Three dimensional geoacoustic inversion on the New Jersey shelf

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Outline

- Methodology
  - Horizontal wavenumber estimation
  - Perturbative inversion
  - Qualitative Regularization
- The Shallow Water 2006 Experiment
  - Acoustic and oceanographic measurements
  - Inversion results
  - Three dimensional model for the environment
  - Validation of results: comparison to core data and ability to predict the acoustic field

Horizontal Wavenumber Estimation

The Hankel Transform Pair using the far field approximation

$$p(r;z,z_0)=\frac{e^{-i\pi/4}}{\sqrt{2\pi r}}\int_{-\infty}^{\infty} g(k_r;z,z_0)\sqrt{k_r}\,e^{ik_r r}\,dk_r$$

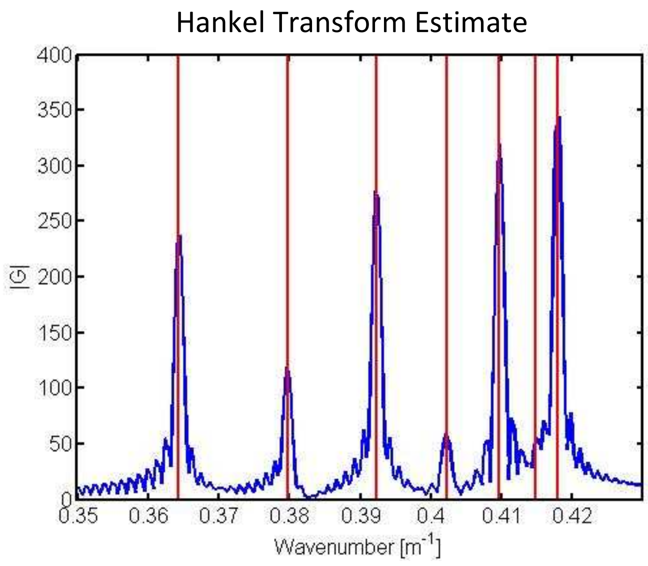
$$g(k_r;z,z_0)=\frac{e^{i\pi/4}}{\sqrt{2\pi k_r}}\int_{-\infty}^{\infty} p(r;z,z_0)\sqrt{r}\,e^{-ik_r r}\,dr$$

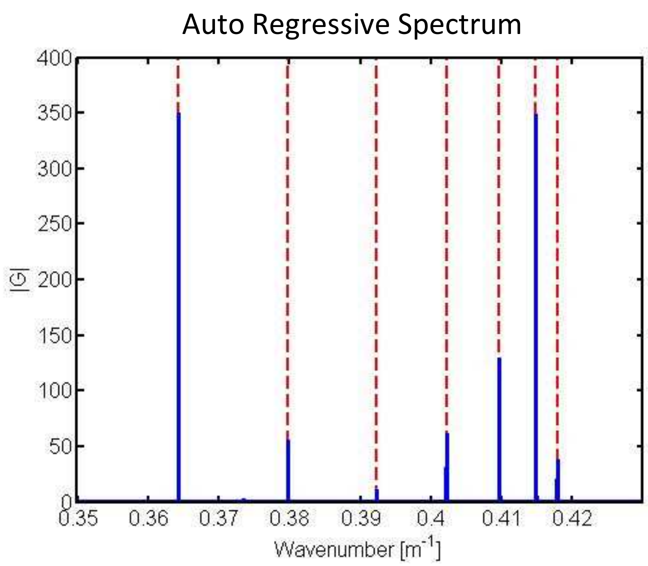
The Short-Time Fourier Transform (STFT)

$$g(k_r;\hat{r},z,z_0)=\frac{e^{i\pi/4}}{\sqrt{2\pi k_r}}\int_{-\infty}^{\infty} w_L(r;\hat{r})\,p(r;z,z_0)\sqrt{r}\,e^{-ik_r r}\,dr$$

Auto Regression (AR)

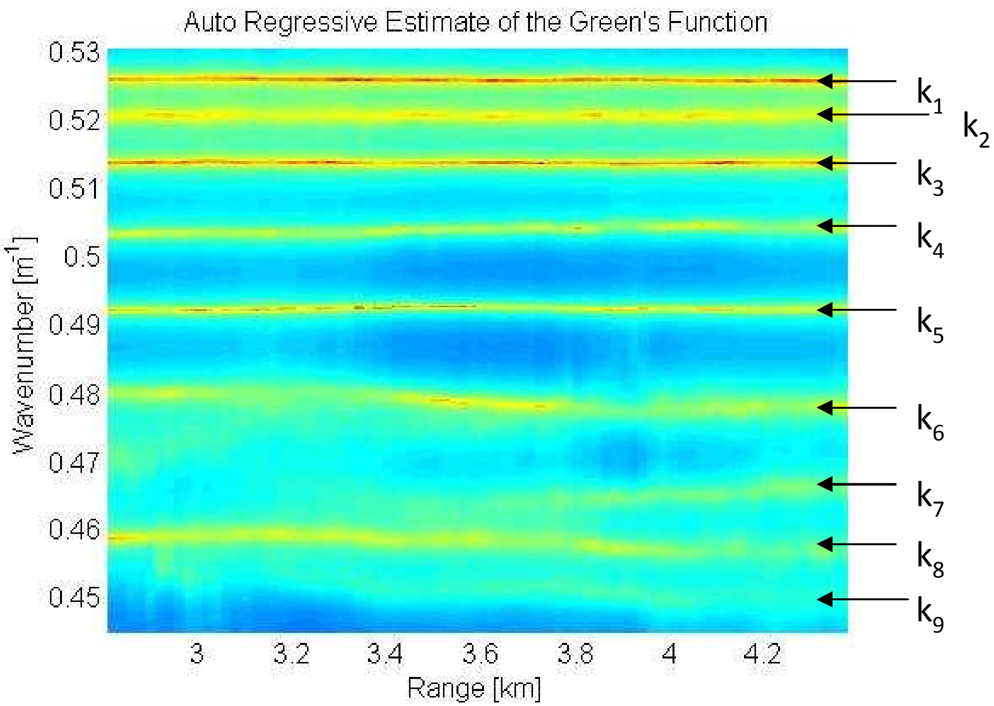
$$x_n=\sum_{k=1}^{p} a_k x_{n-k} \longrightarrow P_{AR}=\frac{\sigma^2 T}{\left|1+\sum_{k=1}^{p} a_k e^{-i2\pi fkT}\right|^2}$$

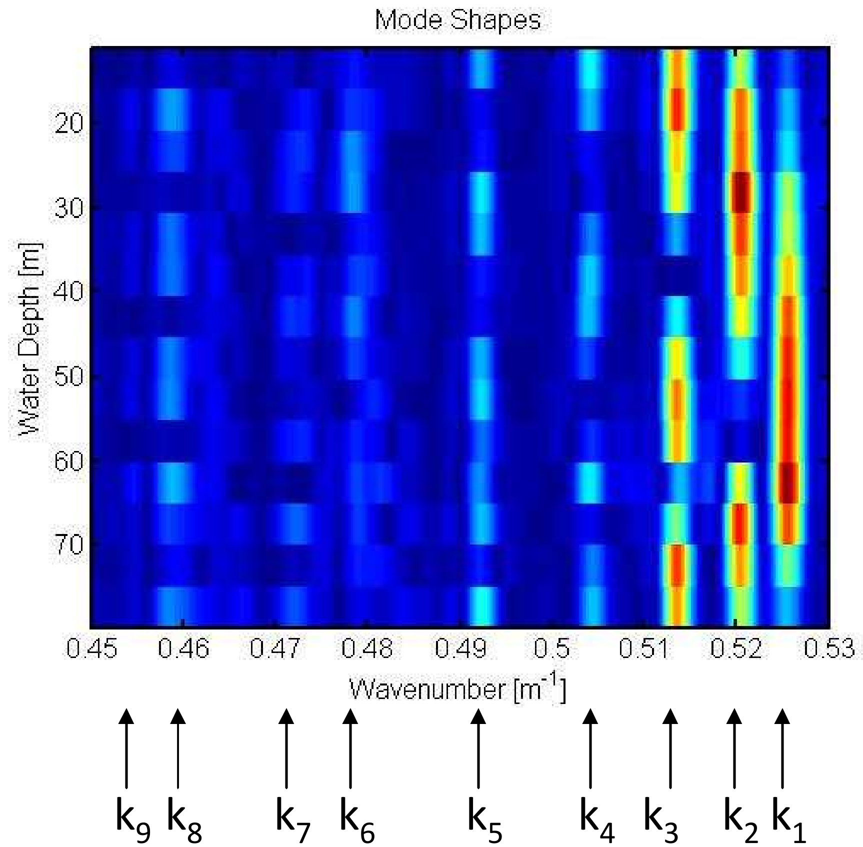




Horizontal Wavenumber Estimation

Auto regressive (AR) techniques were used to estimate wave numbers from pressure field data. Mode shapes aided in identifying modes. Across shelf data for 125 Hz shown here.





