Acoustic mode beam effects of nonlinear internal gravity waves in shallow water

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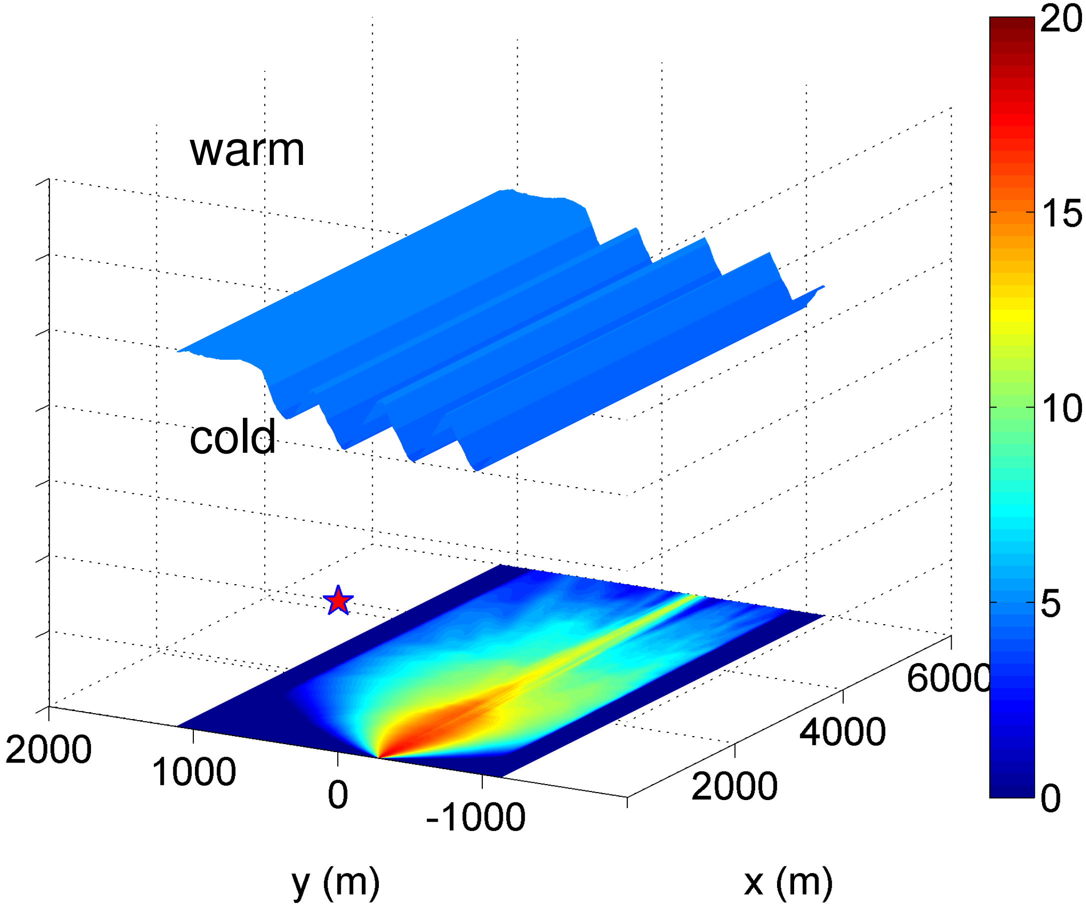
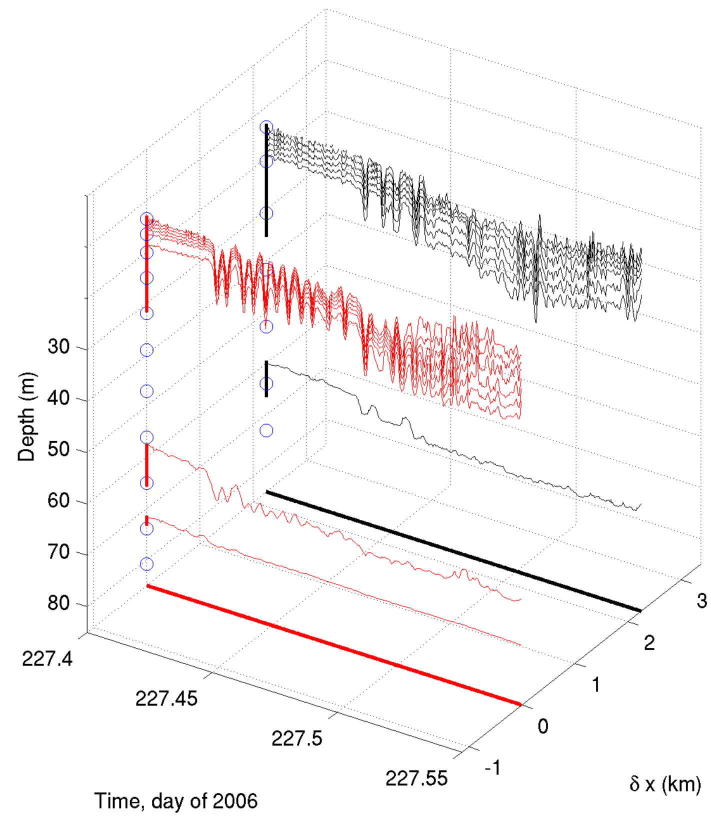
Introduction: Internal-wave acoustic ducts in shelf waters

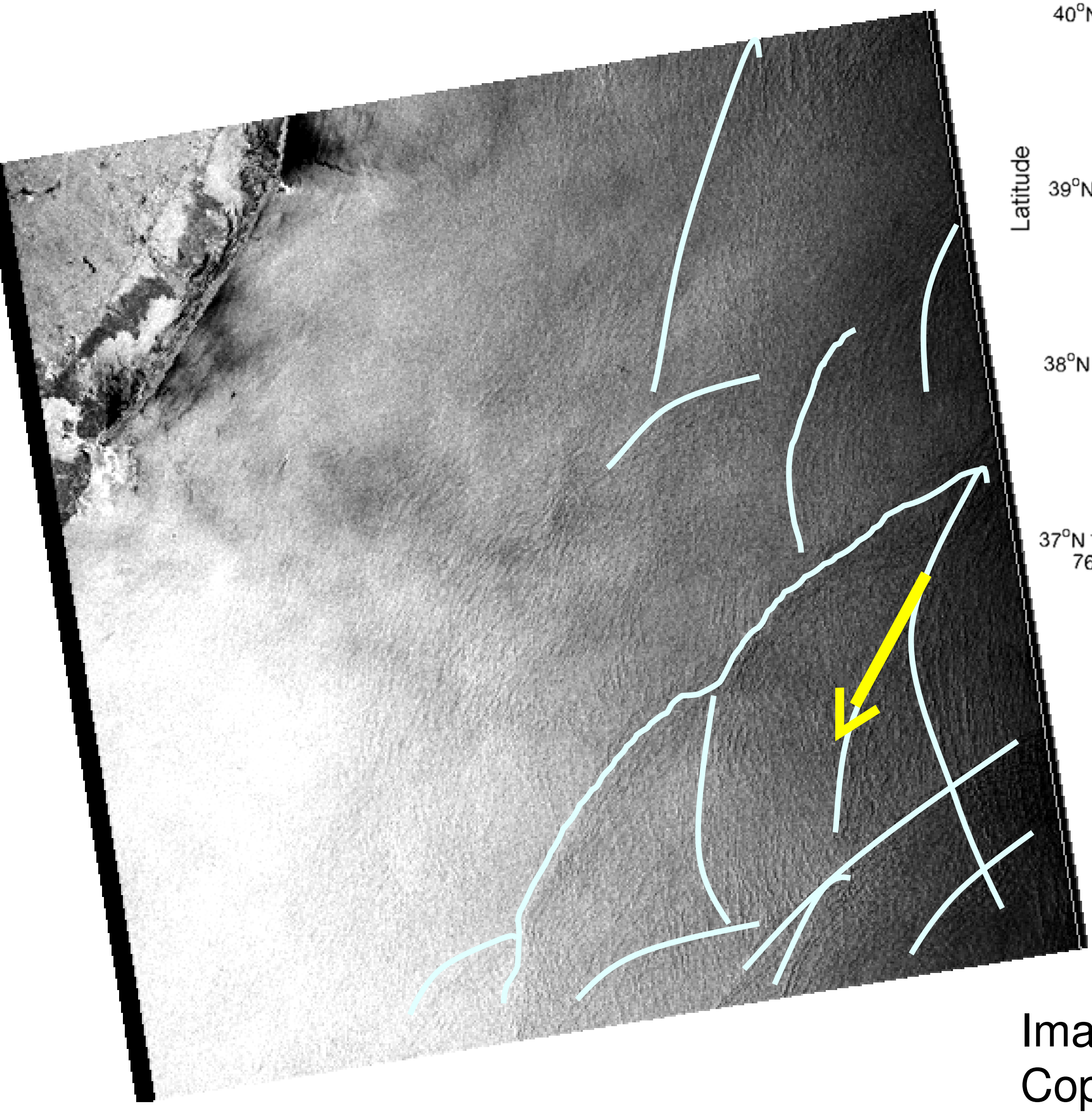
- Low-frequency (50-1000 Hz) sound travels efficiently in shallow-water environments; sound can be exploited, but propagation can be variable.
- Non-linear internal gravity waves (NLIWs) create strong anomalies of sound speed at the thermocline.
- Typically, packets of long-crested waves produce ducts (slow acoustic mode phase velocity) between troughs of NLIW.
- Ducting varies over the set of acoustic normal modes. A unique interference pattern for each mode results.
- Consequences of this on sound propagation are illustrated with two examples: ***(1) moving, terminating internal-wave duct; (2) curved internal-wave duct***.



