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Information theory application to inversions of acoustic data from a continental shelf environment

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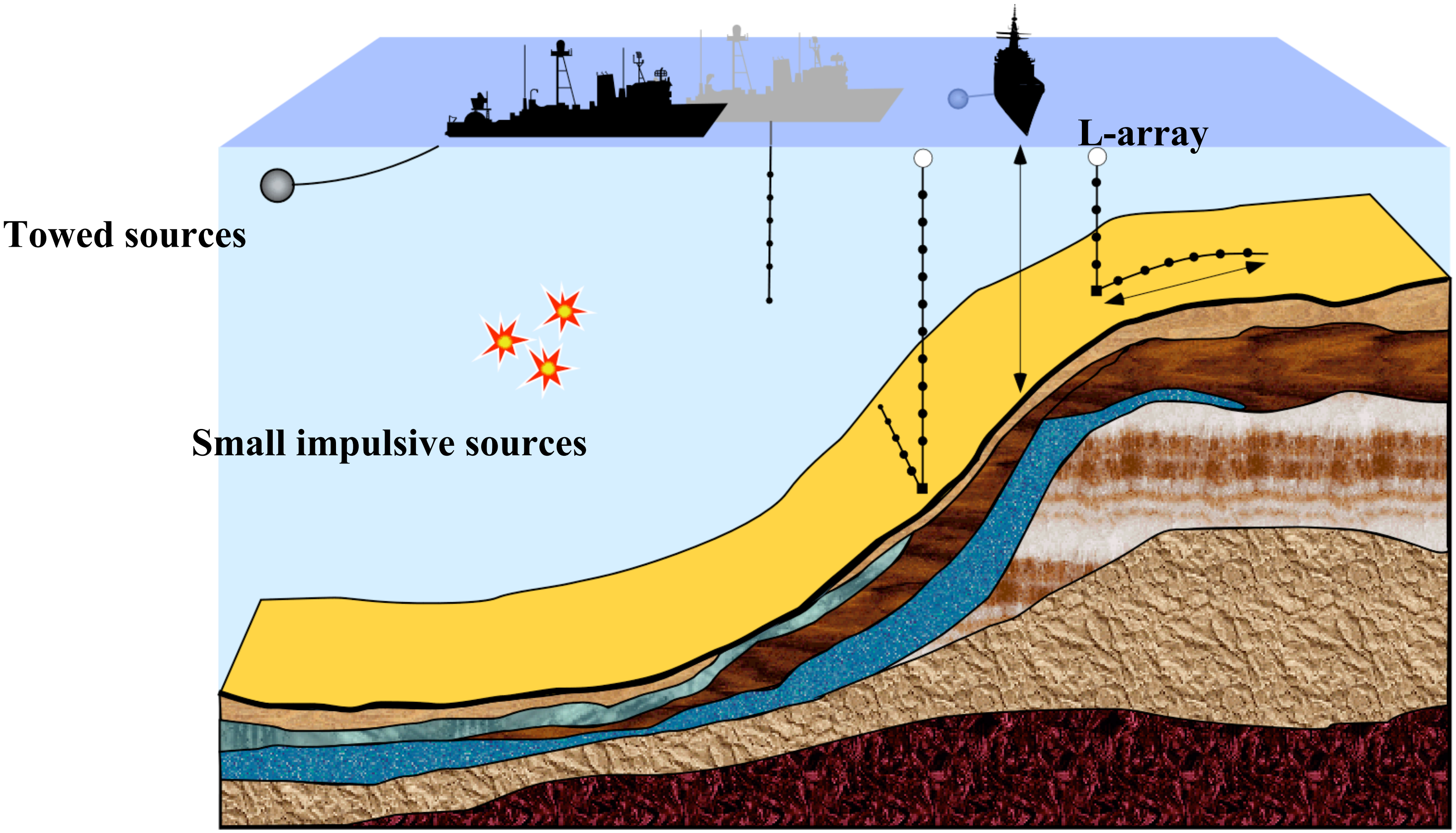
Outline

- Inhomogeneous Oceans: Measurements and Inference
- Maximum entropy versus Bayesian
- Initial Computations
- Example maximum likelihood range-dependent calculations
- Summary

Example of acoustic measurements to infer parameters for inhomogeneous seabed



Measurements and Inferences
Why is inversion by itself not sufficient?

- Scientific objective: Interpret acoustic propagation in inhomogeneous ocean waveguides
  - Mode coupling mechanisms on shelf
  - Kramers Kronig relationships in seabed acoustics
- A maximum likelihood or inversion solution for C_{min} is useful but insufficient
- **Uncertainty** in inferences is a natural consequence of
  - Environmental variability
  - Noise in data
  - Source-receiver motion and source level variability
  - Model errors
- Thus the need for posterior probability $\rho(W|D)$ that given data D the correct solution is W

Two methods of inferring ρ(W|D)

- Bayesian approach designed to solve this problem
  - Requires likelihood function
- Alternative method is maximum entropy principle
  - Requires constraints

Maximum Entropy Approach to Uncertainty in Ocean Acoustics

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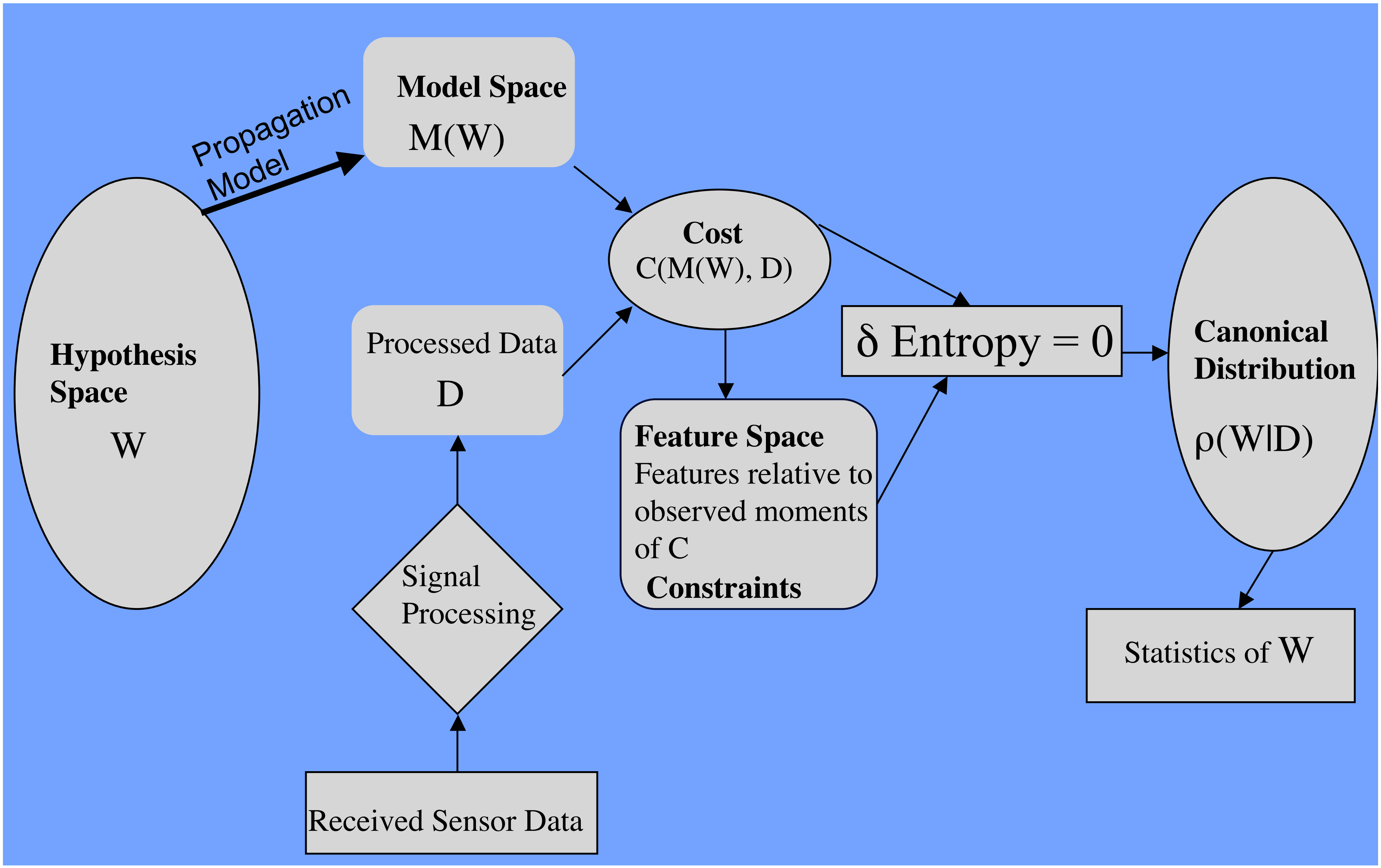