Window length of the AR estimator is 2000m.

Perturbative Inversion

A relation between a perturbation to sound speed in sediment and a perturbation to horizontal wavenumbers is formulated from the depth separated normal mode equation:

$$\Delta k_n = \frac{1}{k_n} \int_0^{\infty} \rho^{-1}(z) Z_n^2(z) k^2(z) \frac{\Delta c(z)}{c_0(z)} dz$$

This equation can be written in the form of a Fredholm integral of the first kind:

$$y_i = \int_0^{D} x(z) A_i(z) dz \qquad i = 1, \ldots, N$$

Which can be written in matrix form as: $\mathbf{y} = \mathbf{A}\mathbf{x}$

$\mathbf{y}$ is a vector representing the data

$\mathbf{A}$ is a matrix representing the forward model

$\mathbf{x}$ is a vector representing the model parameter

Tikhonov Regularization

Solve the ill-conditioned problem: $\mathbf{y} = \mathbf{A}\mathbf{x}$ by choosing the smoothest solution.

$$\min \left\| \mathbf{A}\mathbf{x} - \mathbf{y} \right\|_2^2 + \lambda^2 \left\| \mathbf{L}\mathbf{x} \right\|_2^2$$

where $\mathbf{L}$ is a discrete version of the differential operator $\dfrac{d^n}{d\mathbf{x}^n}$

$n = 1$ favors the flattest solution;

$n = 2$ favors the smoothest solution.

L-Curve Criterion: The Lagrange multiplier λ is chosen such that it both the residual $\left\| \mathbf{A}\mathbf{x} - \mathbf{y} \right\|_2$ and the semi-norm $\left\| \mathbf{L}\mathbf{x} \right\|_2$ are minimized simultaneously.

The Regularization solution is given by:

$$\boxed{\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{A} + \lambda^2 \mathbf{L}^T\mathbf{L})^{-1}\mathbf{A}^T\mathbf{y}}$$

Qualitative Regularization

Solve the ill-conditioned problem: $\mathbf{y} = \mathbf{A}\mathbf{x}$ by choosing the solution that best fits some prior knowledge.

$$\min \left\| \mathbf{A}\mathbf{x} - \mathbf{y} \right\|_2^2 + \lambda^2 \left\| \mathbf{L}_q \mathbf{x} \right\|_2^2$$

where $\mathbf{L}_q$ is created by the user.

$\mathbf{L}_q$ is given by: $\mathbf{L}_q = \mathbf{L}(I - \sum_{i=1}^{r} \mathbf{q}_i \mathbf{q}_i^T)$

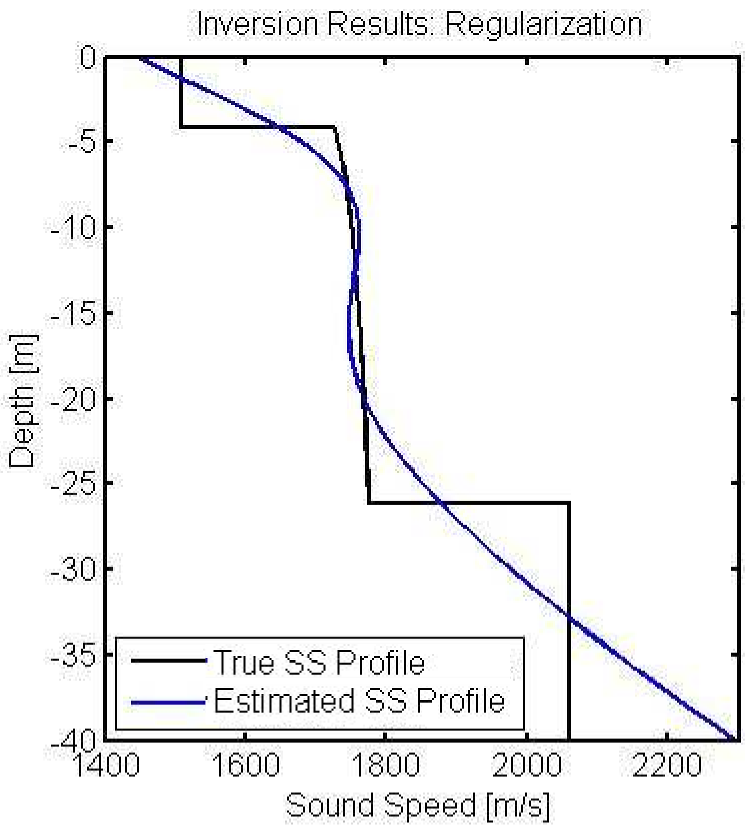
where the set $\{\mathbf{q}_i\}_{i=1}^{r}$ is on orthogonal basis for $\mathbf{Q}$.

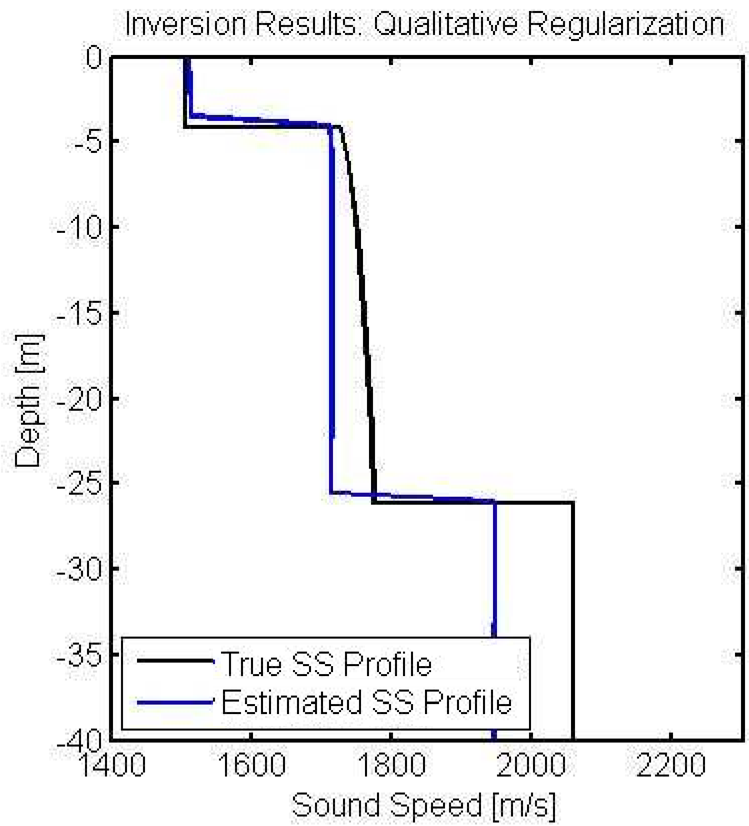
The Qualitative Regularization solution is given by:

$$\boxed{\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{A} + \lambda^2 \mathbf{L}_q{}^T \mathbf{L}_q)^{-1} \mathbf{A}^T \mathbf{y}}$$

