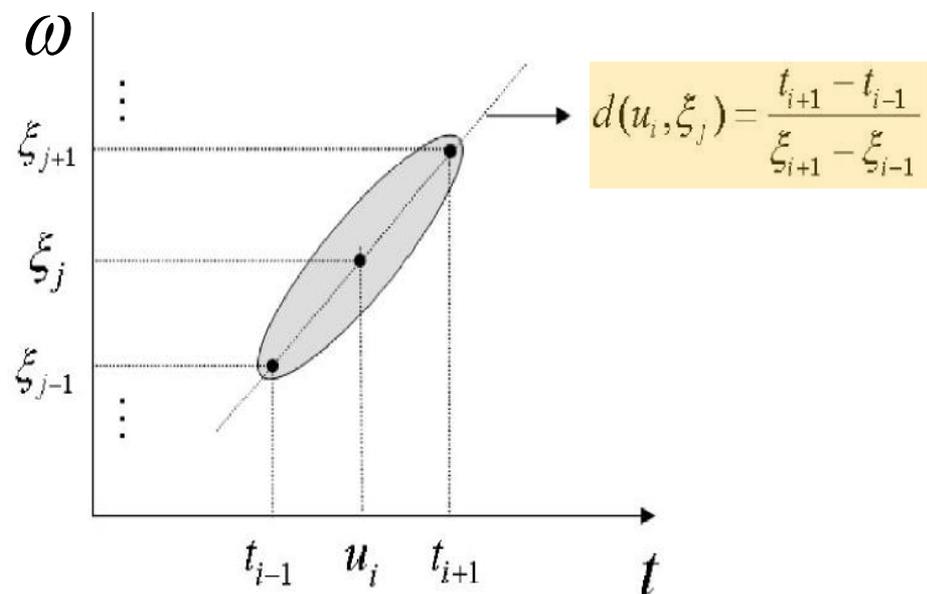
# Dispersion based Short time Fourier transforms

$d$  determines the amount of rotation  
of the time - frequency box in  $(u, \xi)$

$$d = d(u, \xi) = \frac{\Delta u}{\Delta \xi}$$

The group delay is :

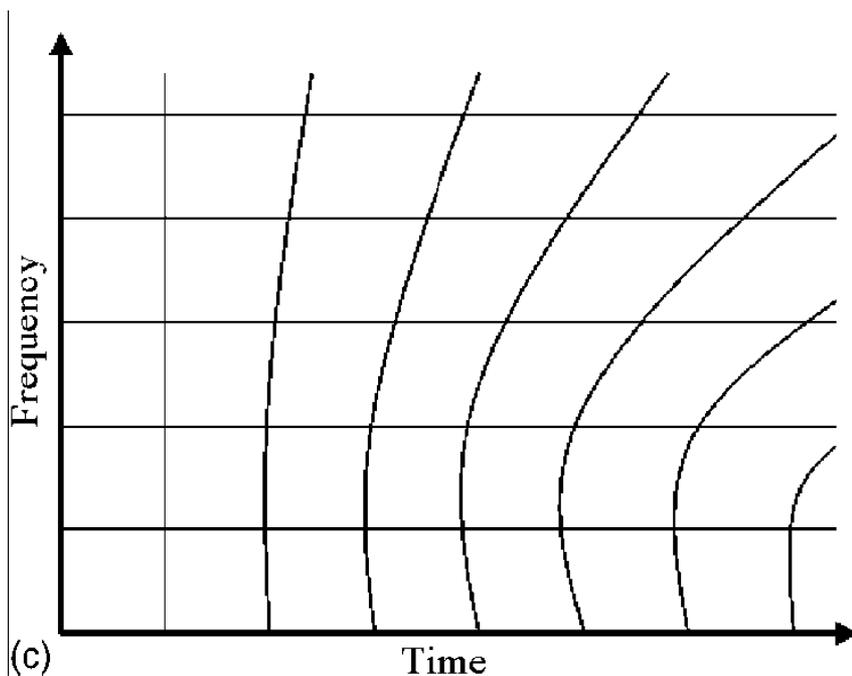
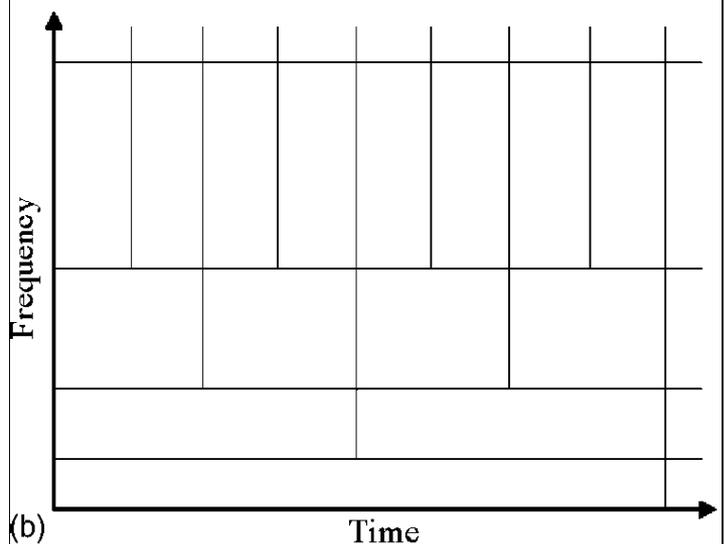
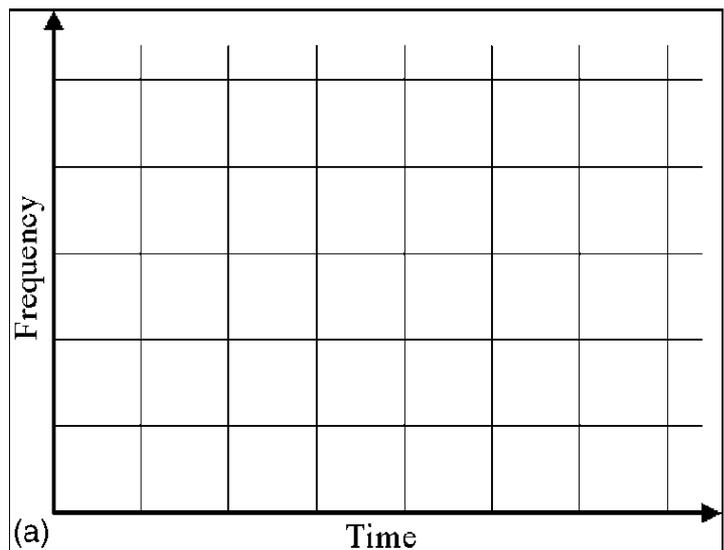
$$\tau(\omega) = u + d(\omega - \xi)$$