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A Canonical Distribution Approach (1)

Claude Shannon

Shannon or Gibbs Entropy

δ S = 0 subject to stated constraints

Edwin Jaynes

$$S = -\int_{\Omega} dW\, \rho(W|D) \ln [\rho(W|D)/ \rho(W)]$$

Analogy with statistical mechanics for a closed system in thermodynamic equilibrium with heat reservoir

Constraints

$$\int_{\Omega} dW\, \rho(W|D) = 1$$

$$\int_{\Omega} dW\, C(W)\, \rho(W|D) = \langle C \rangle \quad = \frac{1}{2}(C_{min} + \bar{C})$$

C_{min} is global minimum determined from simulated annealing

$\bar{C}$ average value of cost function space $= 1/N \sum C(W_i)$

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A Canonical Distribution Approach (2)

$\delta S = 0$ subject to stated constraints

$$\rho(W|D) = \frac{\rho(W)\exp(-C(W, D)/T)}{Z}$$

Canonical Distribution

$$Z = \int_{\Omega} dW\, \rho(W)\exp(-C(W, D)/T)$$

Average <C> constraint determines T

$$\int_{\Omega} dW\, C(W)\, \rho(W|D) = \langle C \rangle$$

Relationship to Bayes formula

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Thomas Bayes

$$\rho(W|D) = \rho(W)\, \rho(D|W) \,/\, \rho(D)$$

$$\rho(D) = \int_{\Omega} dW\, \rho(W)\, \rho(D|W)$$

$$\rho(W|D) = \rho(W)\, \rho(D|W)\, \left\{\int_{\Omega} dW\, \rho(W)\, \rho(D|W)\right\}^{-1}$$

$$\rho(D|W) = \exp(-C/T)$$

Canonical distribution (not normalized) *plays role* of likelihood function

Average, standard deviation, and marginal distributions

$$< X >= \frac{\int_\Omega d\mathbf{W} X(\mathbf{W})\mathbf{exp}[-C(\mathbf{M}(\mathbf{W}),\mathbf{D})/T]}{Z}$$

$$\sigma_X = \frac{\int_\Omega d\mathbf{W}[X(\mathbf{W})- < X >]^2 \mathbf{exp}[-C(\mathbf{M}(\mathbf{W}),\mathbf{D}]/T)}{Z}$$

Continuous formulation

$$P(w_i) = \frac{\int_\Omega d\mathbf{W}' \delta(w_i' - w_i)(\mathbf{exp}[-C(\mathbf{W}']/T)}{Z}$$

$$< X >= \frac{\sum_i^N X(\mathbf{W}_i)\mathbf{exp}[-C(\mathbf{W}_i]/T)}{\sum_k^N \mathbf{exp}[-C(\mathbf{W}_k)/T]}$$

$$\sigma_X^2 = \frac{\sum_i^N [X(\mathbf{W}_i))- < X >]^2 \mathbf{exp}[-C(\mathbf{W}_i)/T]}{\sum_k^N \mathbf{exp}[-C(\mathbf{W}_k)/T]}$$

Monte-Carlo integration

$$P(w_i) = \frac{\sum_{j \in {}_j w_i = w_i}^N \mathbf{exp}[-C(\mathbf{W}_j)/T]}{\sum_k^N \mathbf{exp}[-C(\mathbf{W}_k)/T}$$

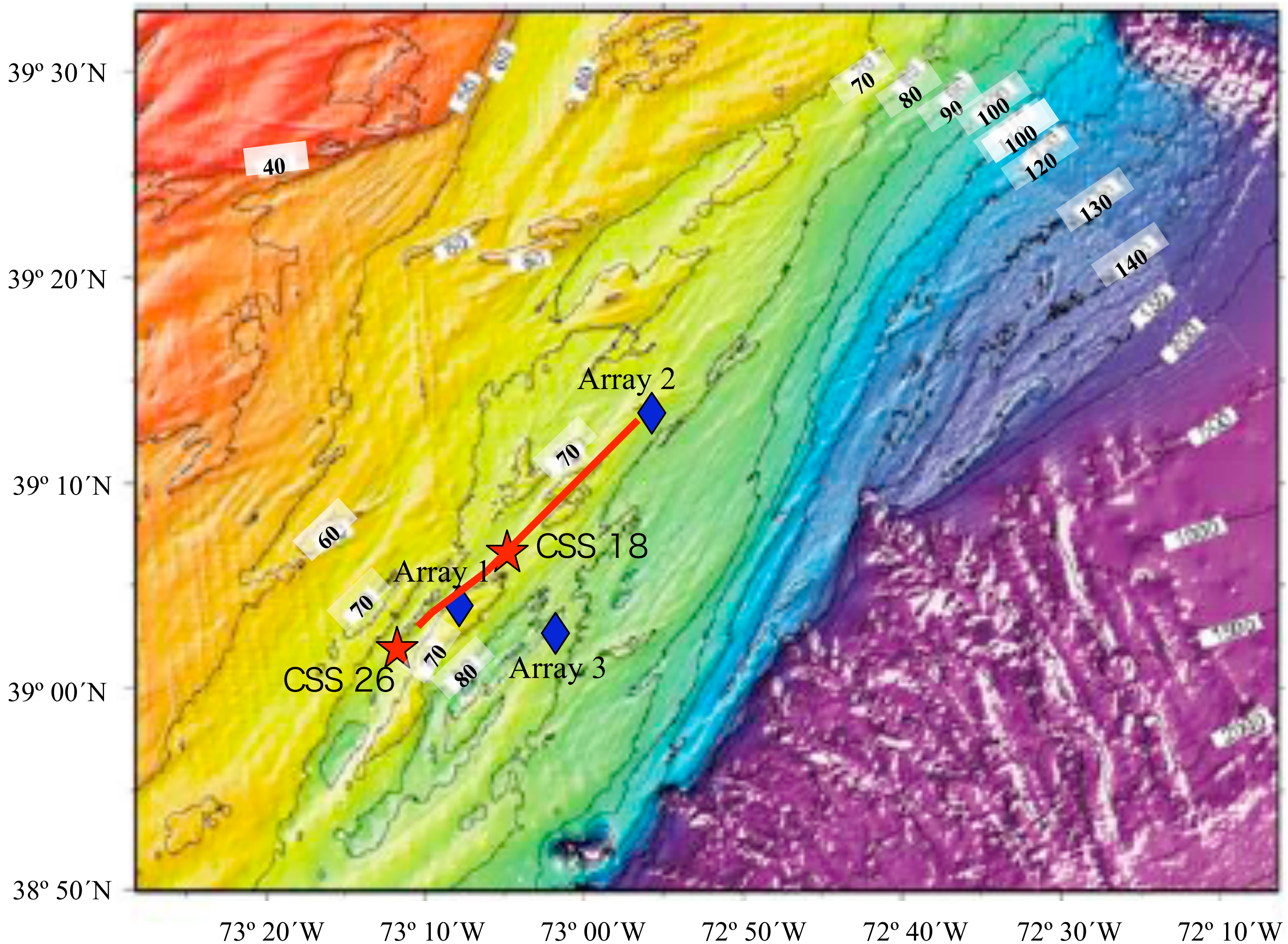
Pros and cons of Bayesian versus canonical distribution approach

- When prior information on noise and model errors is available, Bayesian approach is well justified
- Maximum entropy method appears well suited for problems with sparse data
  - Does not require direct assumptions about model / data errors or noise
    - Indirectly includes such information via constraints from *observed* features of cost
  - Prior information on $\rho(W)$ is included naturally via relative Shannon entropy
  - Leads to most conservative distribution
  - No restrictions on cost functions
    - Posterior distribution depends on cost function
  - Can include higher order moments of features, if available ,via constraints