Comparison: Tikhonov and QR

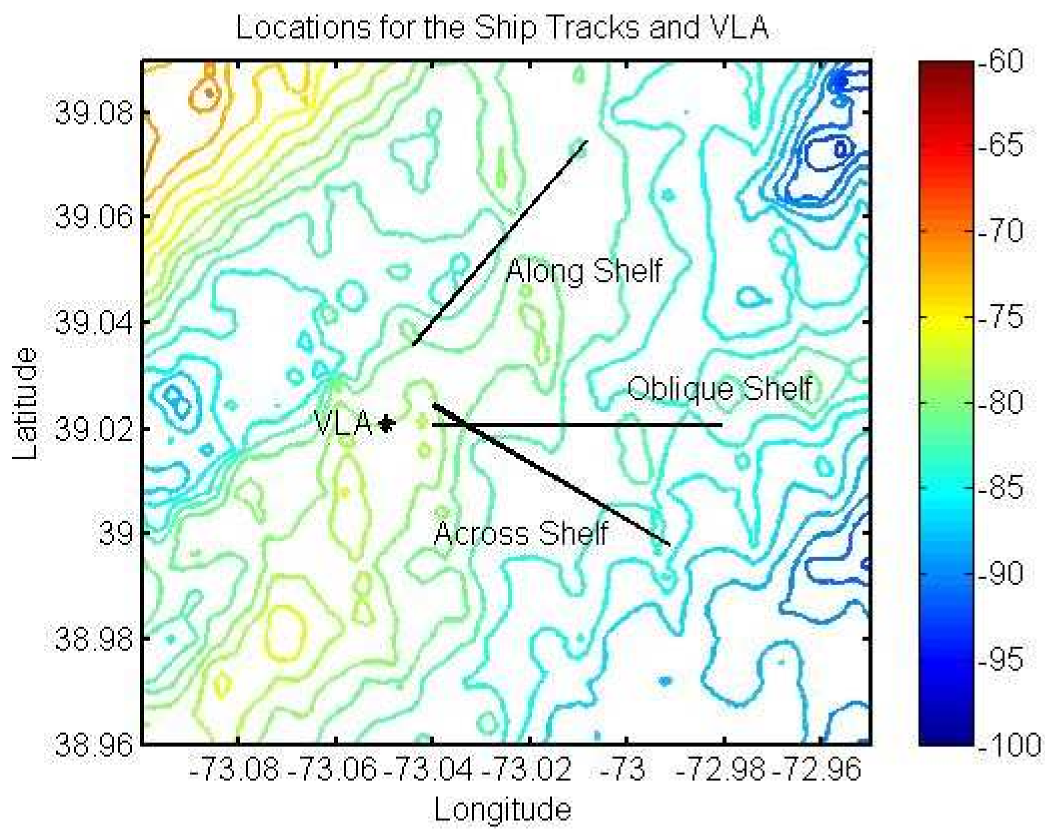
More accurate estimation of the bottom parameters can be made by allowing the solution to be a layered medium instead of a smooth profile.





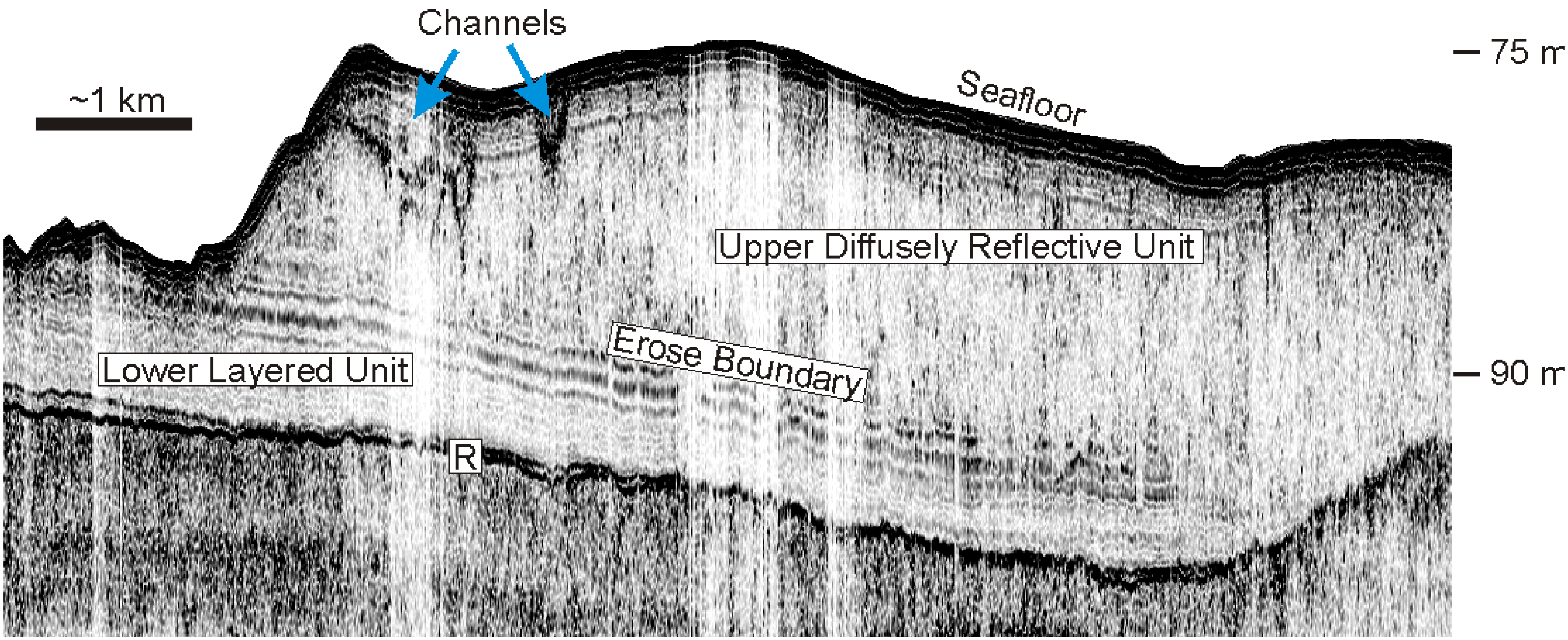
Ship Tracks

Ship tracks oriented along, across, and oblique to the shelf break on radials with respect to the Shark VLA. All ship tracks are about 5km long.