Idealized duct (South China Sea parameters)



Poorly-organized real-world example
(SW06 New Jersey)



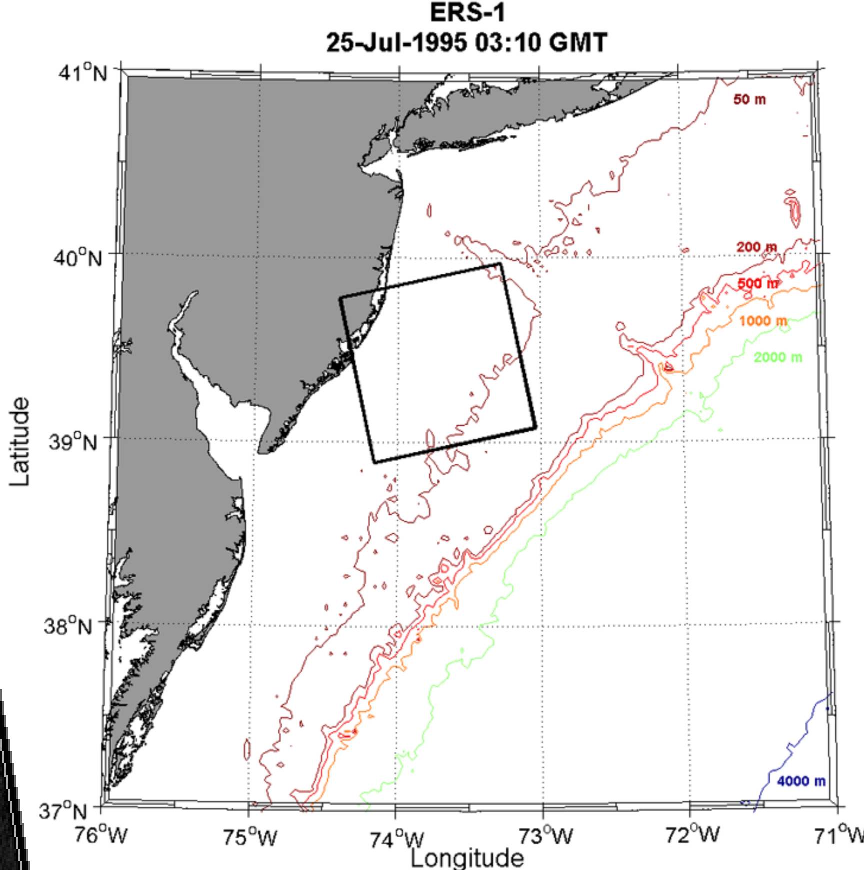
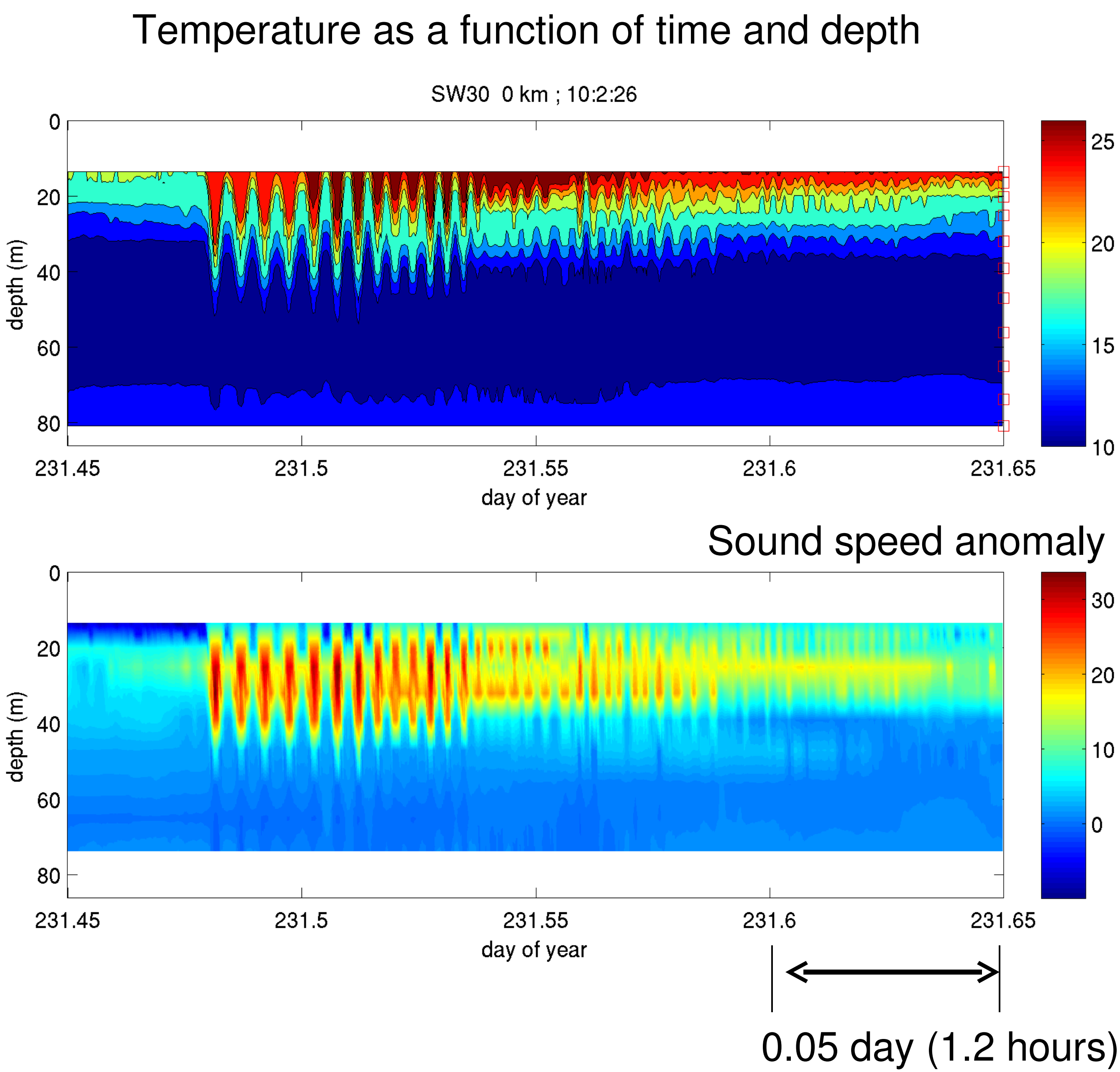
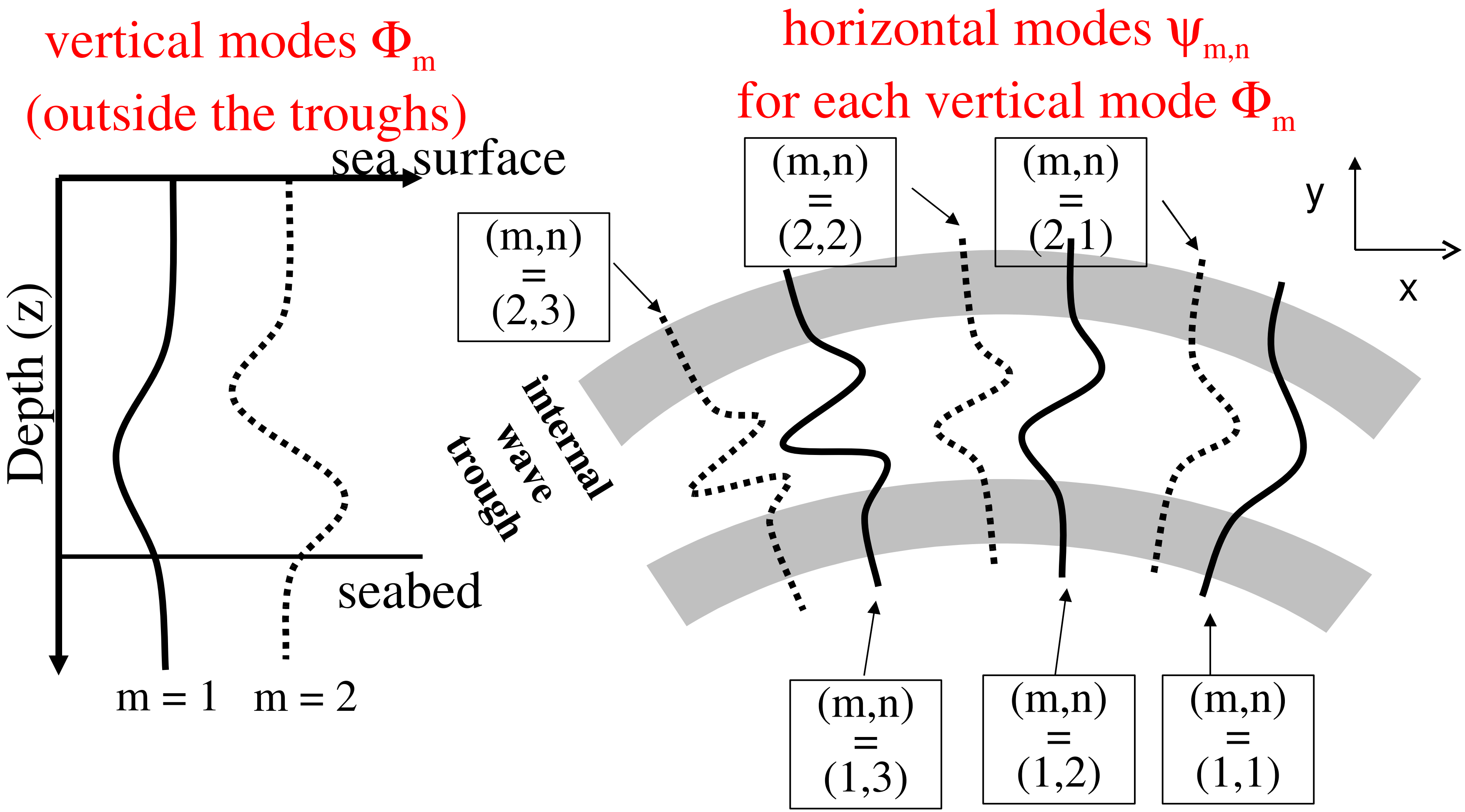