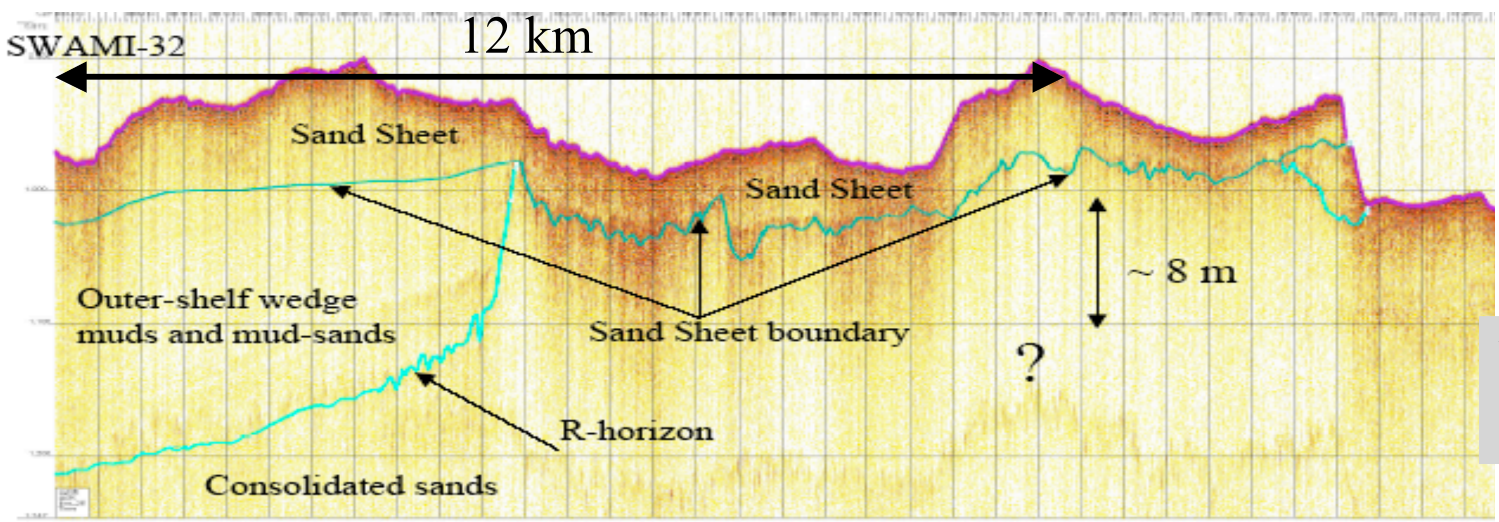
Application to acoustic data taken on continental shelf

- Shallow Water 2006 experiment
  - Data set of interest because of large spatial and temporal inhomogenities on continental shelf
    - Seabed
    - Water column
- Current Work
  - Maximum entropy principle applied to data with small range inhomogenities assuming range independence
  - Cross-slope range-dependent data using knowledge gained from MEP analysis, Goff geophysics characterization, and SSP measurements
  - Representation of range-dependent media - balance of representation versus number of parameters
  - Working to implement faster propagation model than PE

Location of sources and receivers

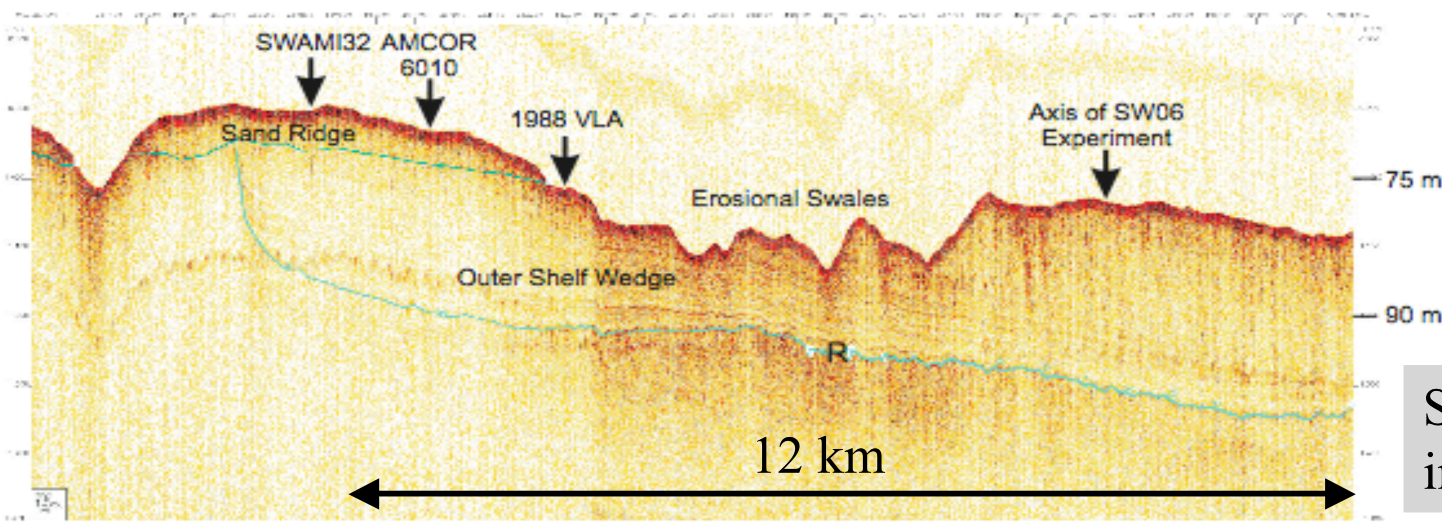


Chirp seismic reflection profiles



Track 1

Weak variations in SSP along track



Track 2

Strong variations in SSP along track

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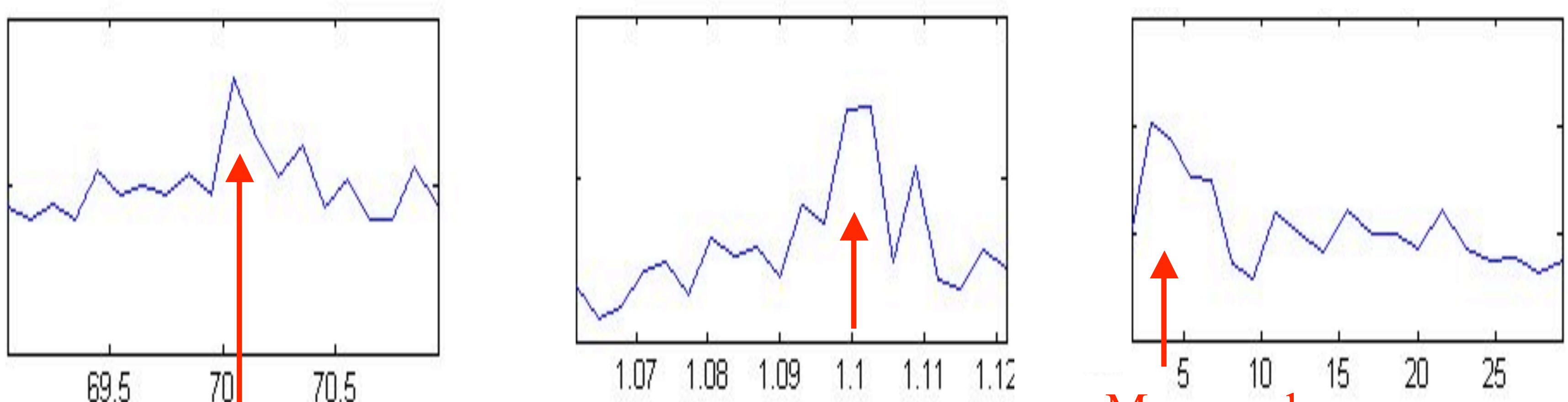
Marginal distributions from MEP for short range data (1-4 km) taken on Array 2

Water depth - m

Ratio(layer 1)