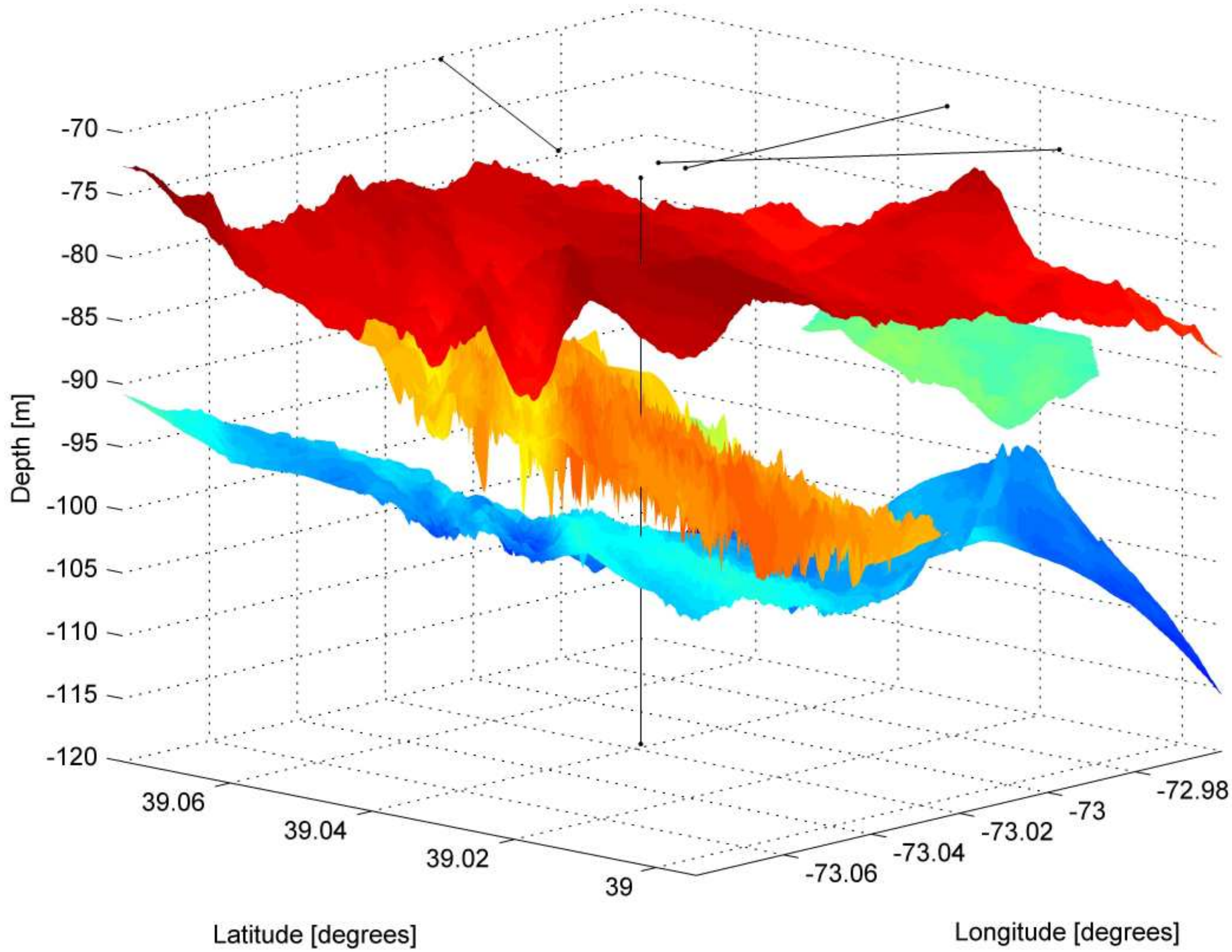
The New Jersey Shelf

chirp seismic reflection data

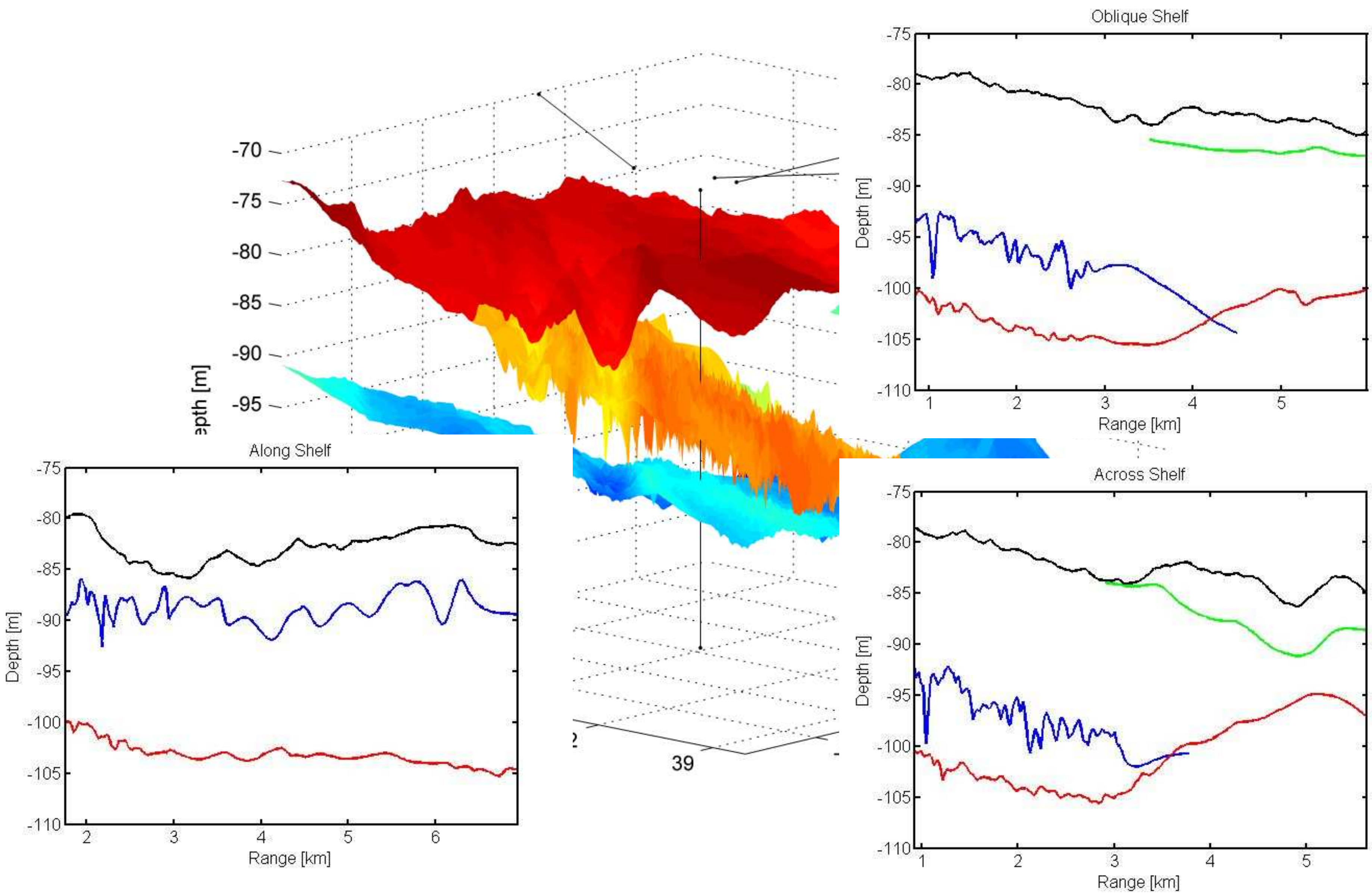


Chirp data provided by John Goff

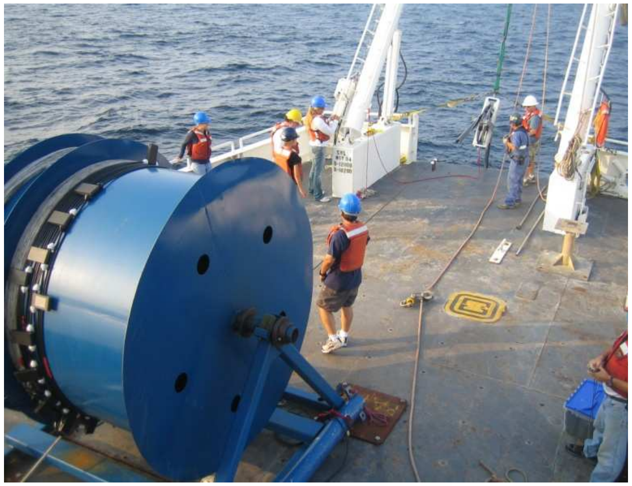
Layering Information from Chirp Data

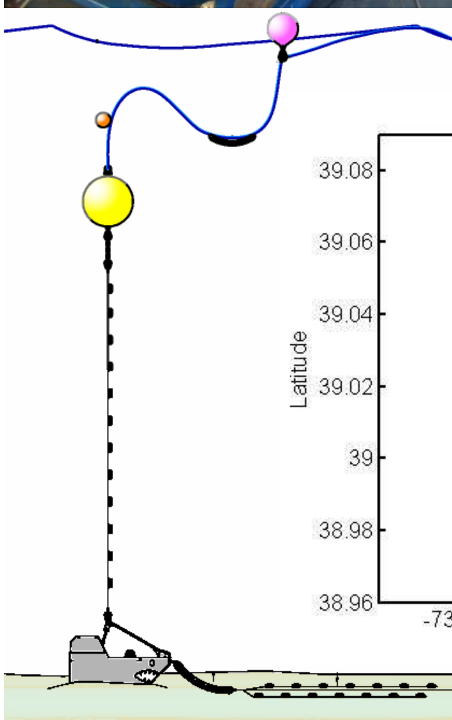


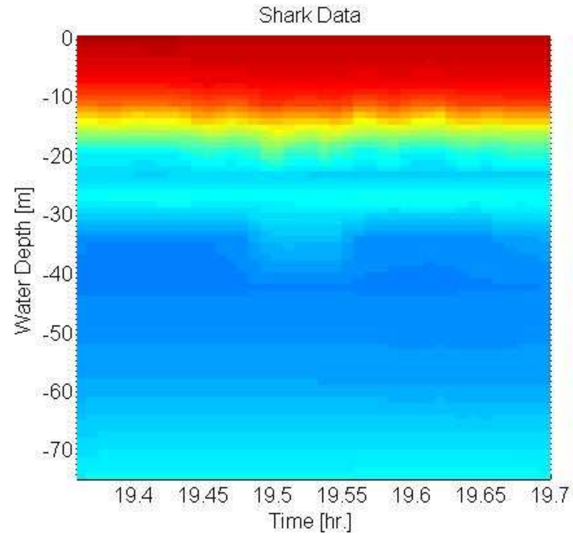
Layering Information from Chirp Data