This implies that the time-frequency box in  $(u, \xi)$  can be obtained by rotating or shearing the time frequency box of standard STFT using the parameter  $d(u, \xi)$

If  $d(u, \xi)$  is chosen based on the local wave dispersion, then the resulting time frequency tiling will correspond to the entire wave dispersion behavior.

**Time-frequency tiling in D-STFT is performed by adaptively rotating each of the analysis atoms with respect to the dispersion relationship**



*A comparison of time-frequency tilings.*

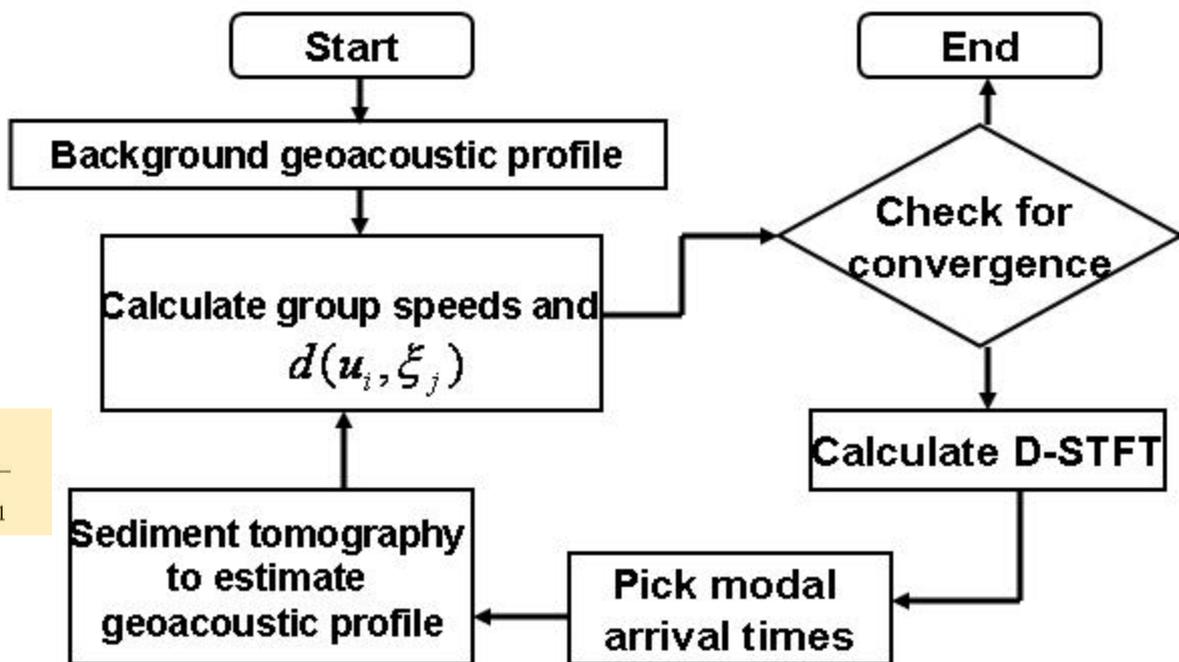
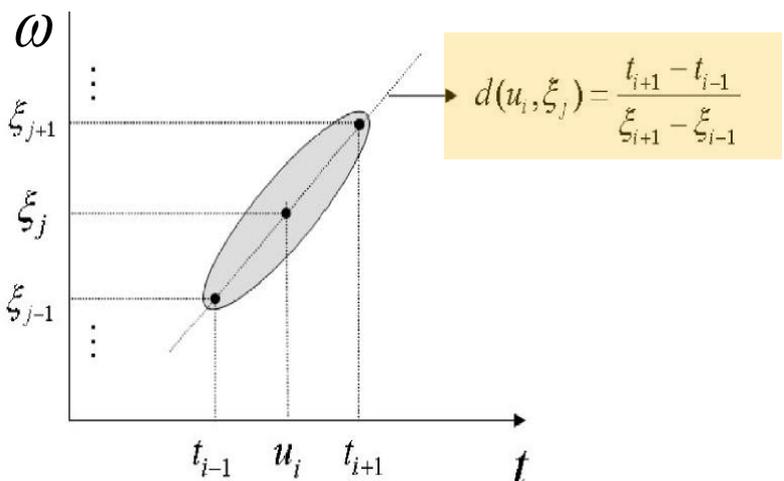
*a. Short-time Fourier transform*