Thickness(layer1) - m



Measured

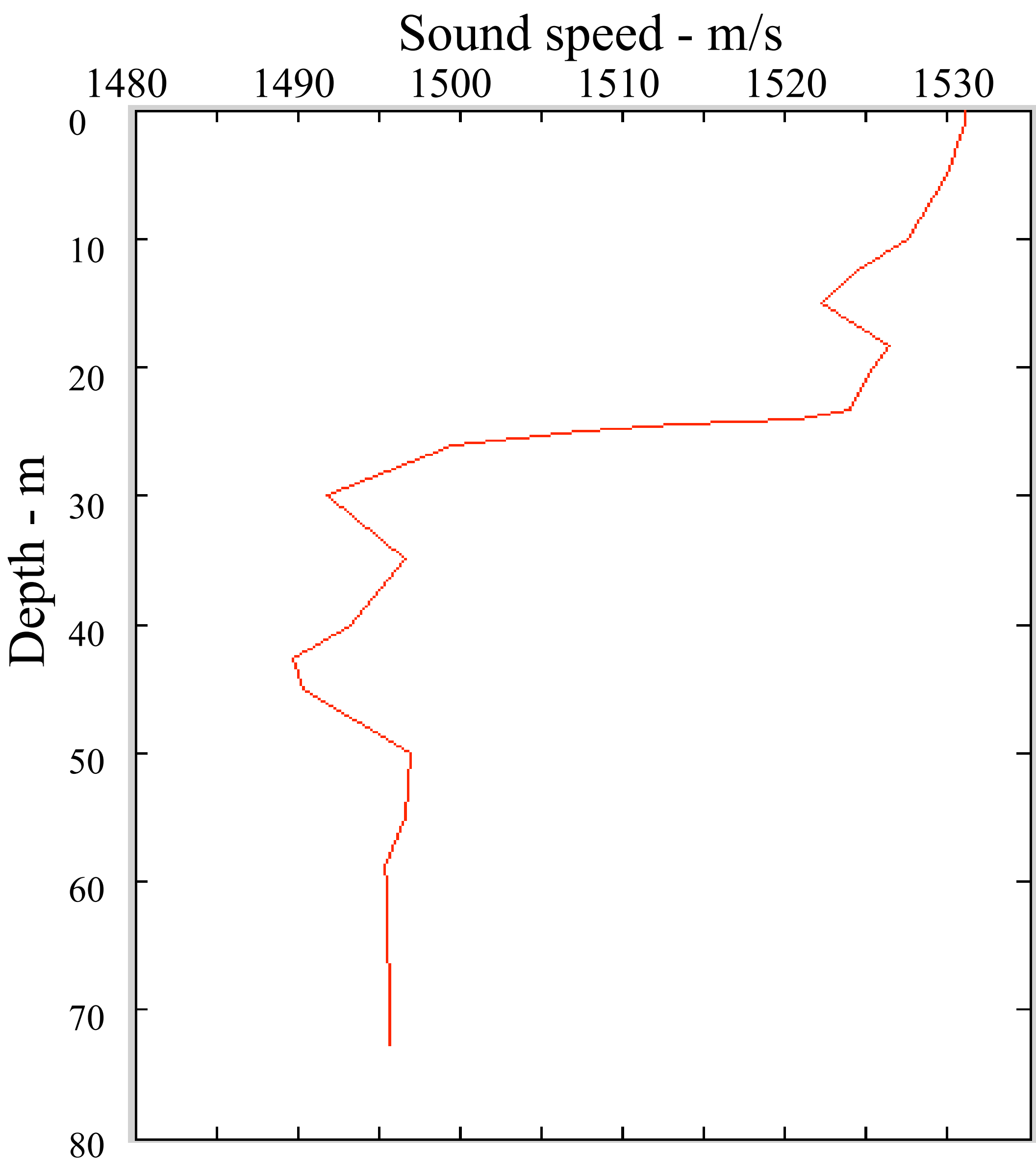
Course sand agreement with cores

Measured

$$P(w_i) = \frac{\sum_{j \in_j w_i = w_i}^{N} \mathbf{exp}[-C(\mathbf{W}_j)/T]}{\sum_{k}^{N} \mathbf{exp}[-C(\mathbf{W}_k)/T]}$$

$2x10^6$ samples

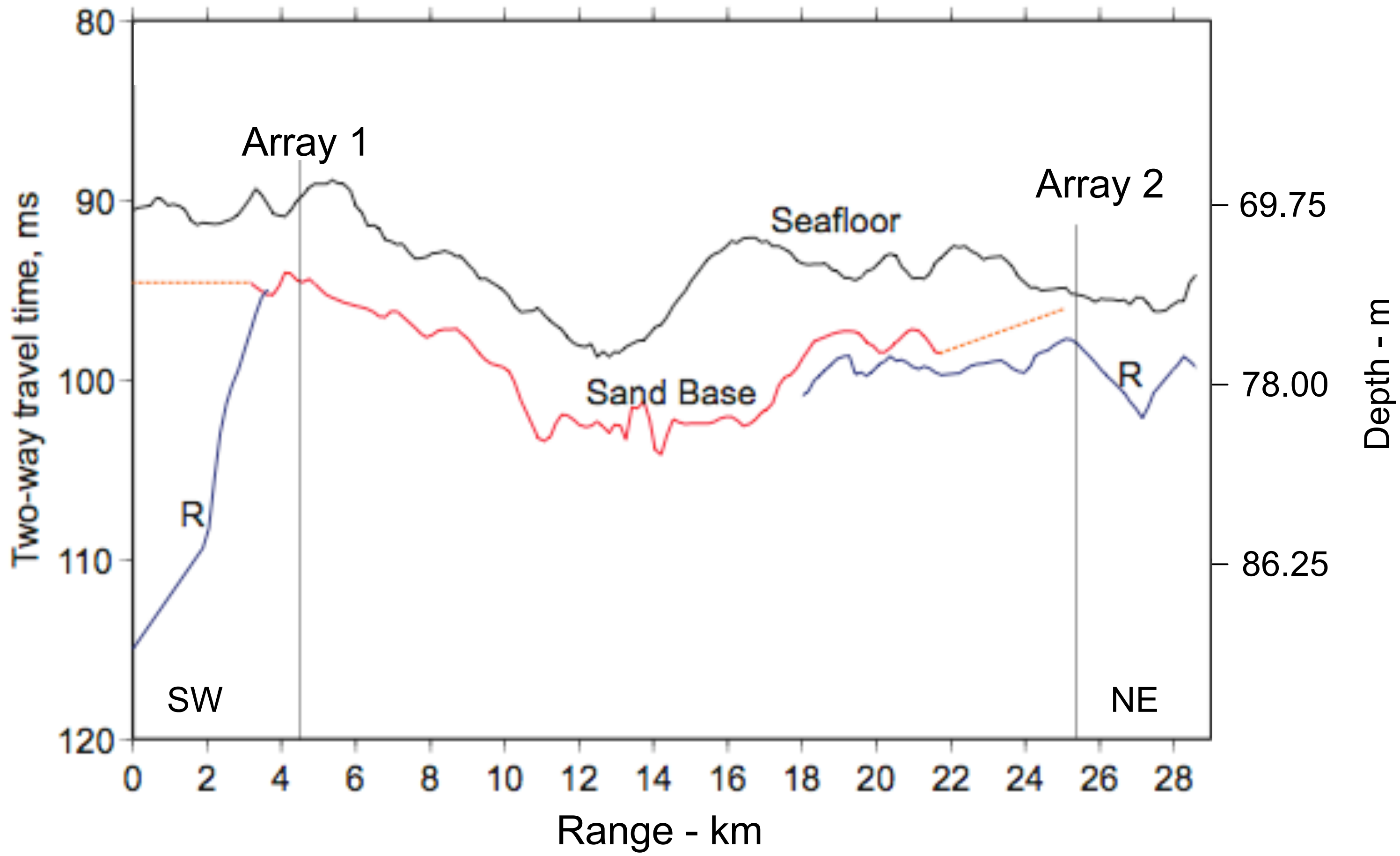
Sound speed profile along Track 1



SSP had small variations along isobath

Geophysical structure of propagation track 1 from chirp reflection sonar

Weakly range-dependent track



CSS event 26 model-data comparison for weakly range dependent track range 26.3 km, 35-325 Hz band