Image from Global Ocean Associates
Copyright ESA


Temperature as a function of time and depth
SW30 0 km ; 10:2:26
depth (m)
day of year
Sound speed anomaly
depth (m)
day of year
0.05 day (1.2 hours)

Mode-by-mode internal-wave ducting of acoustic modes



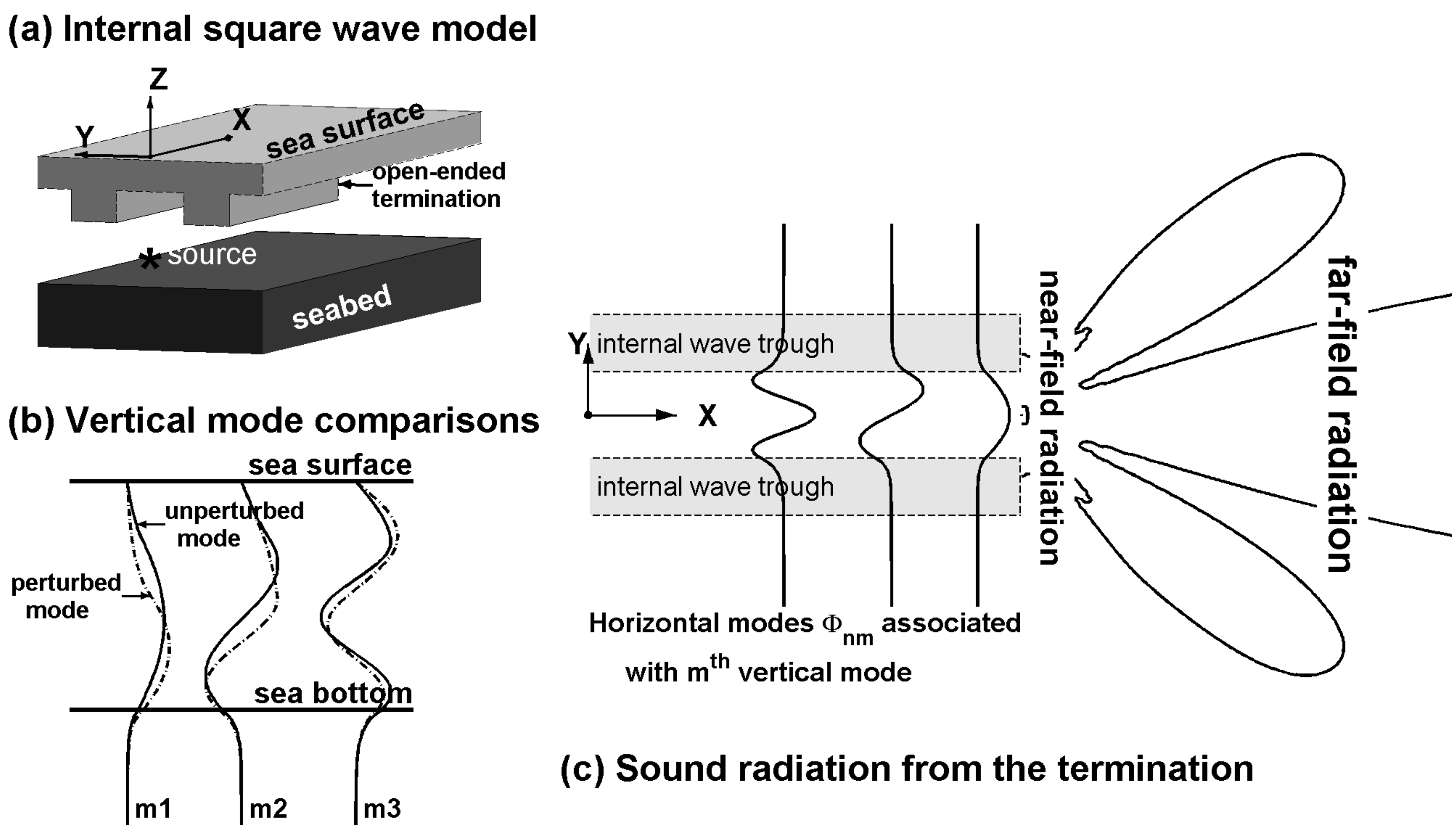
Pressure field: $P(x,y,z) = \Sigma_m \, \Sigma_n \; \alpha_{m,n}(x,y) \; \psi_{m,n}(y;\, x) \; \Phi_m(z;\, x,y)$

for straight waves: $P(x,y,z) = \Sigma_m \, \Sigma_n \; \alpha_{m,n}(x) \; \psi_{m,n}(y) \; \Phi_m(z;\, y)$

Two methods are used to study fields in and around ducts:

Analytic (tabulated with computer): Horizontal modes in a (straight) duct are derived, for each vertical mode. Coefficients evaluated for a source in the duct. The radiated field from an open duct is calculated with **Huygens' Principle**.

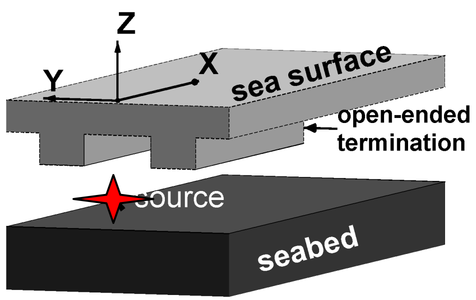
Computational: Fully 3D parabolic equation (later slide)



(c) Sound radiation from the termination

Idealized (analytic) terminating duct: Unique time-evolving radiation beam pattern for each vertical mode as wave duct sweeps past a stationary source

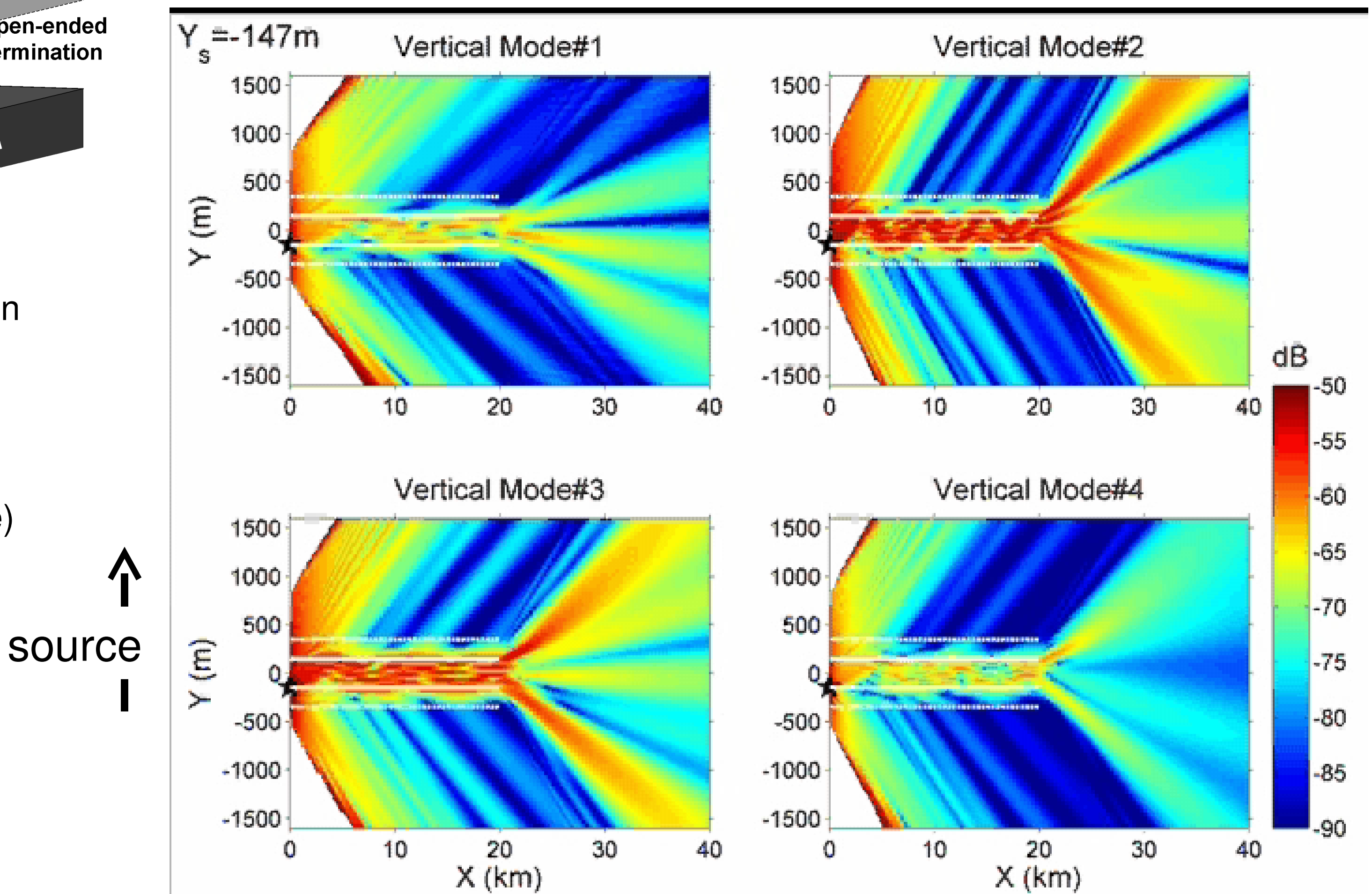
100Hz source @ 70m depth in duct, top view



Two-layer water column
80m water depth
c1 1520 m/s
c2 1480 m/s
upper layer thickness:
20 m (no internal wave)
40 m (internal wave)