*b. continuous wavelet transform*

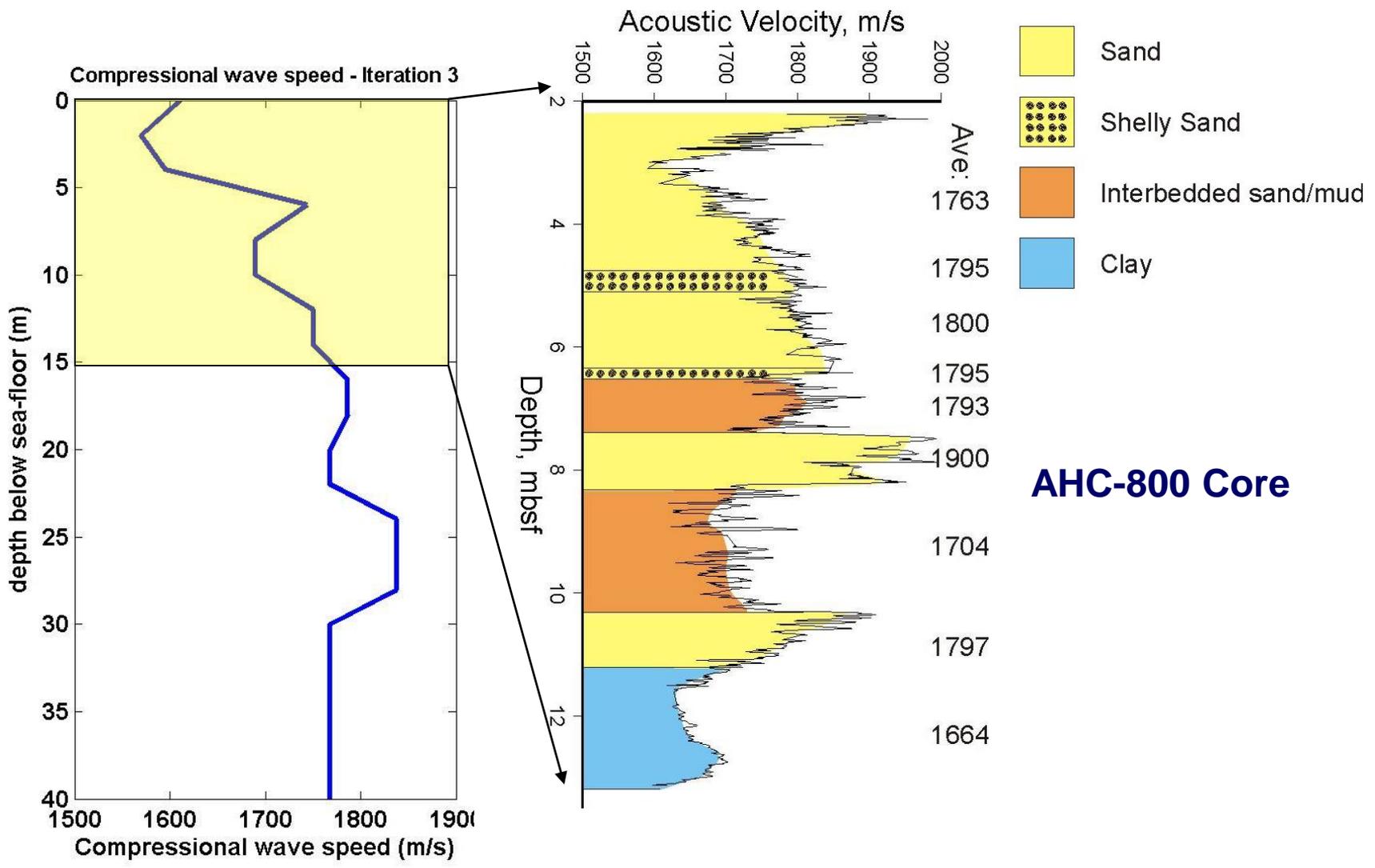
*c. dispersion-based short-time Fourier transform.*

# Iterative Scheme for estimating modal group speeds

The key step in the algorithm is to connect each of the rotation parameters  $d(u, \xi)$  to the actual dispersion relationship