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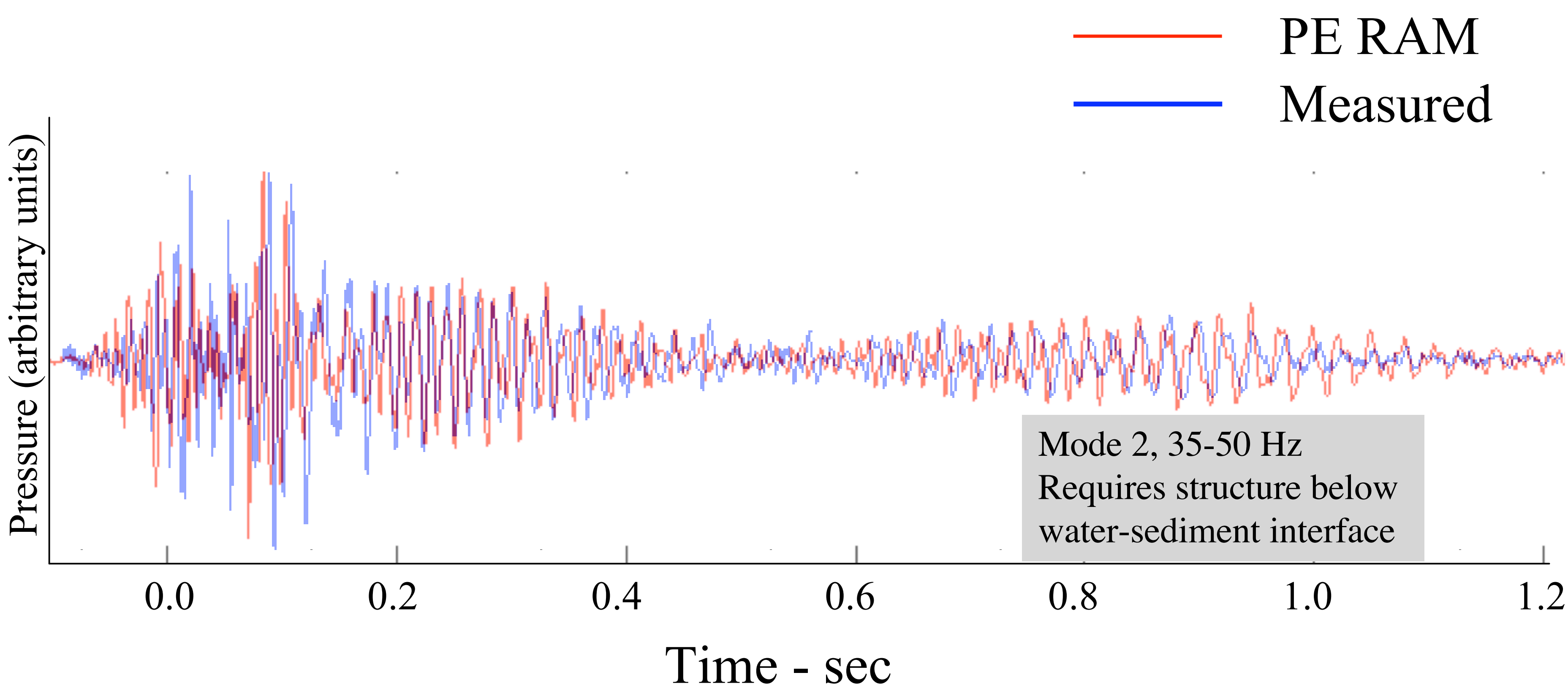
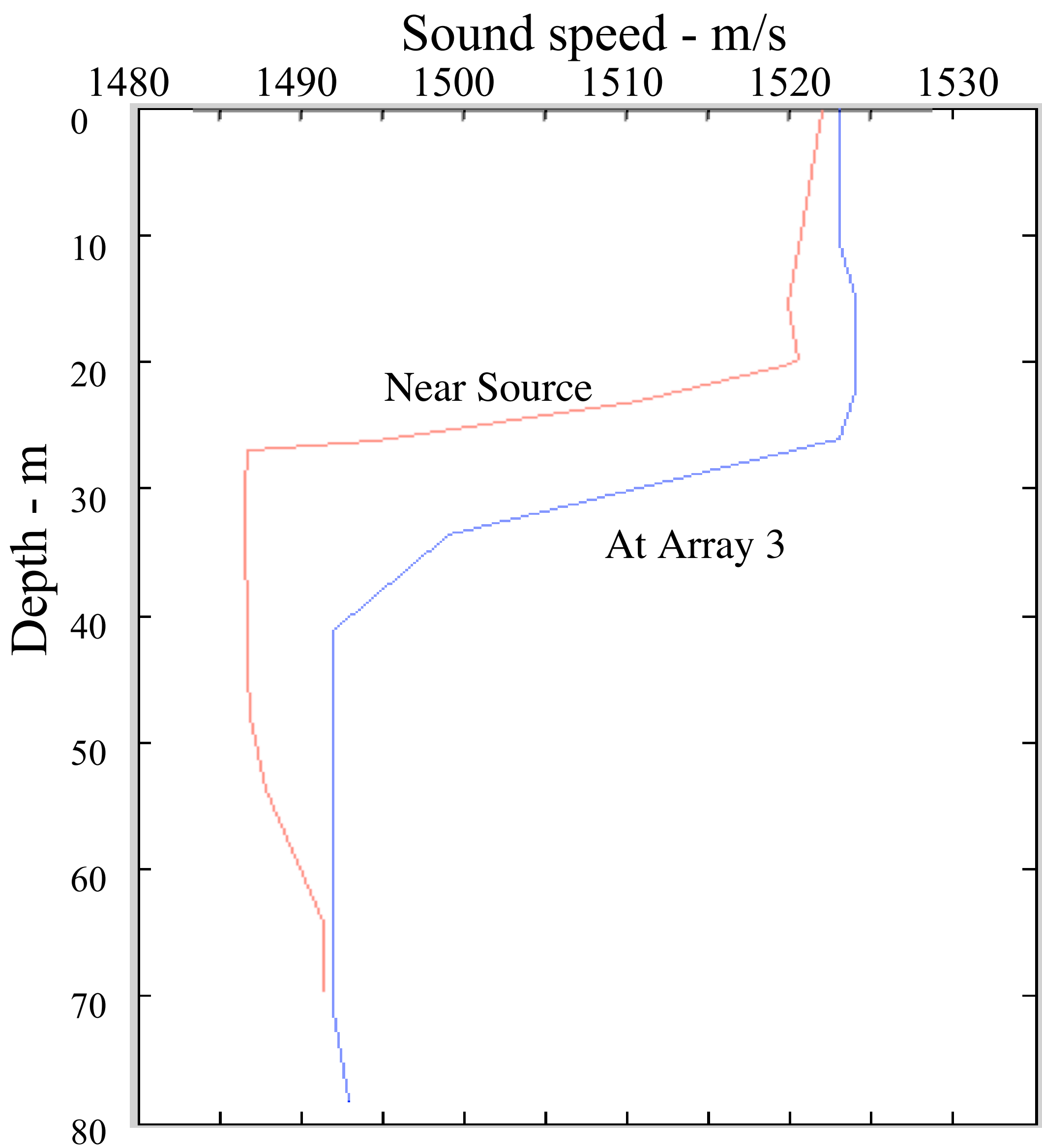


Information from short-range data with additional information provided by direct geophysical and sound speed measurements is sufficient for acoustic prediction along ~ isobath