Homogenous bottom
cb 1700 m/s
density 1.5 g/cm³

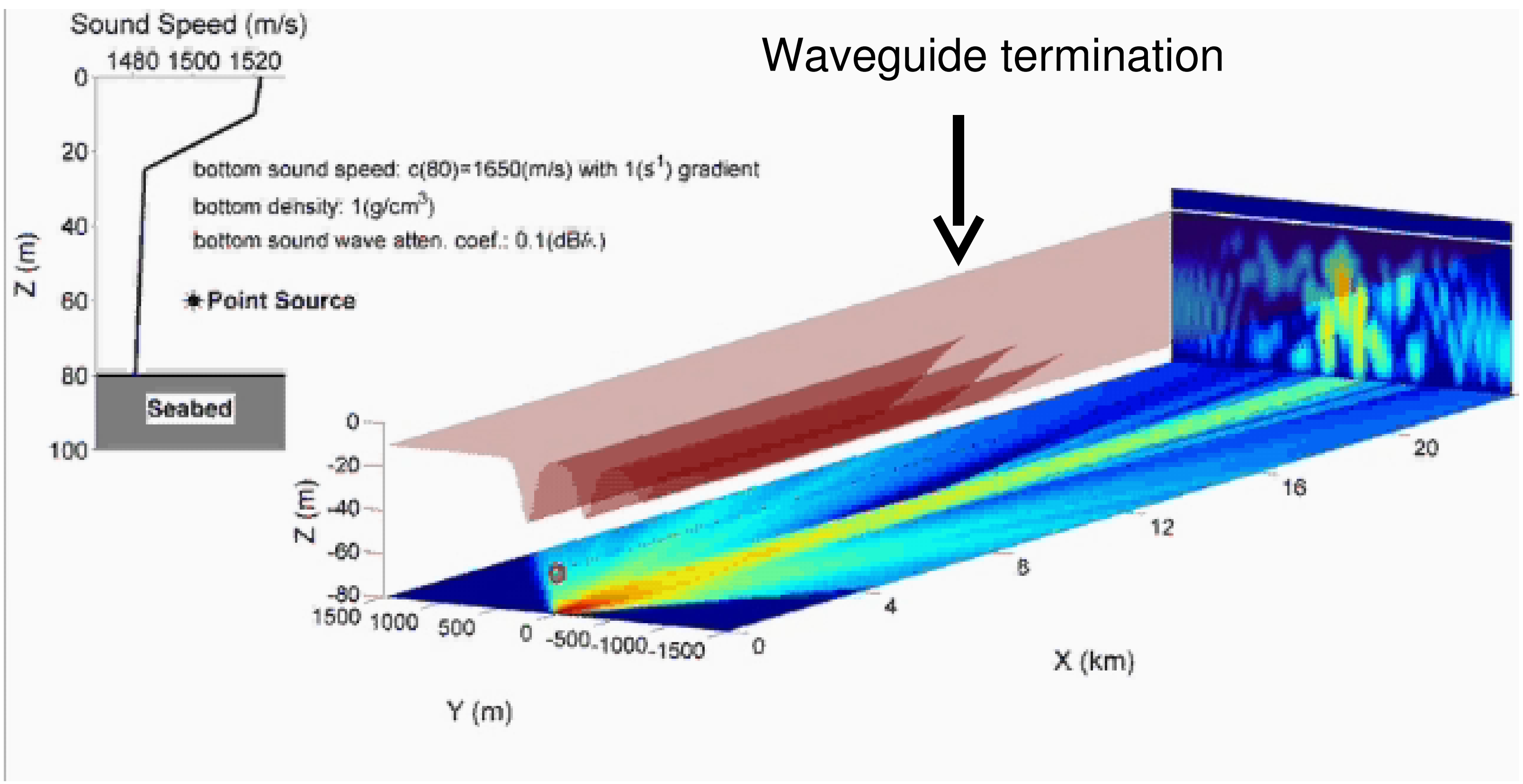
Internal wave field
duct width 300m
Int. wave width 200m



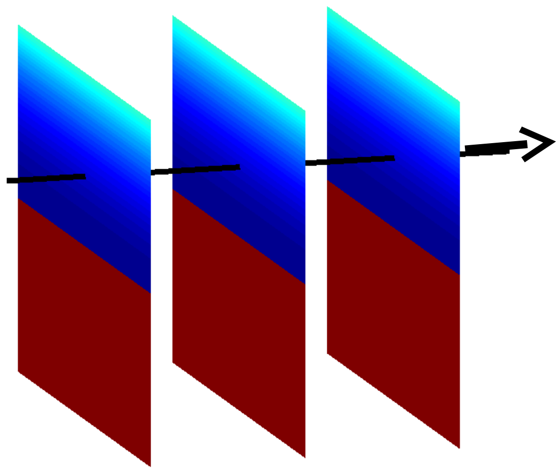
Cartesian 3D PE computational method:

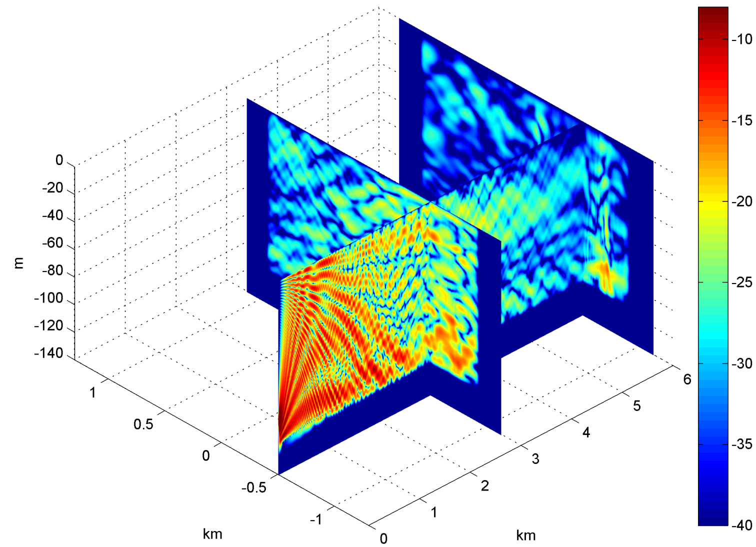
- Allows more realistic internal-wave structures.
  Smoothly shaped waves, curved waves, etc.
- More time-consuming to compute.

200Hz source @ 80m depth



Three-Dimensional Computational solution





Well-known Tappert/Hardin Fourier/split-step parabolic equation (PE) solution

$$\Psi(x+\delta) = \boldsymbol{F}^{-1}\,[\,G\cdot(\,\boldsymbol{F}\,[\,P\cdot\Psi(x)\,]\,)\,]$$

$P = A_P \exp(-ik_0 U\delta)$	operator in the spatial domain
G	propagator in the wavenumber domain
F	Fourier transform operator (2D in this case)

Cartesian coordinates. Resolution constant throughout domain.

Cartesian 3D PE computational method:

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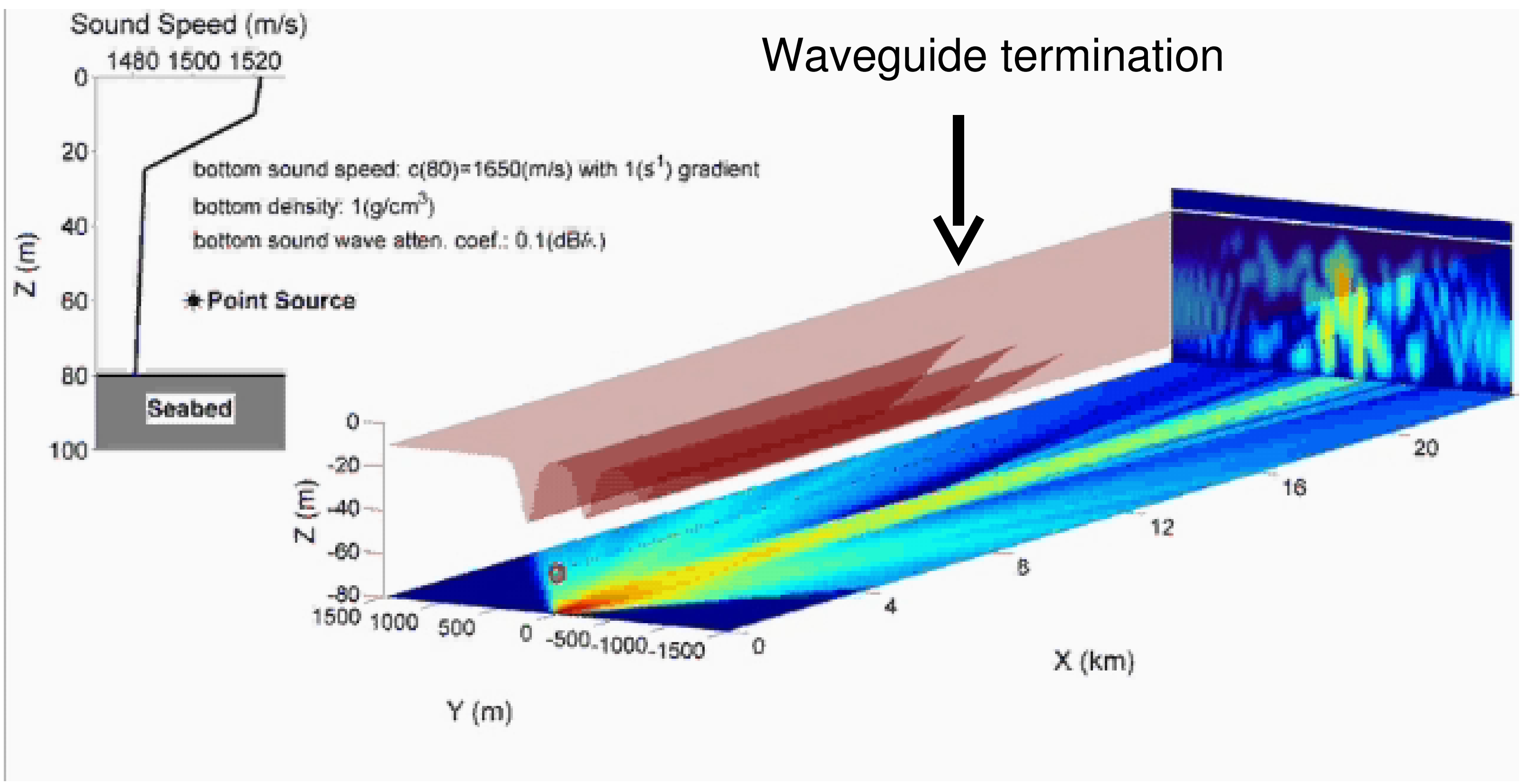
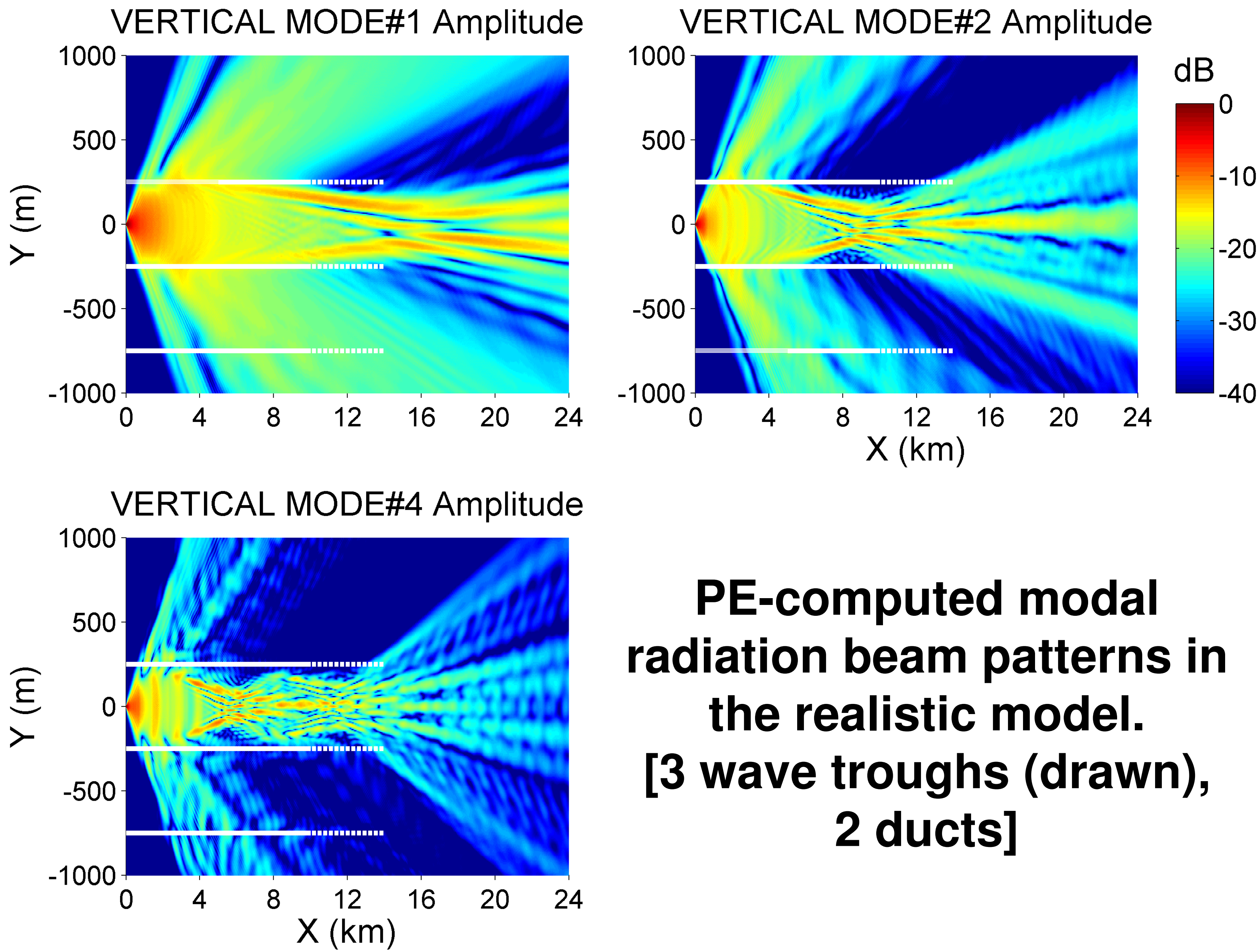
- More time-consuming to compute.

200Hz source @ 80m depth

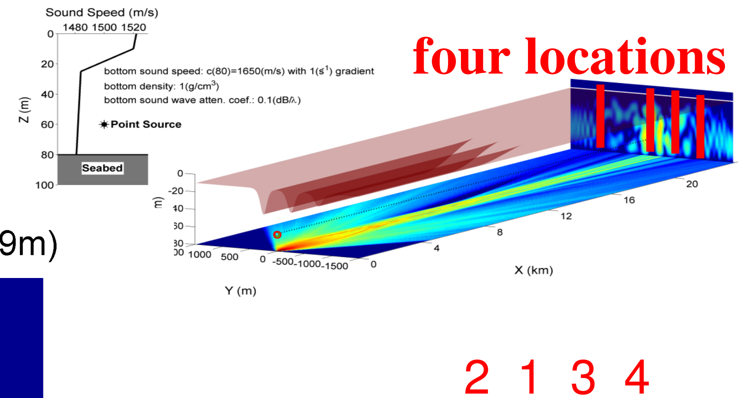


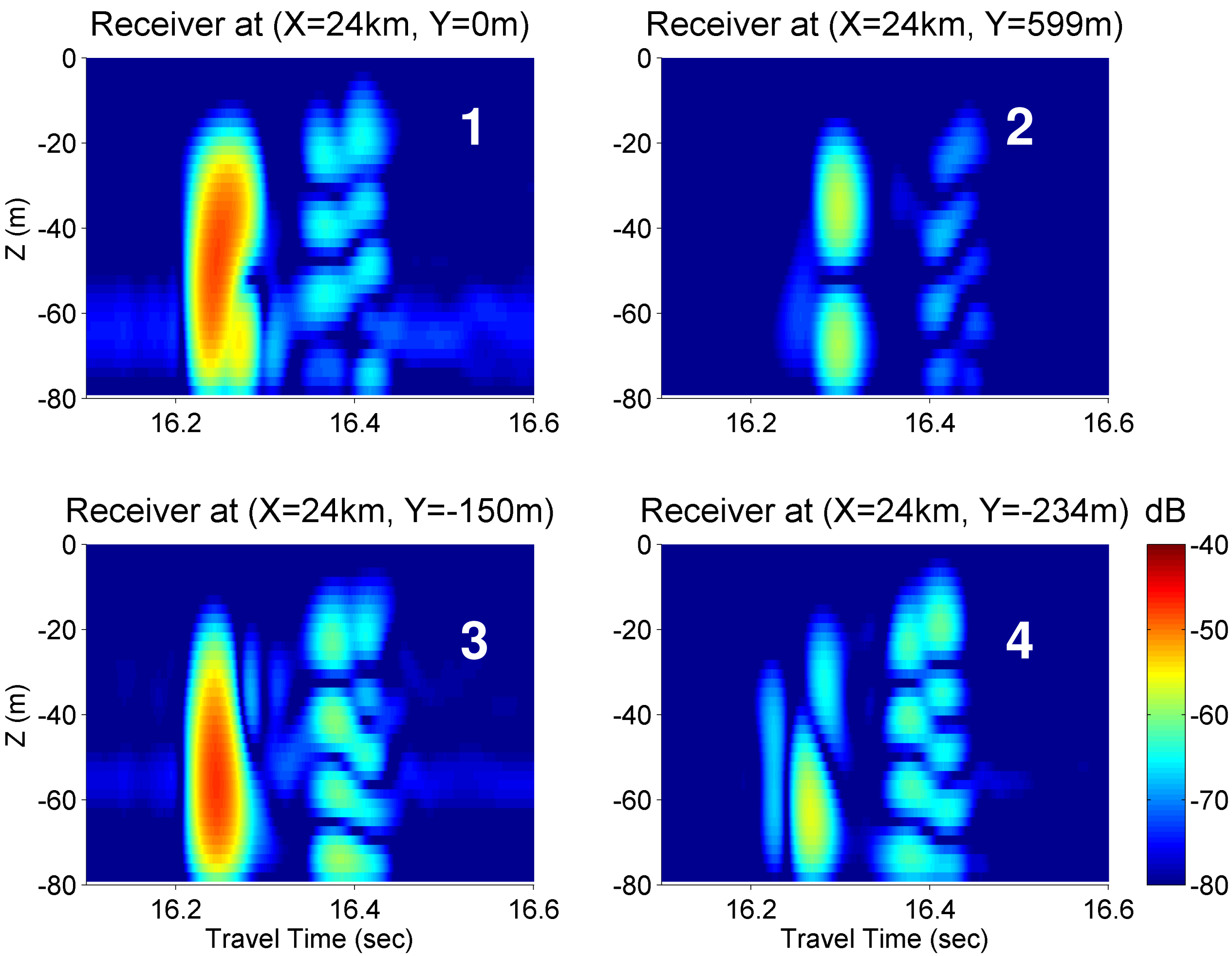


PE-computed modal radiation beam patterns in the realistic model. [3 wave troughs (drawn), 2 ducts]

Broadband source (PE Computational solution)