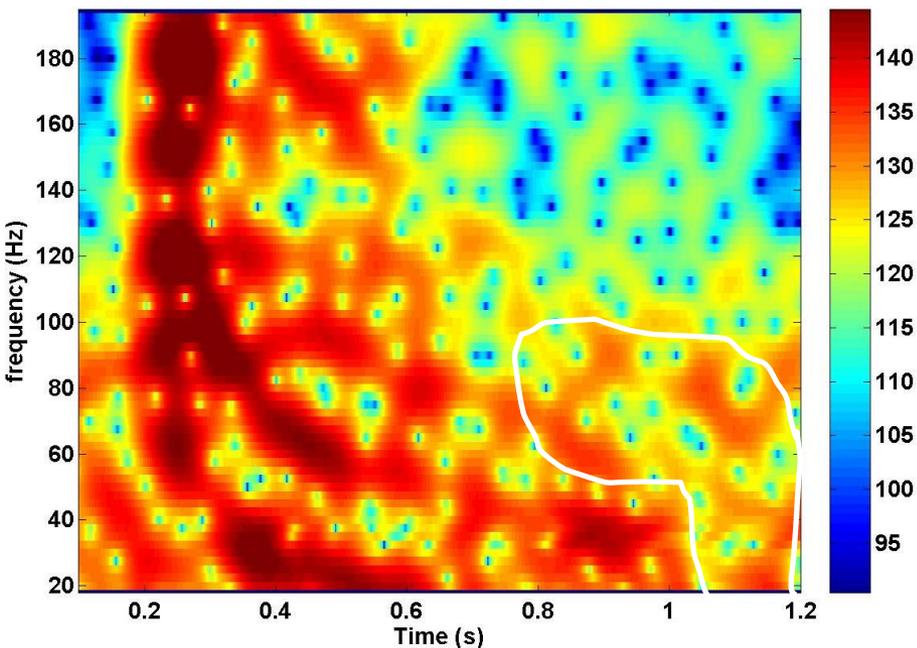
# D-STFT- Iteration: 3



# Comparison – D-STFT Vs Wavelet Scalogram

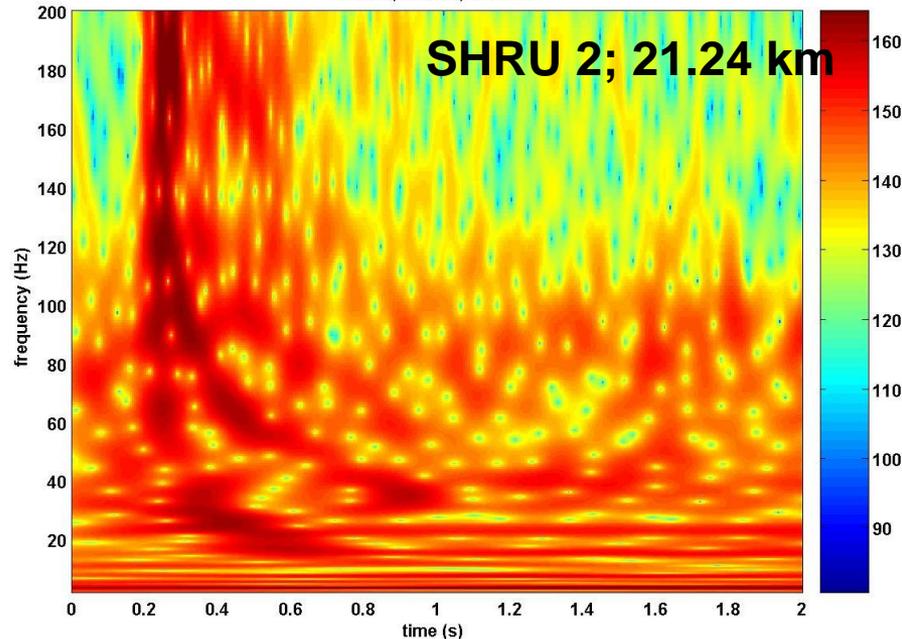
- Modes 1, 2 and 3 D-STFT produces similar information
- Mode 4 – possibly on a null
- Mode 5 – D- STFT offers some promise as opposed to Scalogram

Dispersion based STFT - using a rough estimate of group speed



D-STFT

Shru#2,CSS-20,Rec 109



Wavelet Scalogram

# Empirical Mode Decomposition

**Empirical mode decomposition (EMD), is used to generate a set of intrinsic mode functions (IMF). EMD is a method of breaking down a signal without leaving the time domain.**

**The objective of the EMD is to empirically separate a signal into several subsignals of varying, and possibly overlapping, frequency content.**

**Each of the sub-signals is referred to as an intrinsic mode function because it is empirically derived from the data i.e., there are no user-specified filters.**

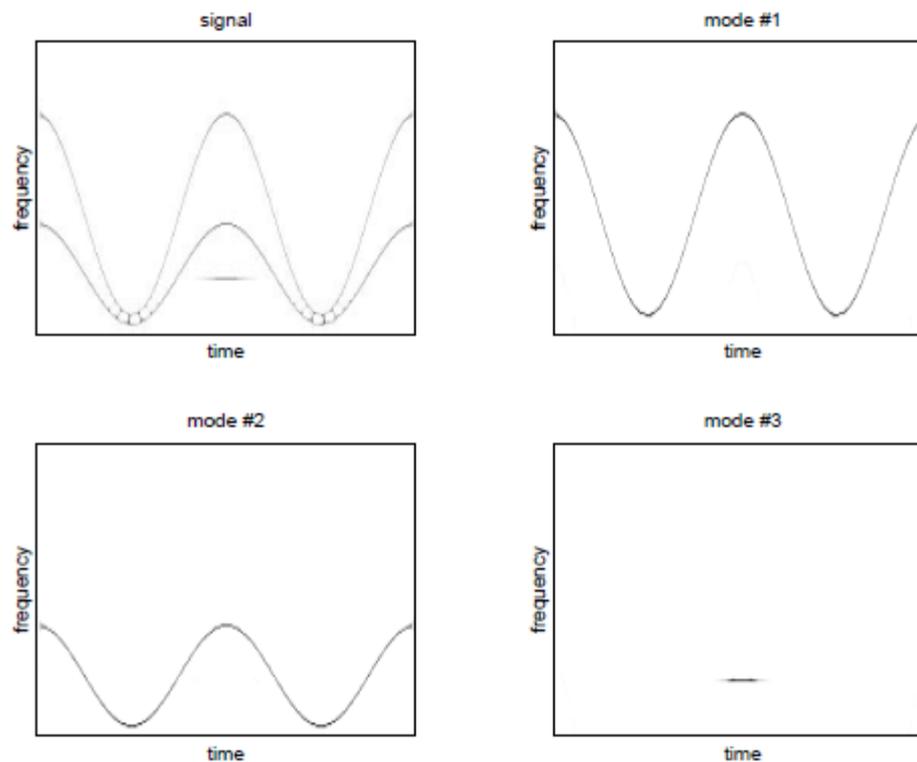
**The EMD produces a bank of IMFs whose sum yields the original signal.**

**The first IMFs produced contain the highest frequency components of a signal while the latter contain the lowest frequency components.**

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**N.E. Huang, Z. Shen, S.R. Long, M.L. Wu, H.H. Shih, Q. Zheng, N.C. Yen, C.C. Tung and H.H. Liu, “The empirical mode decomposition and Hilbert spectrum for nonlinear and nonstationary time series analysis,” *Proc. Roy. Soc. London A*, Vol. 454, pp. 903–995, 1998.**

# EMD – Example (Rilling et al.)



**EMD of a 3-component signal. *The analyzed signal is the sum of 2 sinusoidal FM components and 1 Gaussian wavepacket. The time frequency analysis of the total signal (top left) reveals 3 time-frequency signatures which overlap in both time and frequency. The time-frequency signatures of the first 3 IMF's extracted by EMD evidence that these modes efficiently capture the 3-component structure of the analyzed signal.***