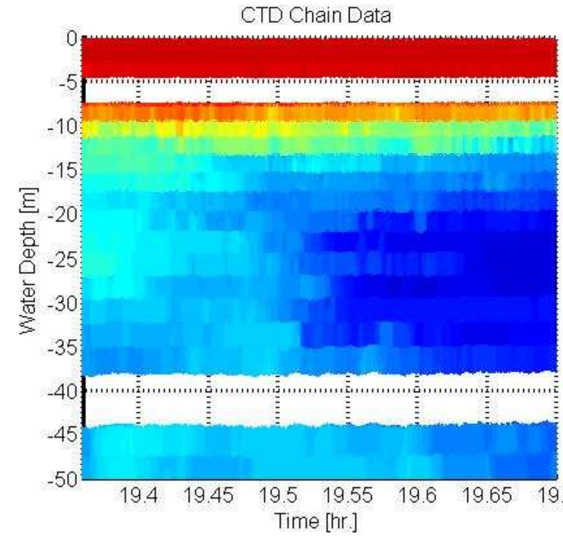


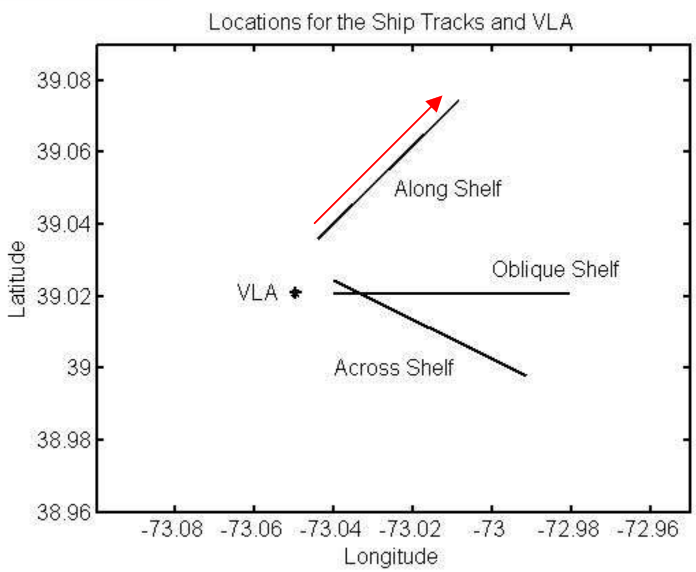
Water Column Sound Speed

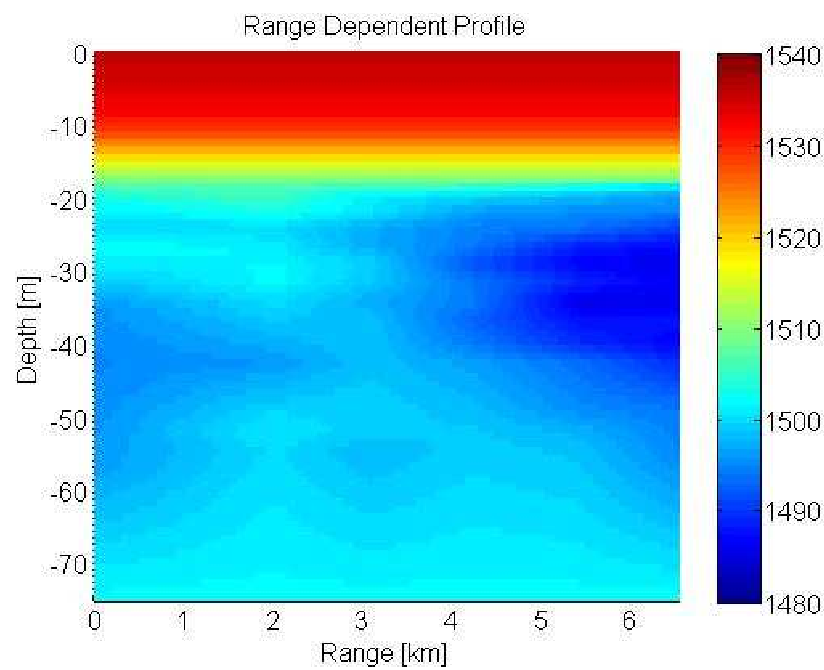






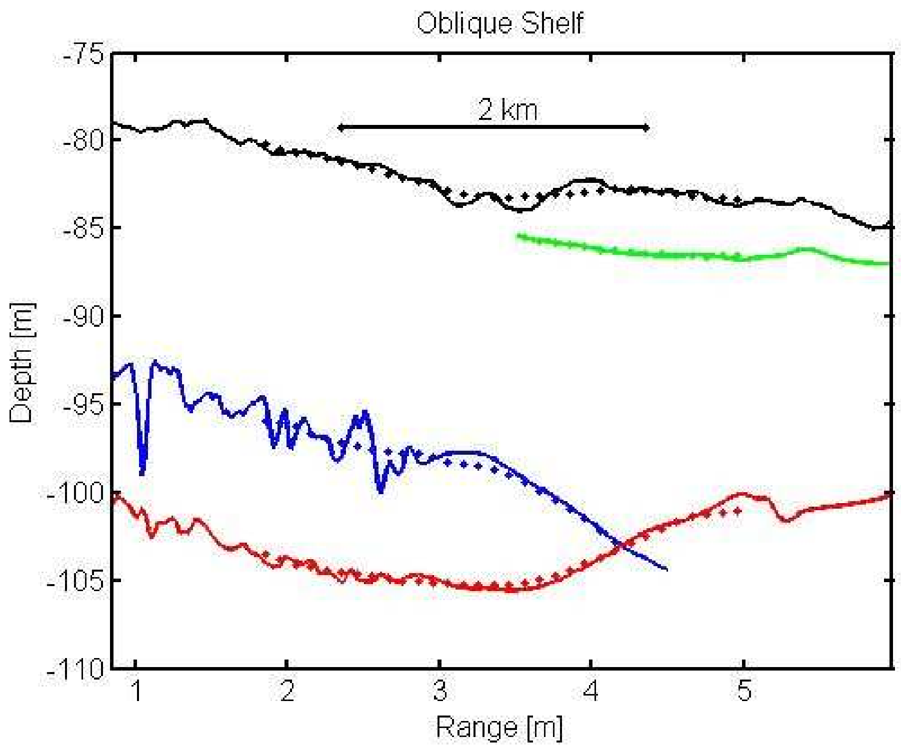


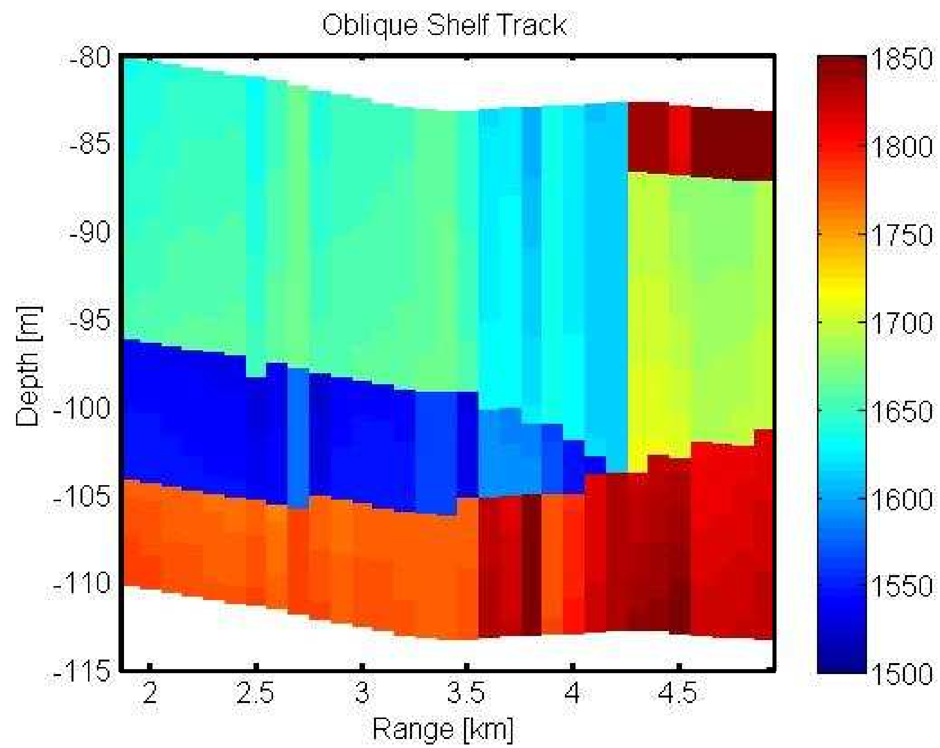




Inversion Results: Oblique Shelf Track

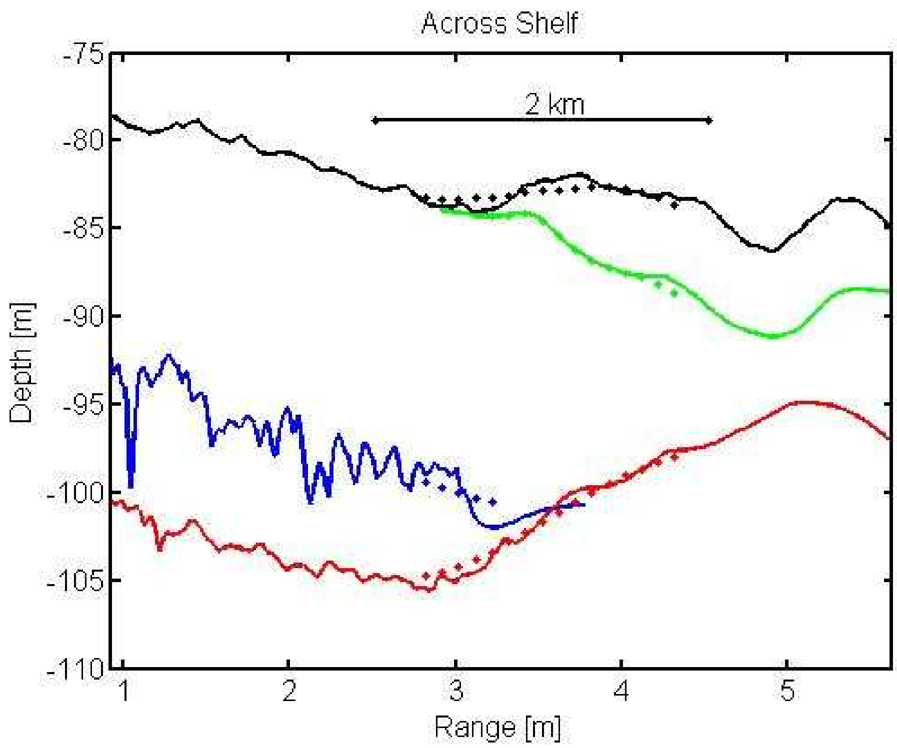
Input data to the inversion scheme: wavenumber estimates from 125 and 175 