Central frequency 200Hz, bandwidth 50 Hz

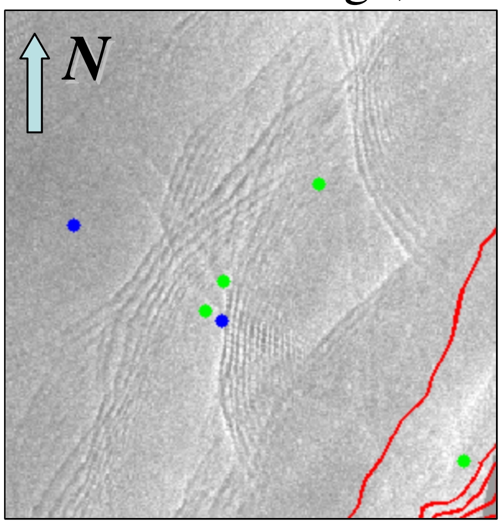


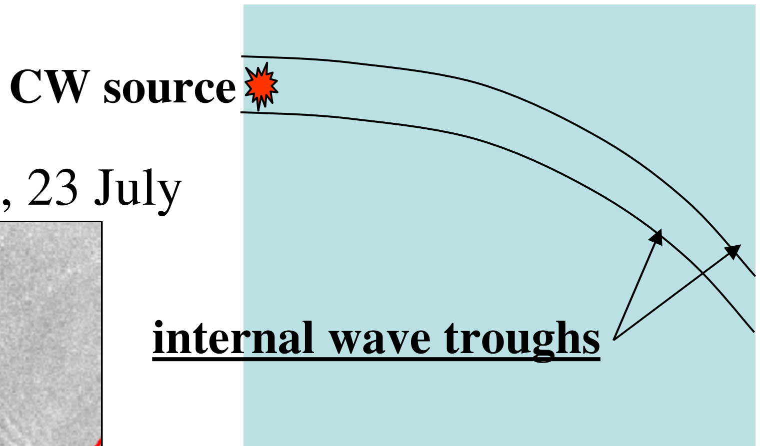


Acoustic mode focusing in a curved internal wave duct

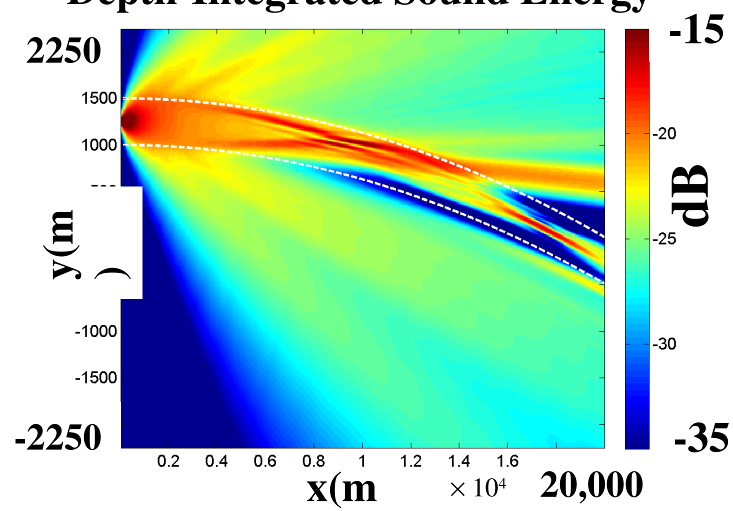
- Radius of curvature=135km, frequency = 100Hz
- Mode 1 penetrates through internal wave duct, but mode 2 focuses in the duct.