Sound speed profile along Track 3

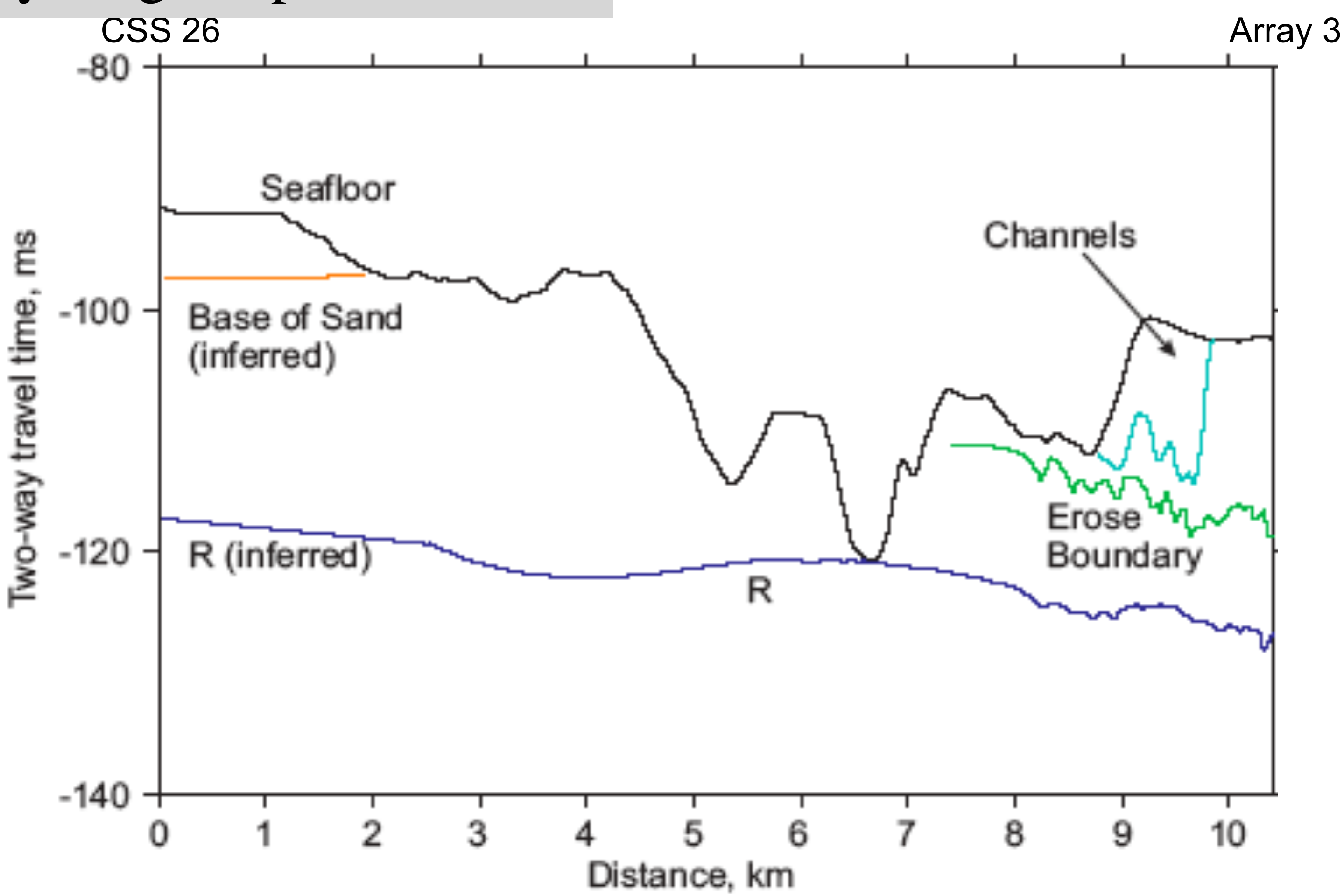


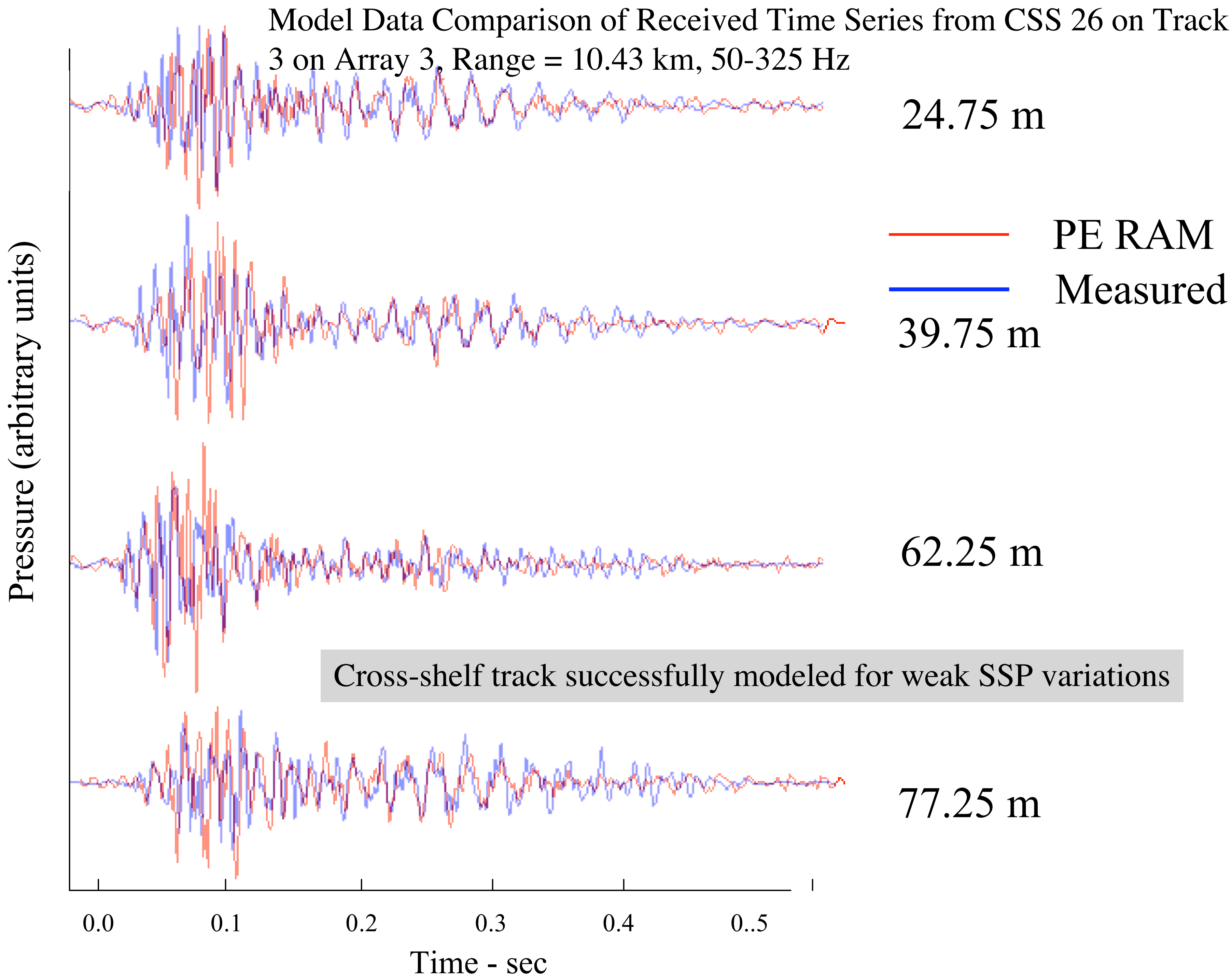
Stronger SSP variation across shelf. Similar SSPs - but displacement of thermocline

Hypothesis:
Place measured SSPs at source and receiver and interpolate for points between source and receiver