Hz data. Over lapping regions are inconsistent due to noise on the wavenumber estimates.

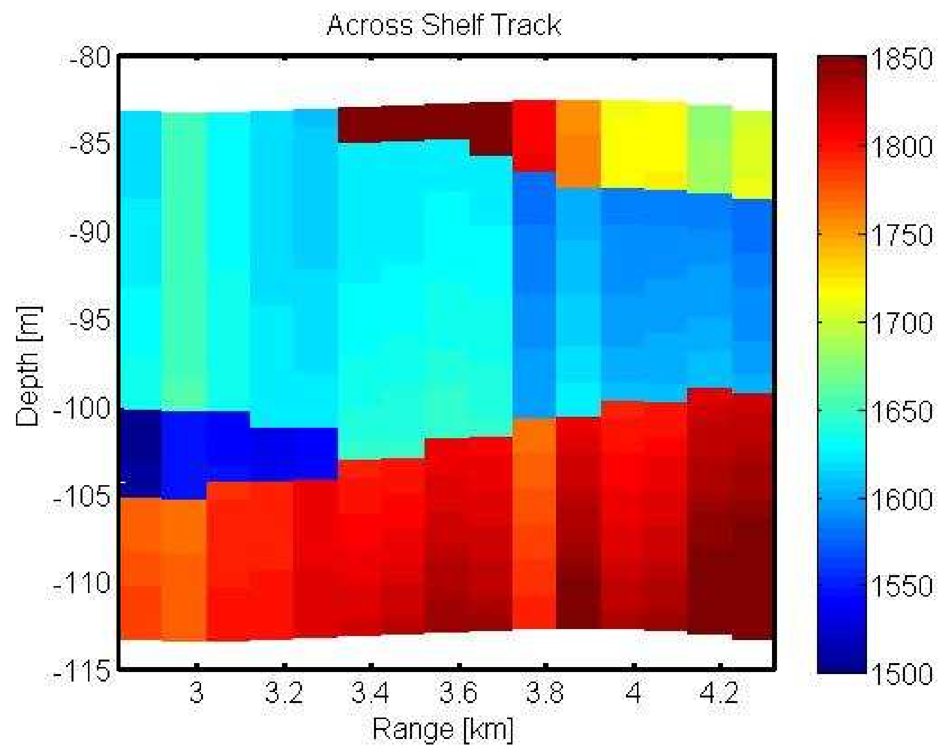




Inversion Results: Across Shelf Track

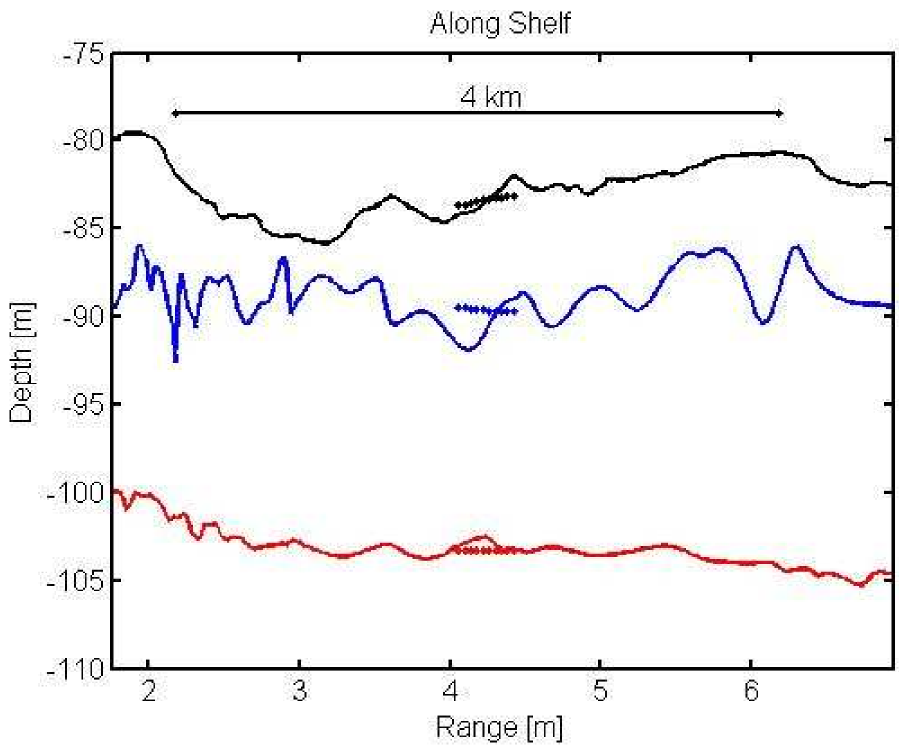
Wavenumber estimates could not be obtained for ranges less than 1.5 km because ship speed could not be approximated as constant along a radial with respect to the VLA.