SW06 SAR image, 23 July

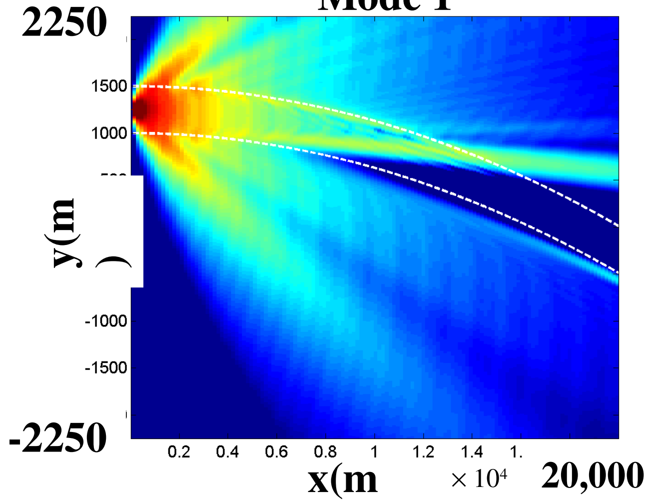




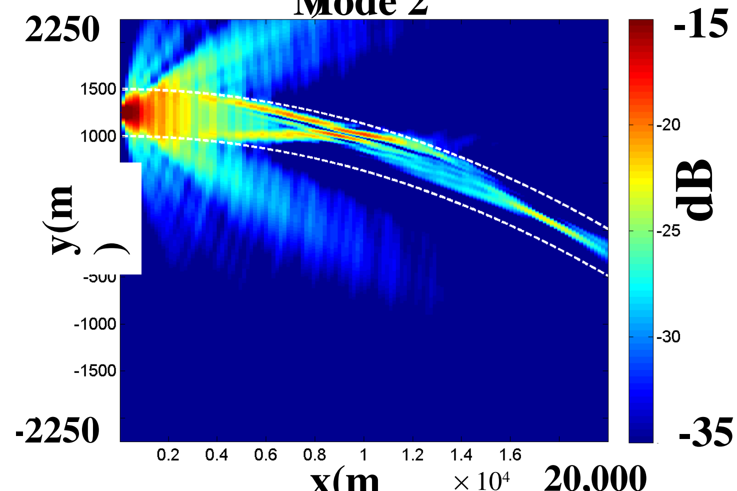
Depth-Integrated Sound Energy



Mode 1

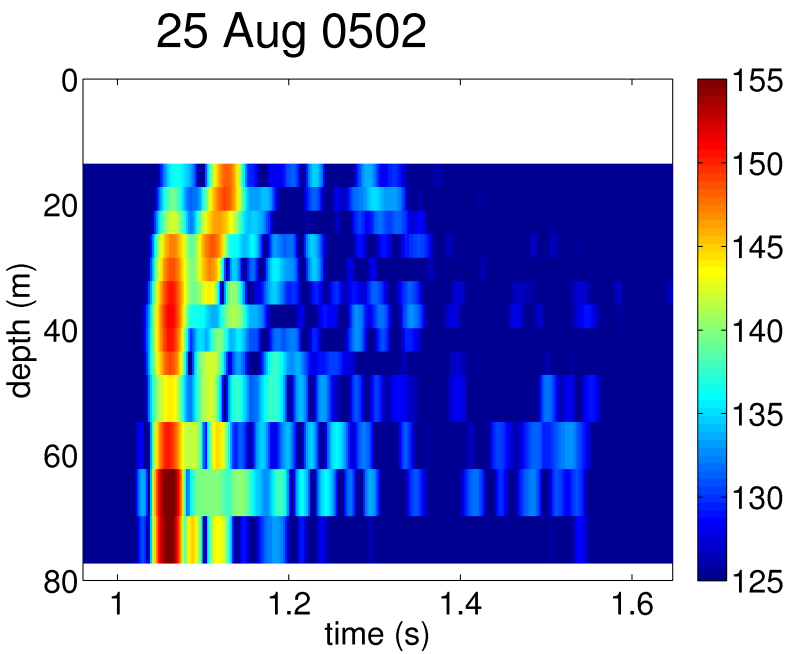


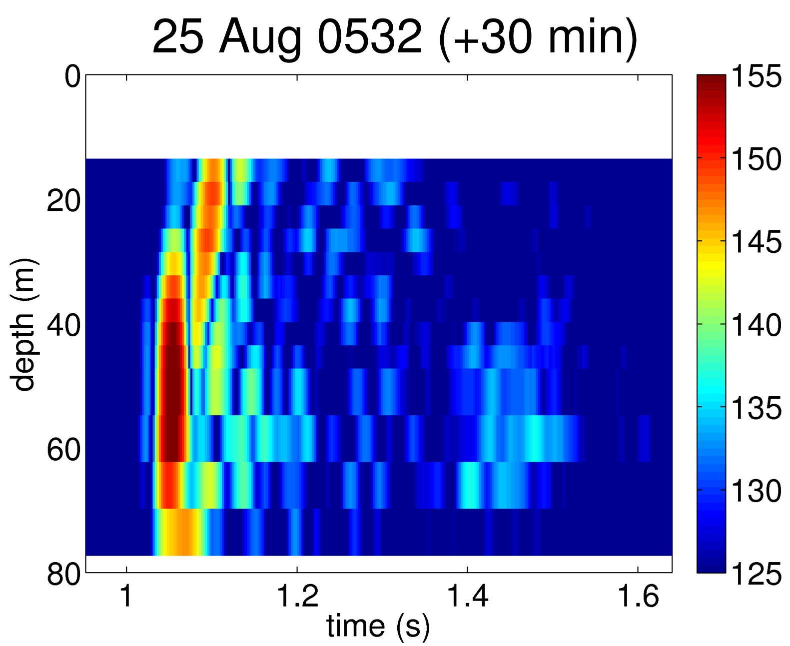
Mode 2

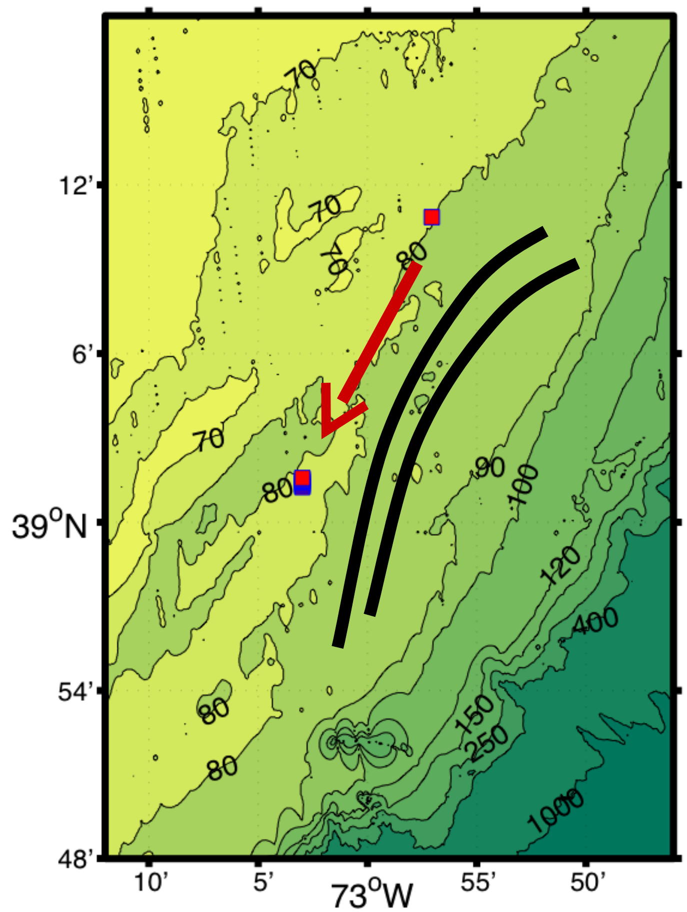


Example of highly variable mode content. SW06 experiment

200 Hz
19 km source to VLA
Prop. roughly along wave crests







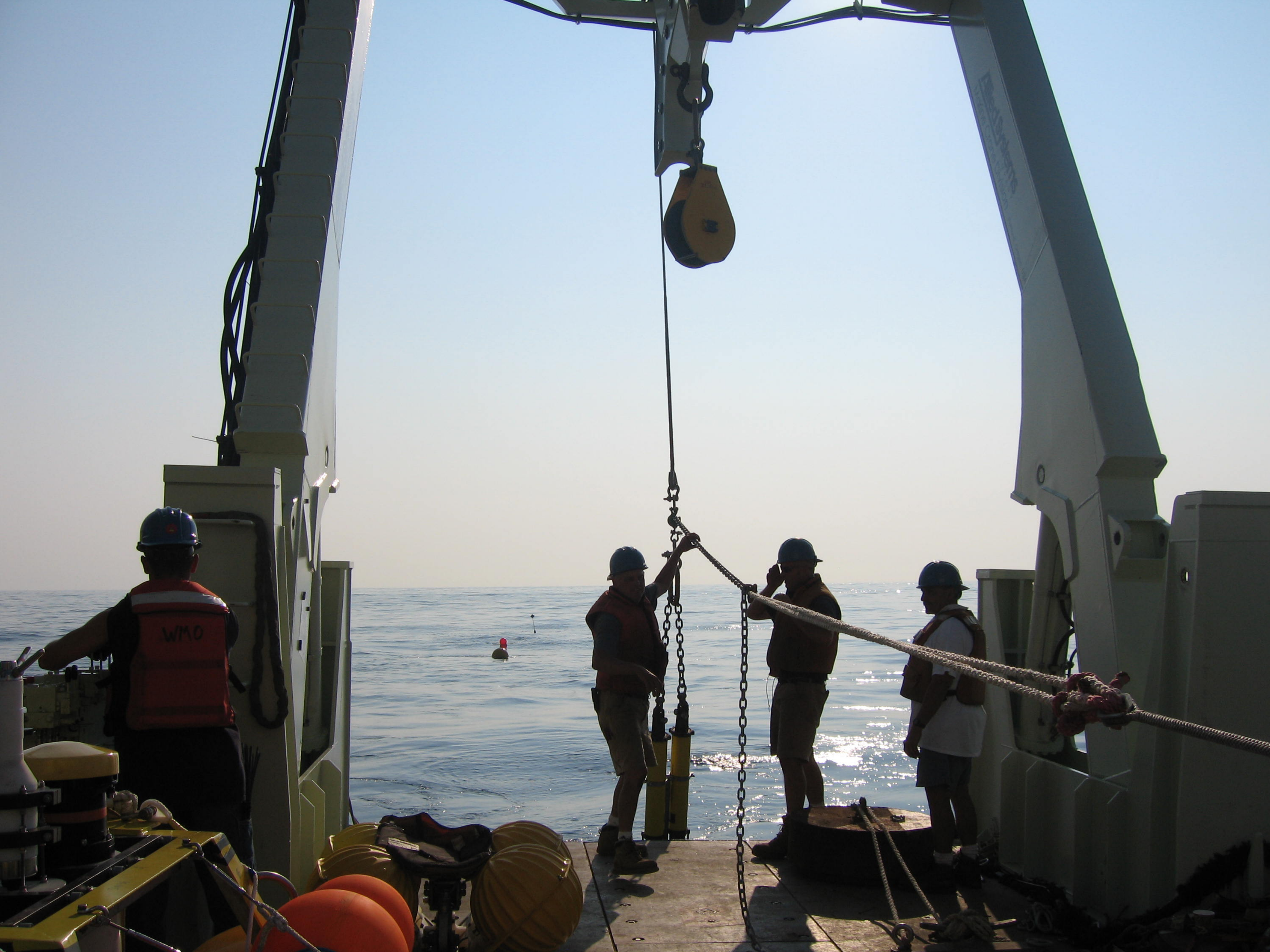
Conclusions

Terminating duct

1. Time-varying mode content observed in the SW06 experiment is consistent with time-varying modal radiation beam pattern.
2. Modes can temporarily disappear.
3. Field can have short horizontal coherence scales, in and out of the duct.
4. Time-dependence studied with semi-analytic model.
5. Computation ground-truths semi-analytic study, allows extension to more realistic wave geometry.

Curved-wave duct

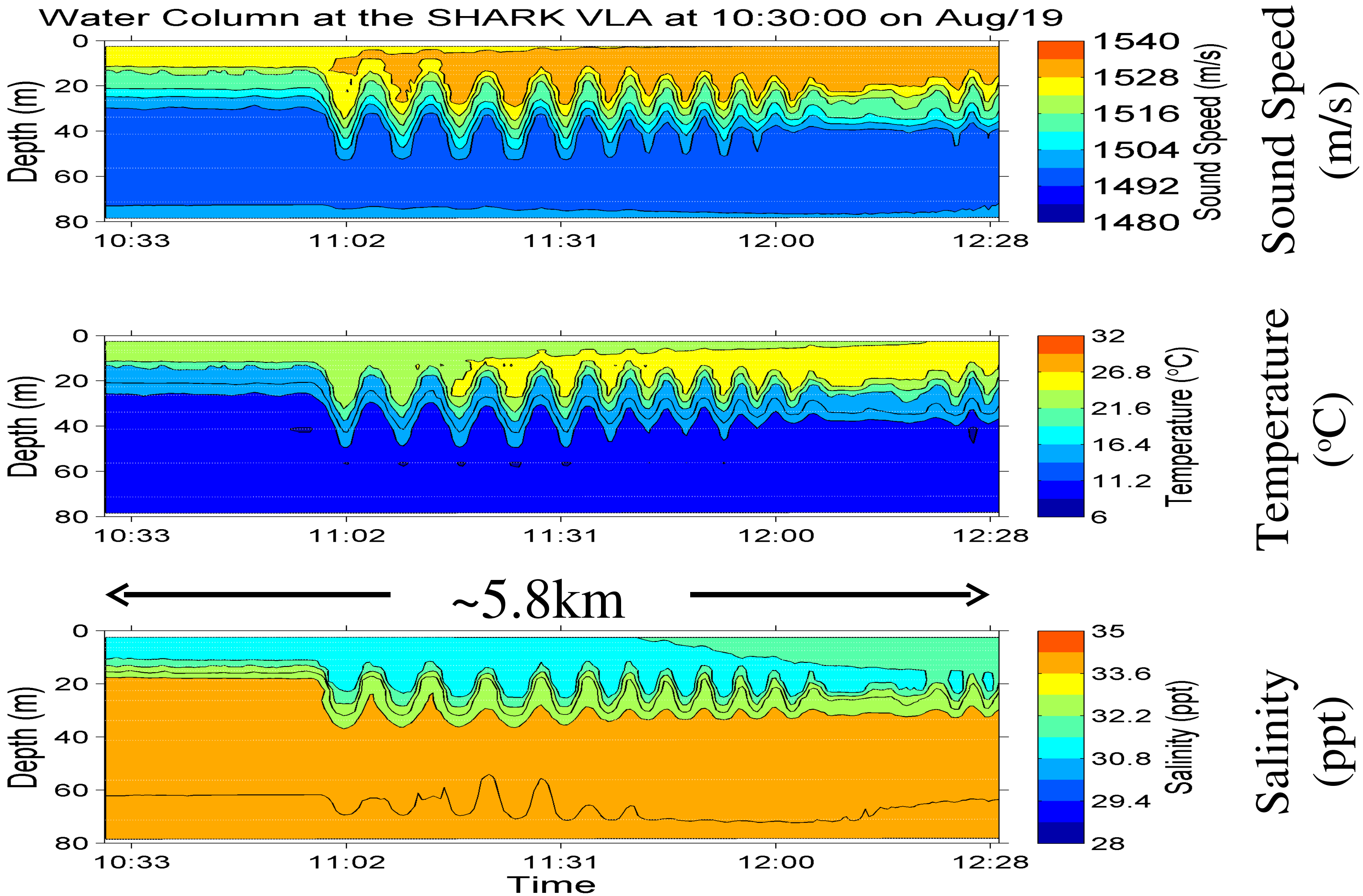
1. Mode beams can be found both inside and outside of a curved NLIW wave packet.