# Hilbert – Huang spectrum

$X_n$  : bank of IMFs

$Y_n$  : bank of their Hilbert transforms

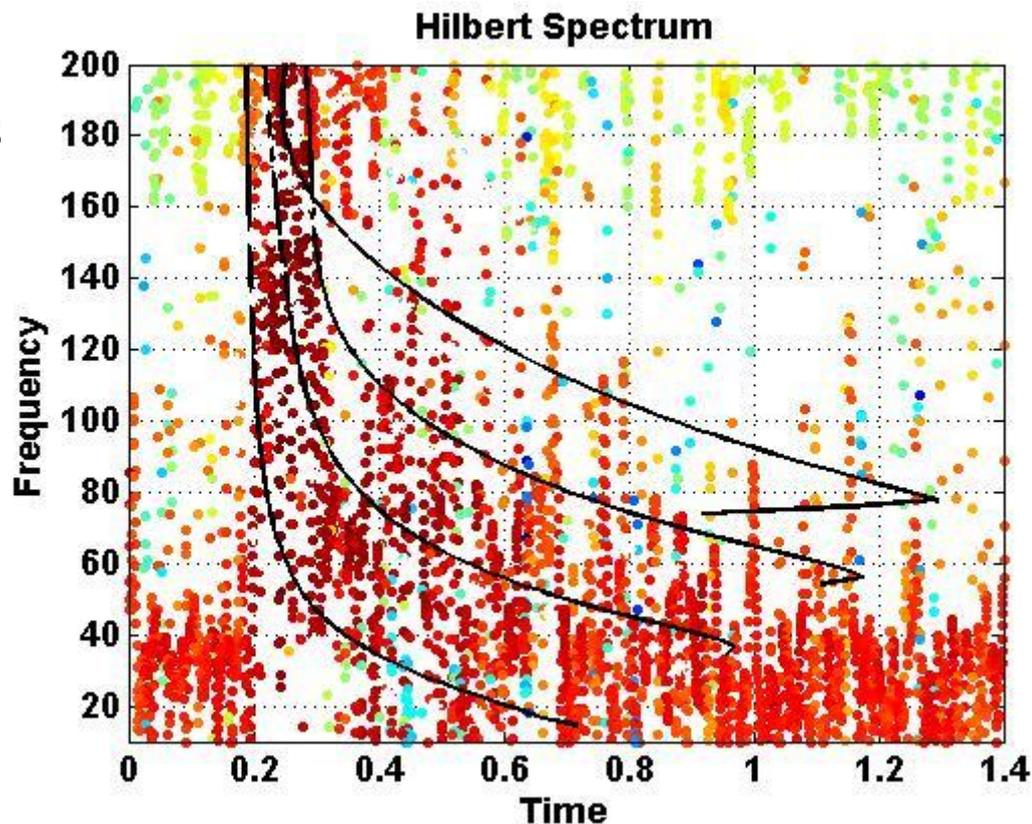
$$\text{Amplitude } a_n(t) = \sqrt{X_n(t)^2 + Y_n(t)^2}$$

$$\text{Phase } \phi_n(t) = \tan^{-1} \left( \frac{Y_n(t)}{X_n(t)} \right)$$

Instantaneous frequency :