Geophysical structure of propagation track 3 from chirp reflection sonar

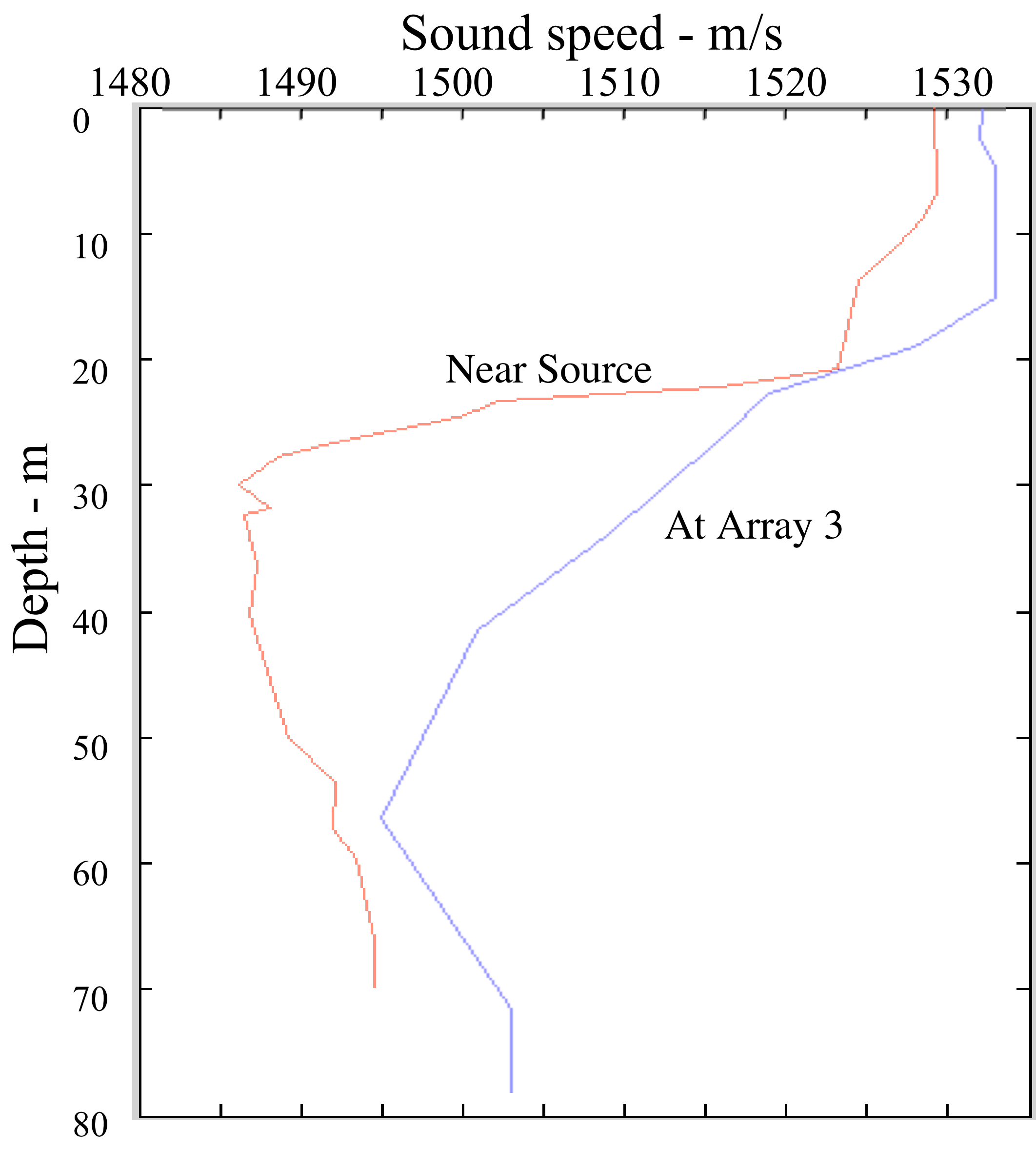
Moderately range-dependent track





Model Data Comparison of Received Time Series from CSS 26 on Track 3 on Array 3, Range = 10.43 km, 50-325 Hz
24.75 m
PE RAM
Measured
39.75 m
62.25 m
Cross-shelf track successfully modeled for weak SSP variations
77.25 m
Pressure (arbitrary units)
0.0
0.1
0.2
0.3
0.4
0..5
Time - sec

Sound speed profile along Track 2



Strong SSP variation across shelf for track 2

Hypothesis 1:
Place measured SSP at source and receiver and interpolate for points between source and receiver