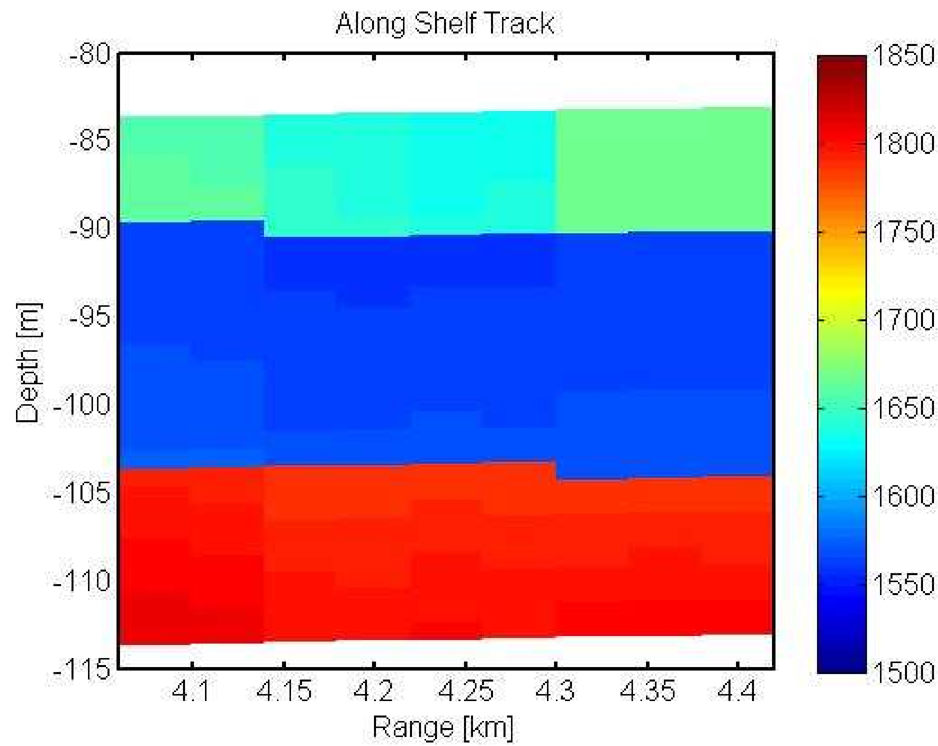




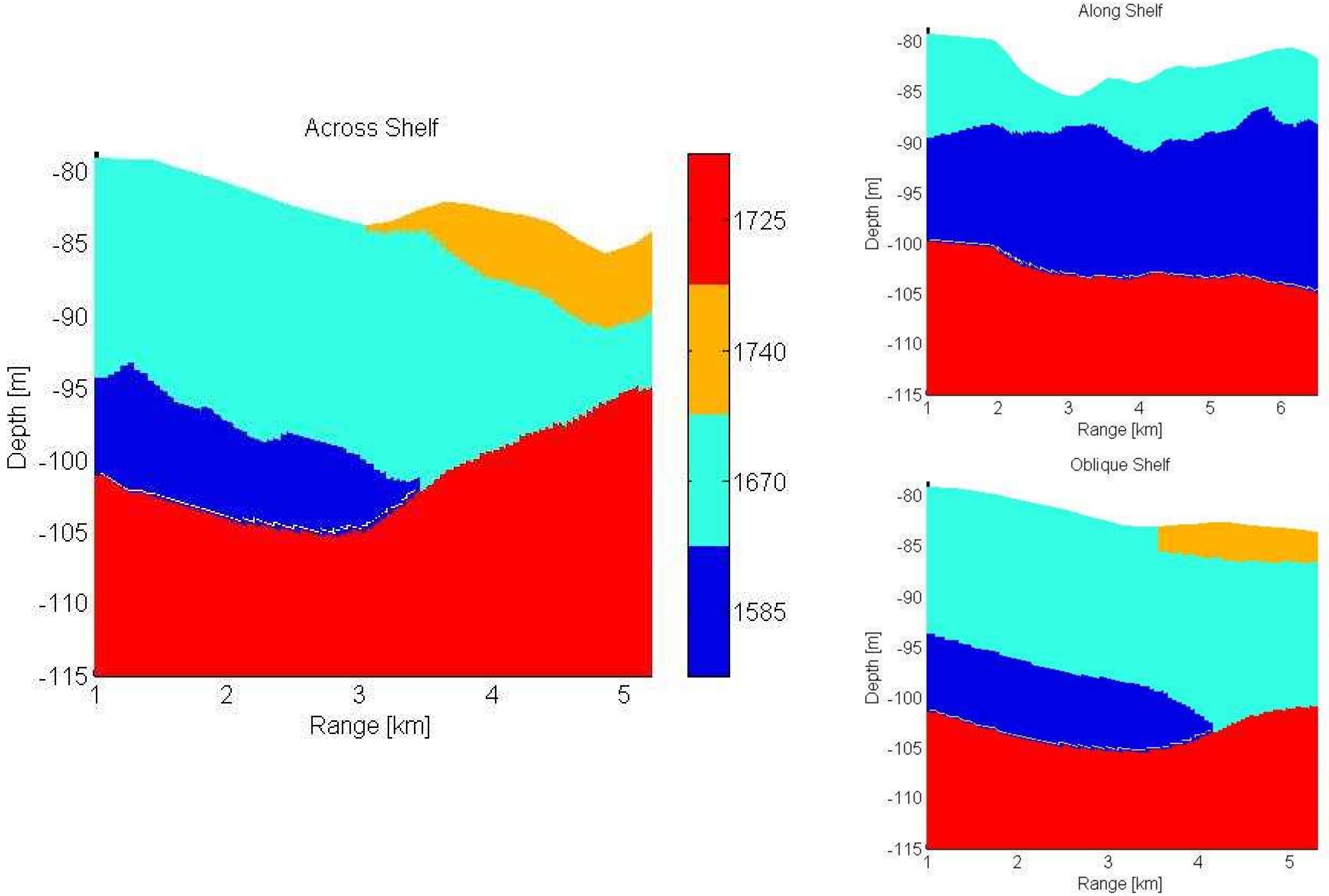
Inversion Results: Along Shelf Track

Longer apertures were required for wavenumber estimation to account for closely spaced wavenumbers and average over the effects of the range dependent water column sound speed profile.