$$f_n(t) = \frac{1}{2\pi} \frac{d\phi_n}{dt}$$

**Amplitude, phase and frequency can be time-sorted and displayed in a time-frequency fashion.**



**Time – frequency structure not clear !!!!!**

# Intrinsic Mode Function (IMF) # 8

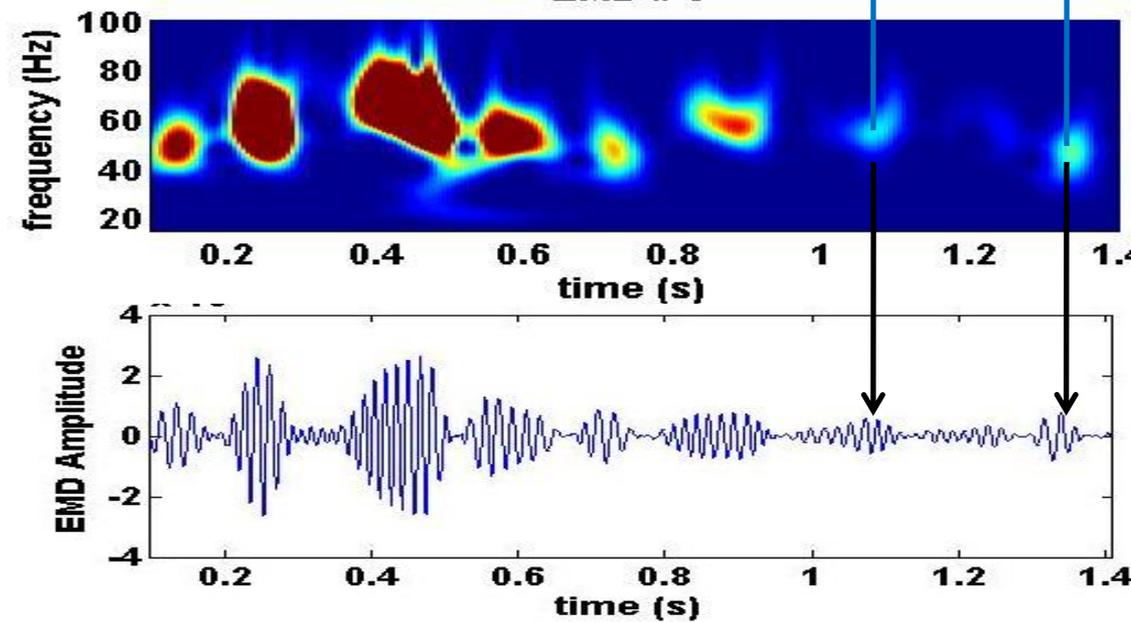
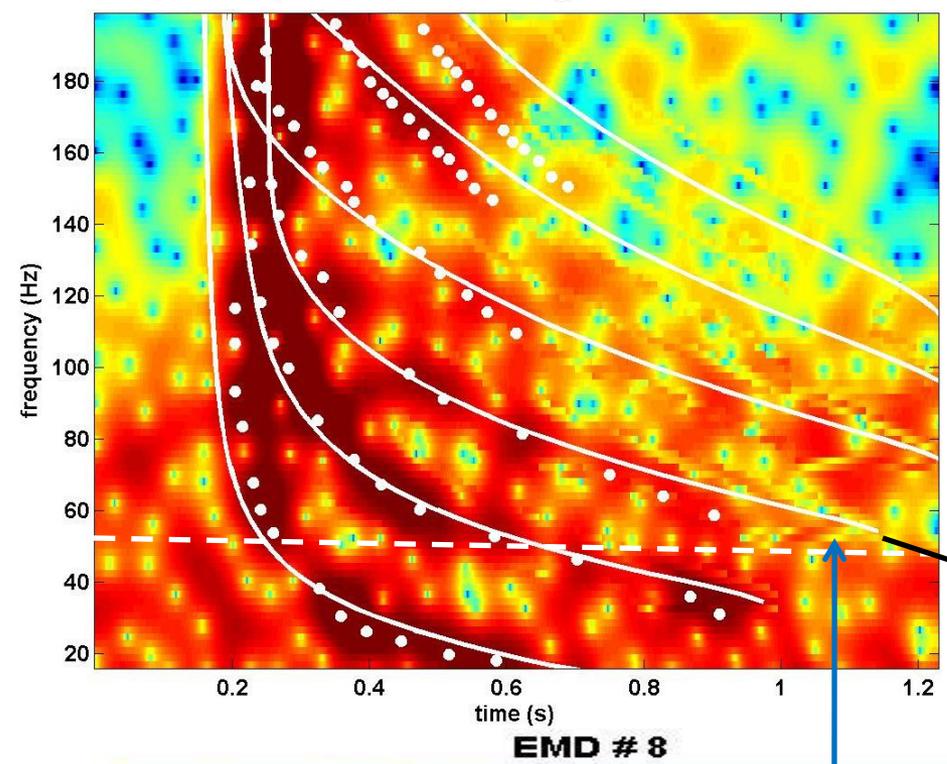
IMF # 8 – 40 to 60 Hz

Mode 1 and 2 dominant (0.4 and 0.6 sec respectively)

Mode 3 energy at 1.1 sec (~50 Hz) and 1.35 sec (~40 Hz)

Arrivals before mode 1 (50 Hz)

Dispersion based STFT - using Mode 1-6- Iteration 2



# Intrinsic Mode Function (IMF) # 7

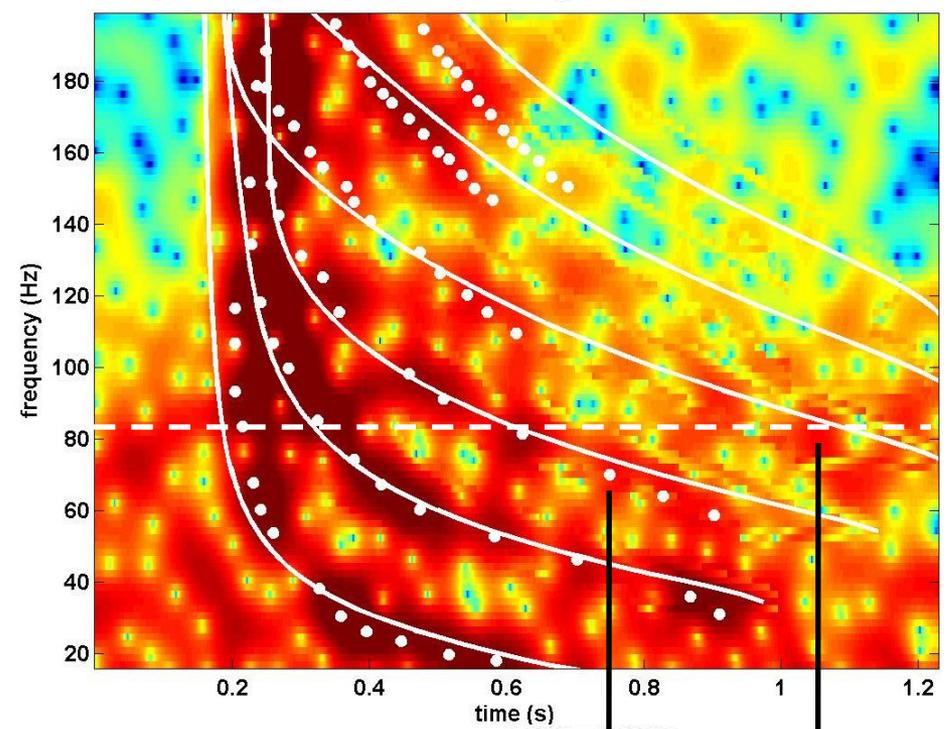
IMF # 7 – 60 to 100 Hz

Mode 1 and 2 dominant (0.2  
and 0.4 sec respectively)