WMO

Backup Slides

Experimental Data

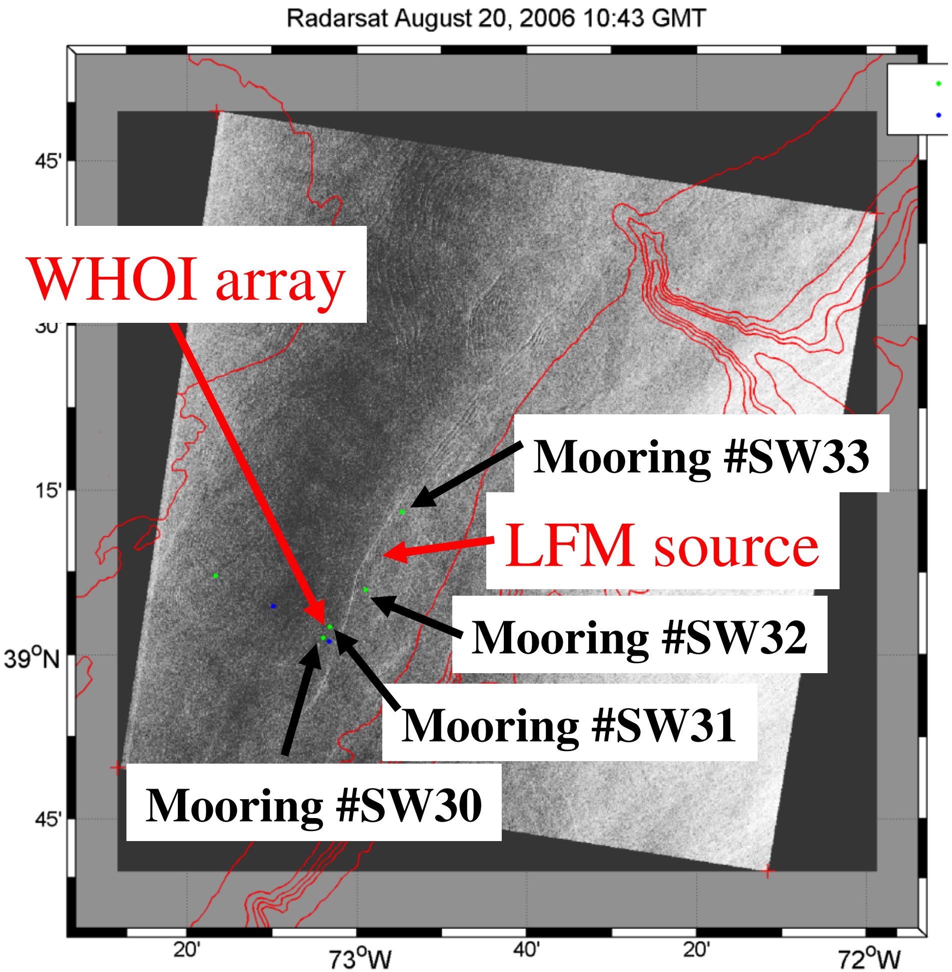
Time/space structure of train of nonlinear internal waves, SW06 program.

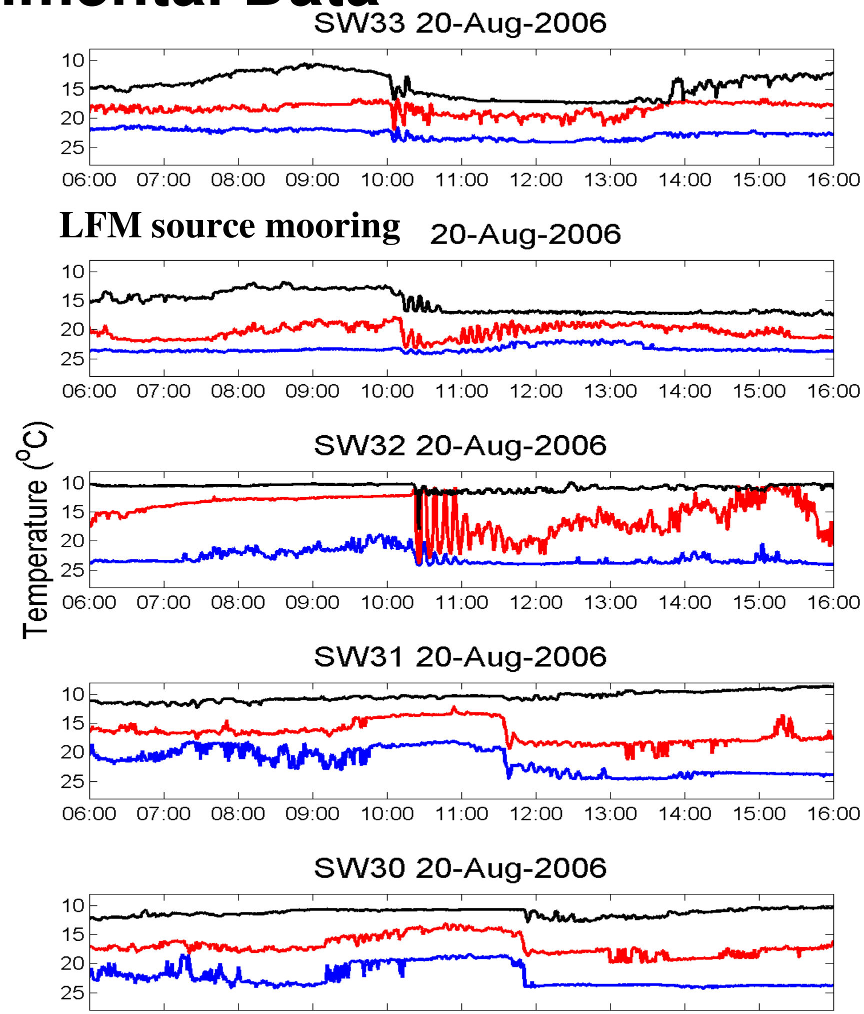


Effects of nonlinear internal waves on 3-D sound propagation

— SW06 Experimental Data

Nonlinear internal wave observation



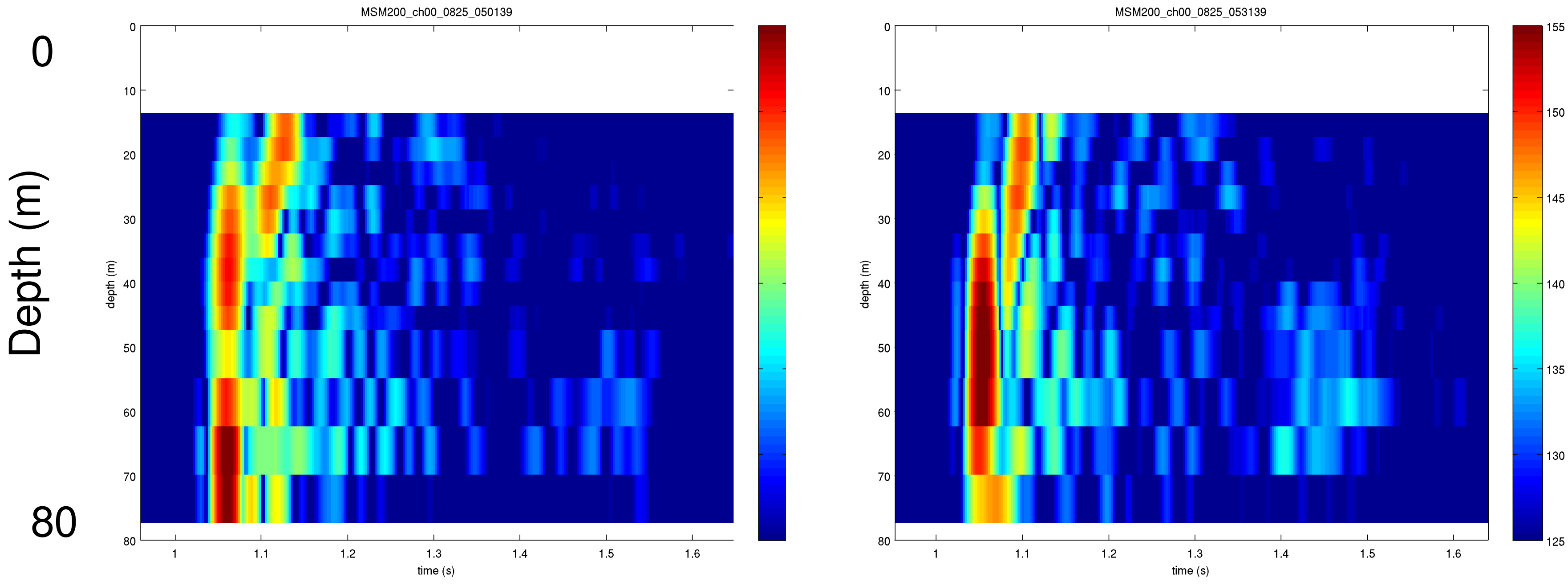


SW06 Temperature Data along the shelf
(15m depth: **blue**, 20m depth: **red**, 30m depth: **black**)

200 Hz, 19 km source to VLA, roughly along wave crests

25 Aug 2006 0501

25 Aug 2006 0531



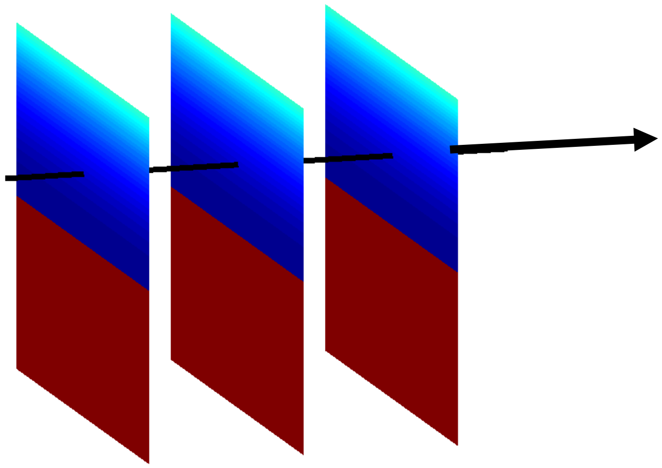
Time (~ 700 ms)

Investigation of the effects of nonlinear internal waves on 3-D sound propagation in shallow-water ocean

- **Theoretical analysis**: Solving the 3-D wave equation with the adiabatic mode assumption

Illustration!!

Three-Dimensional Computational solution



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$$\Psi(x + \delta) = \boldsymbol{F}^{-1} [G \cdot (\boldsymbol{F} [P \cdot \Psi(x)])]$$

$P = A_P \exp(-ik_0 U\delta)$	operator in the spatial domain
G	propagator in the wavenumber domain
F	Fourier transform operator (2D in this case)
$U = (c - c_0) / c_0$;	c_0 : reference sound speed
	c : sound speed,
	$k_0 = \omega / c_0$: wavenumber,
	ω : frequency
	δ : distance increment, x direction.