Geophysical structure of propagation track 2 from chirp reflection sonar

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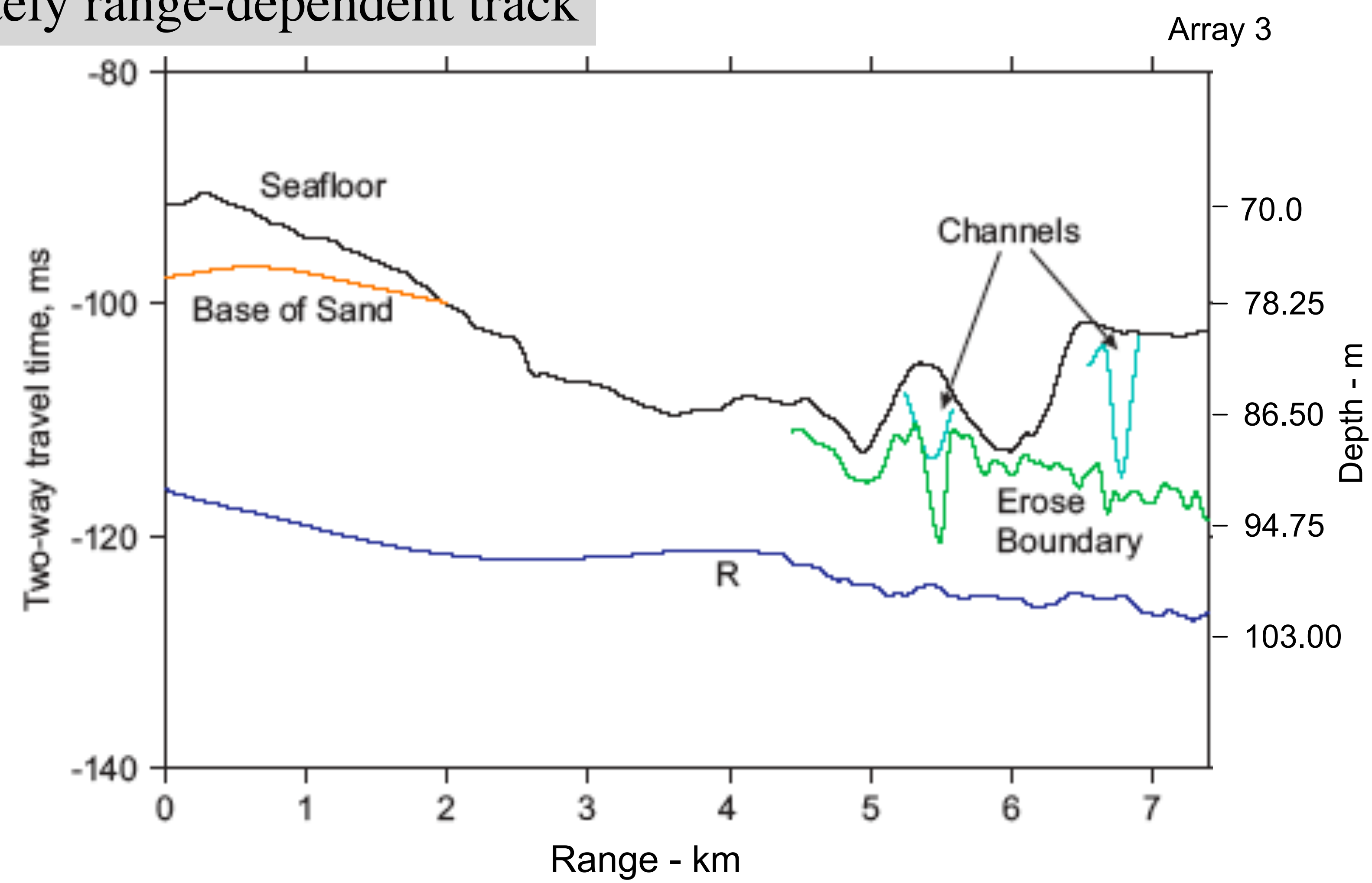
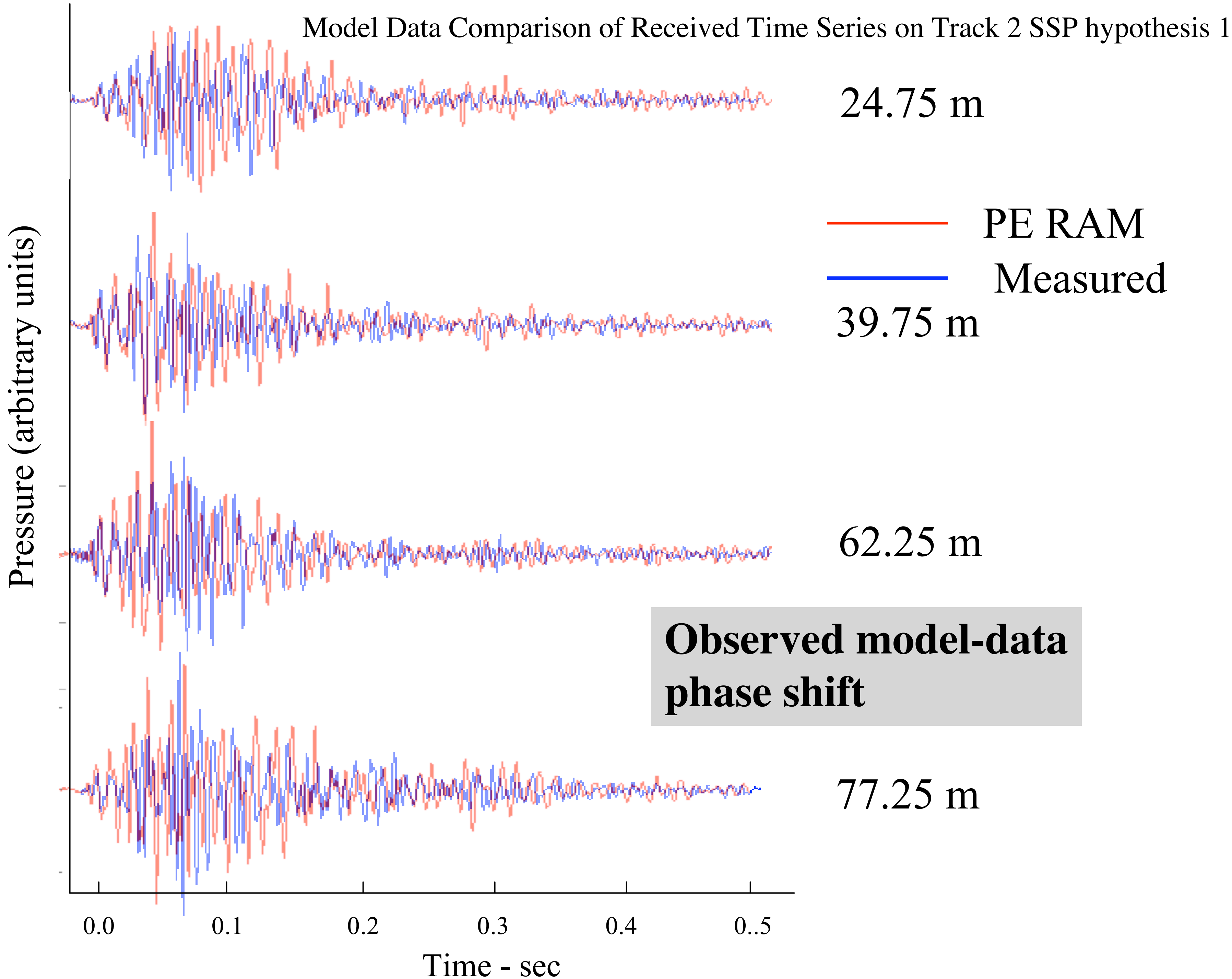
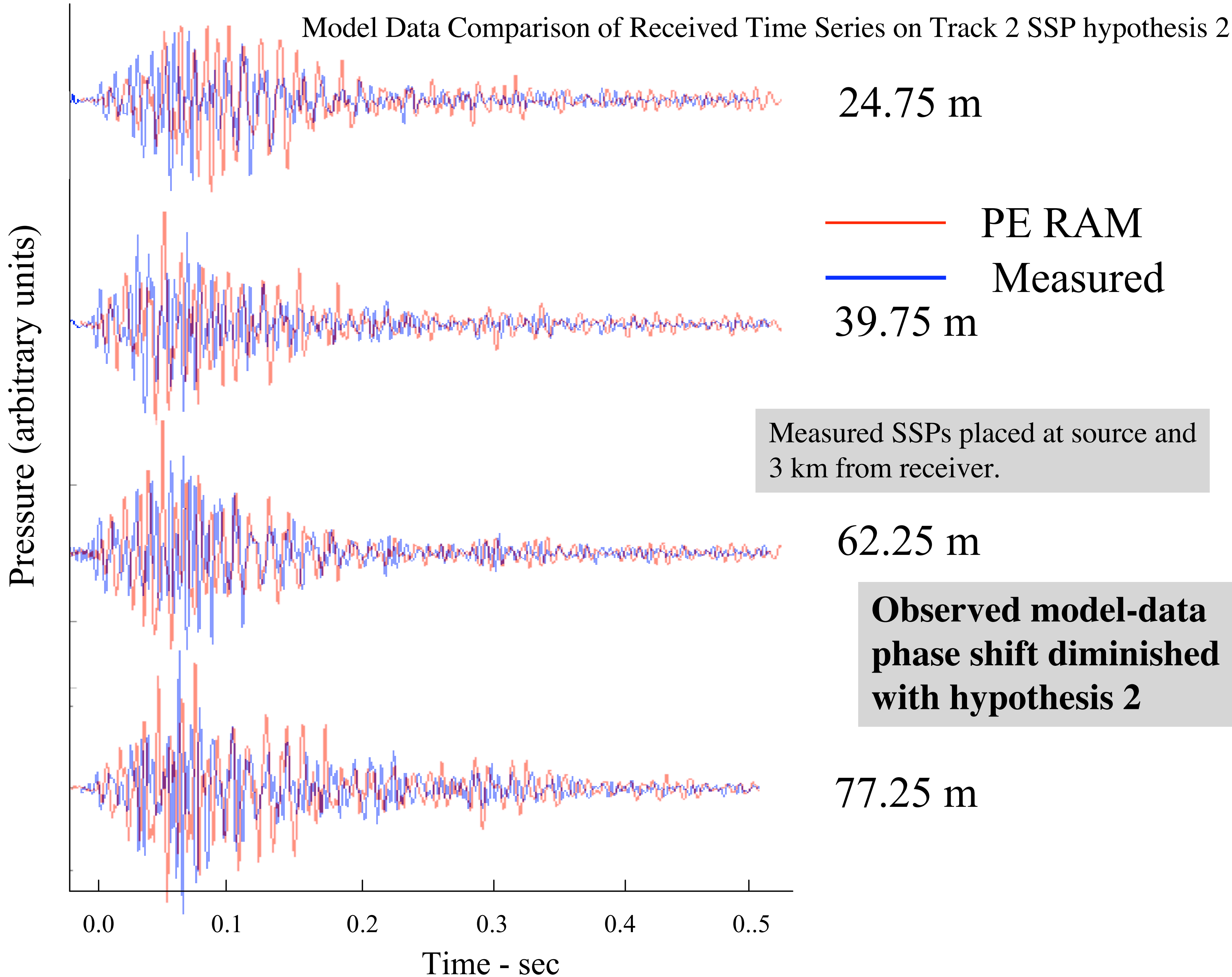
Moderately range-dependent track




Model Data Comparison of Received Time Series on Track 2 SSP hypothesis 1
24.75 m
PE RAM
Measured
39.75 m
62.25 m
Observed model-data
phase shift
77.25 m
Pressure (arbitrary units)
0.0
0.1
0.2
0.3
0.4
0..5
Time - sec


Model Data Comparison of Received Time Series on Track 2 SSP hypothesis 2
24.75 m
PE RAM
Measured
39.75 m
Measured SSPs placed at source and
3 km from receiver.
62.25 m
Observed model-data
phase shift diminished
with hypothesis 2
77.25 m
Pressure (arbitrary units)
0.0
0.1
0.2
0.3
0.4
0..5
Time - sec

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Summary and Implications

- Learning probability distribution from measured acoustic data, while not the physics problem, is the technical problem
- Continued study of connections between Bayesian and Maximum Entropy Principle approach
- Details of coherent time structure of received time series for cross-shelf propagation sensitive to both the range dependence of the
  - Geoacoustic profile
  - SSP profile
- Resolving ambiguities
  - Include range-dependence of SSP and geoacoustics in $\rho(W|D)$
    - Balance of information gain and loss from increased number of parameters
  - Sampling of water column in long range LF shelf experiments needs to be not greater than 5 km
    - Implications for 50x20 km^2 future shelf experiment

Acknowledgements: (1) WHOI for making Array 3 data and CDT data available