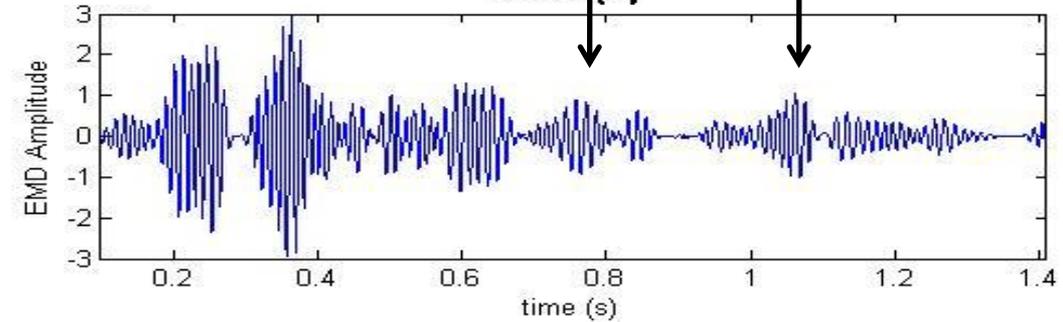
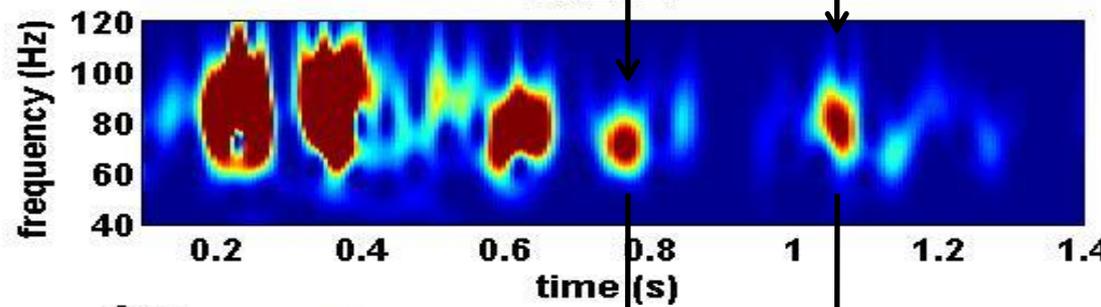
Mode 3 energy at 0.6 sec  
(~80 Hz) and 0.8 sec (~ 65  
Hz)

Mode 4 energy at 1.05 sec  
(75 Hz)

Dispersion based STFT - using Mode 1-6- Iteration 2



EMD # 7



## Intrinsic Mode Function - (IMF) # 6

IMF # 7 – 60 to 160 Hz

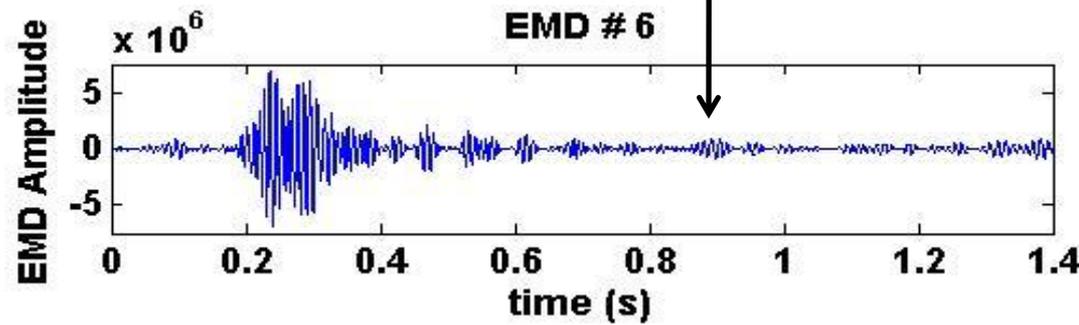
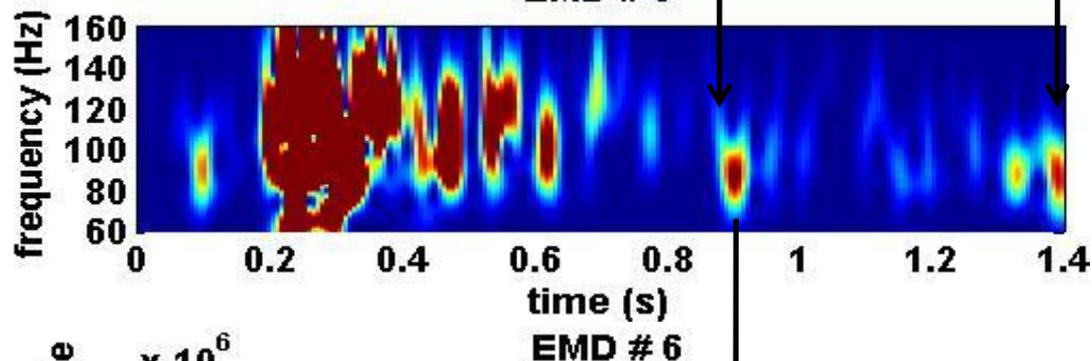
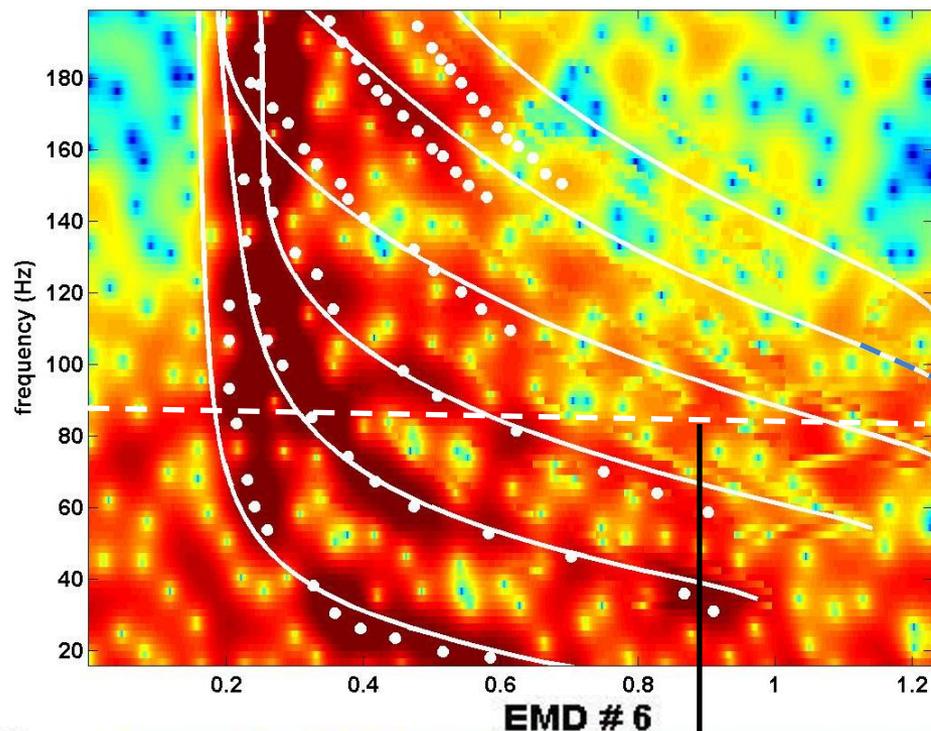
Mode 1 and 2 dominant (0.2  
and 0.3 sec respectively)

Mode 3 energy smeared  
between 0.4 and 0.6 sec

Mode 4 energy at 0.9 sec  
(80 Hz)

Mode 5 energy at 1.4 sec  
(80 Hz)

Dispersion based STFT - using Mode 1-6- Iteration 2



# Intrinsic Mode Function (IMF) # 9

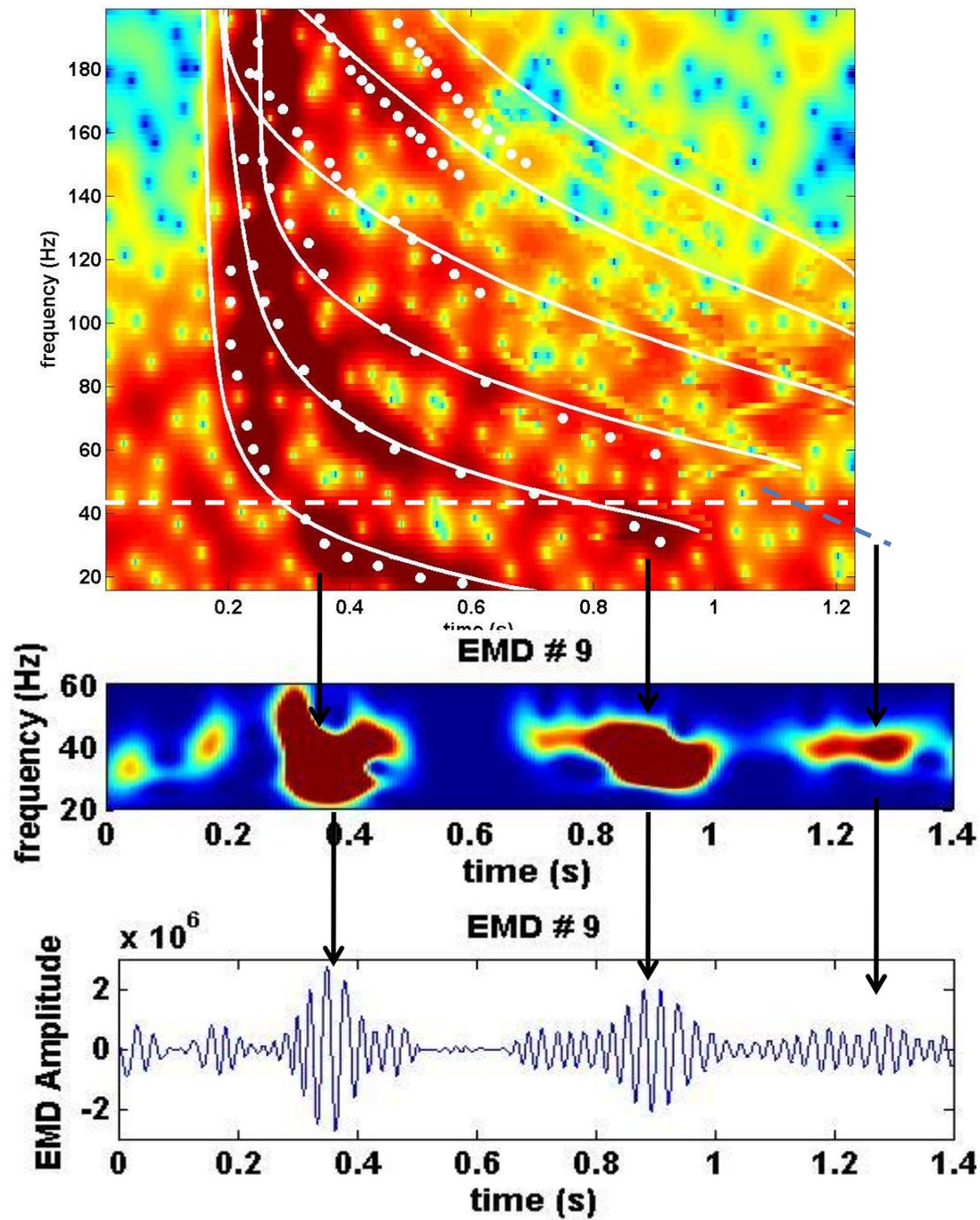
IMF # 9 – 60 to 160 Hz

Mode 1 energy(0.35 sec)

Mode 2 energy at 0.9 sec (40 Hz)

Mode 3 energy at 1.3 sec (40 Hz)

Dispersion based STFT - using Mode 1-6- Iteration 2



# Summary and Future Work

- **D-STFT was applied to CSS data to improve the performance of time-frequency data.**
- **Individual EMFs provide insights into the modal arrivals at specific frequency bands.**
- **EMFs can improve the D-STFT (or wavelet) dispersion information by identifying or confirming mode arrival information especially at low frequency region.**

# Questions ??

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