Simple Model

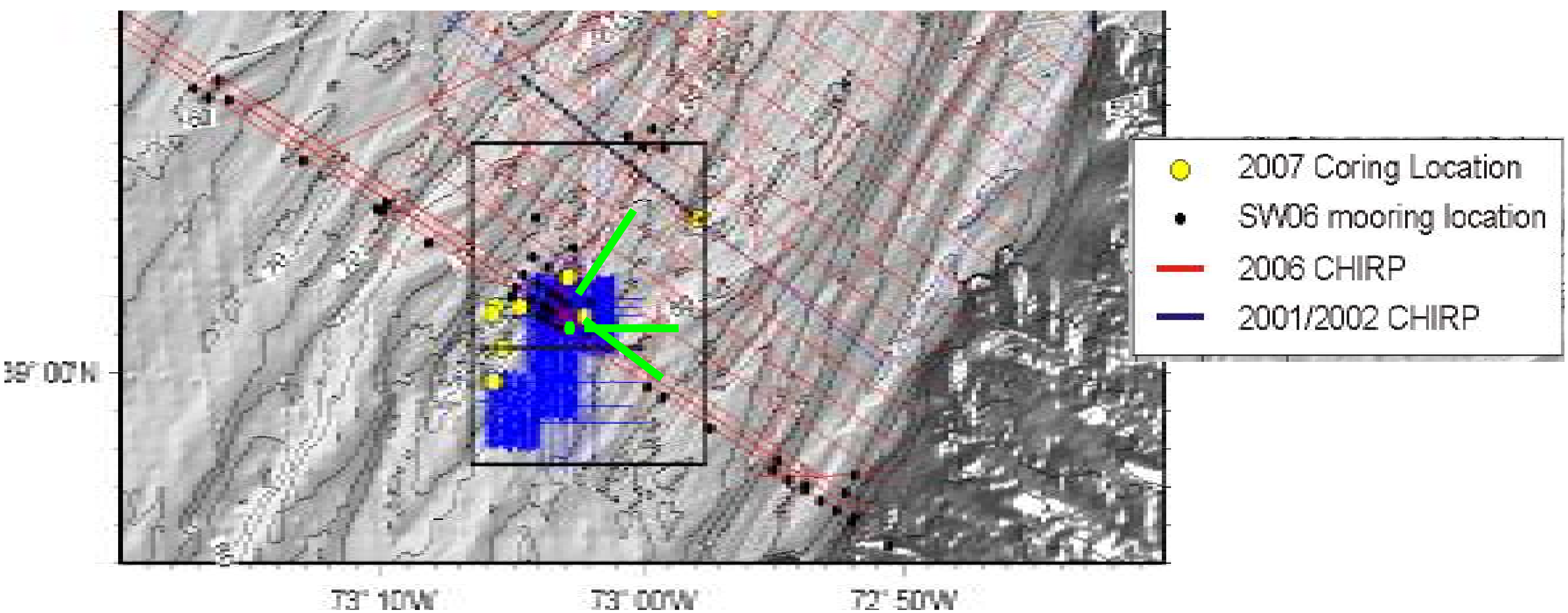


Model Agreement: SW06 Cores

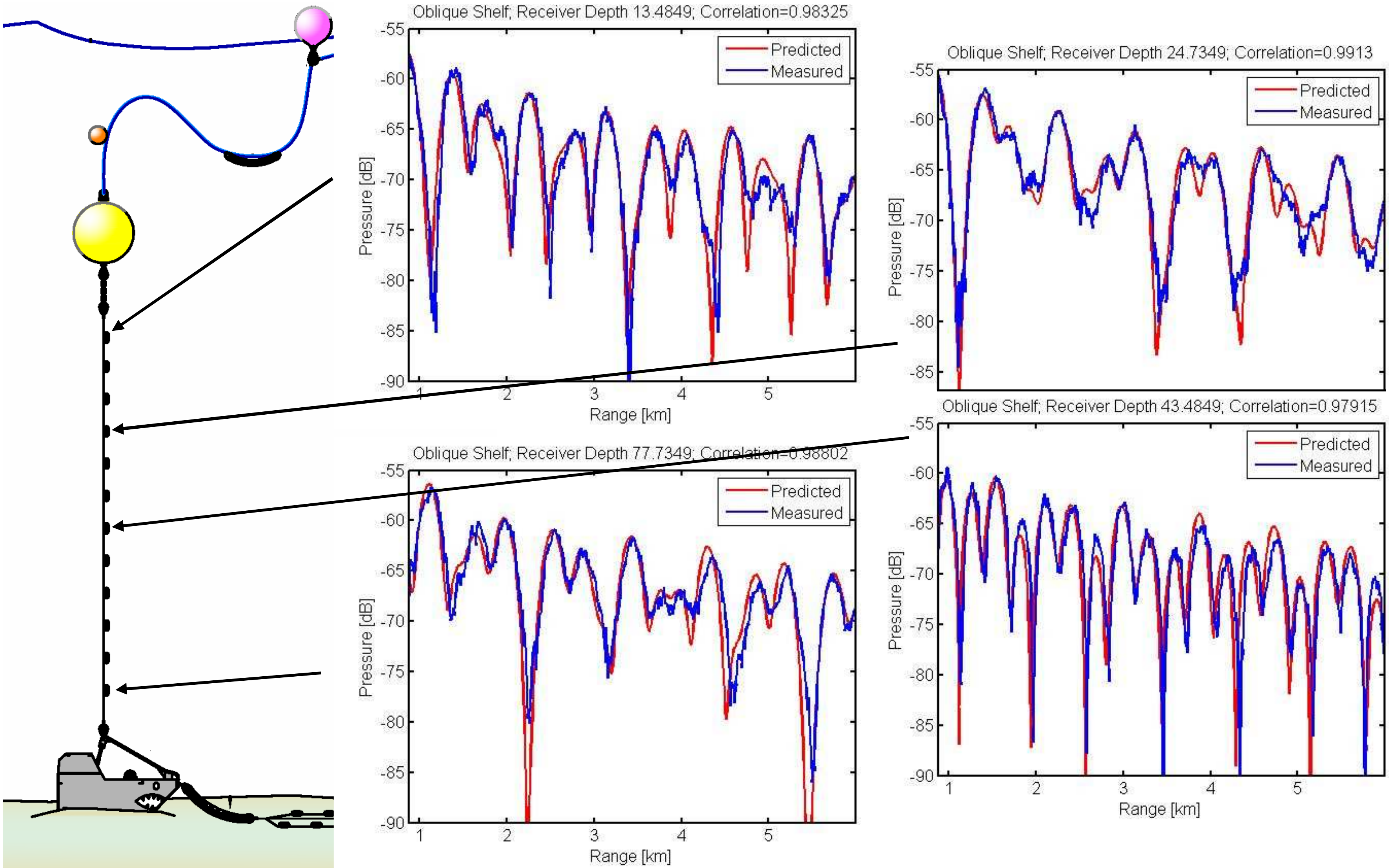
Upper Unit: 1639m/s, 1624m/s, 1657m/s

Lower Unit: 1554m/s, 1652m/s (single values)

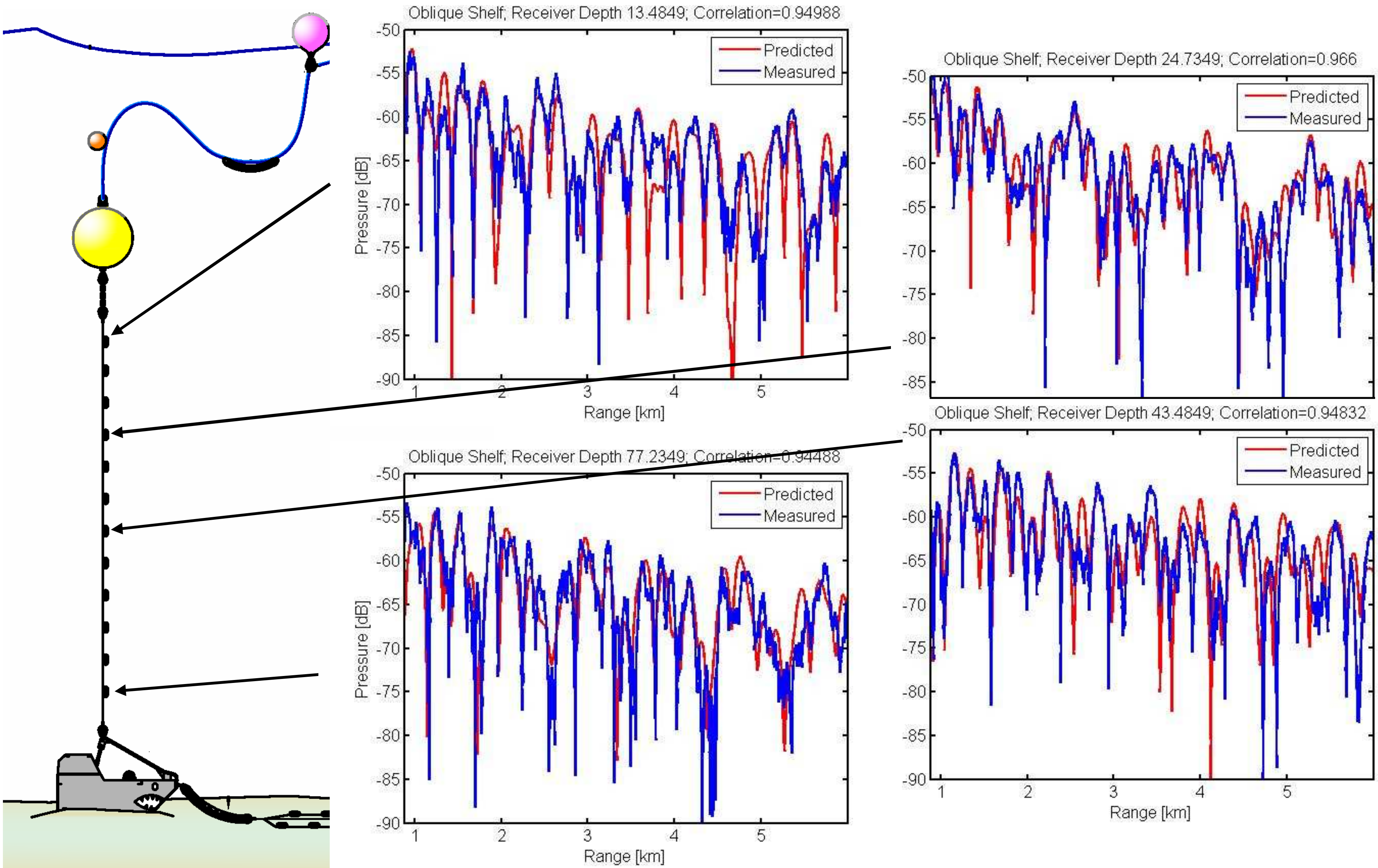
Below R: 1850m/s (Upper range of very erratic measurements)



Evaluation of Results: TL Prediction 50 Hz



Evaluation of Results: TL Prediction 125 Hz



Evaluation of Results: Correlation

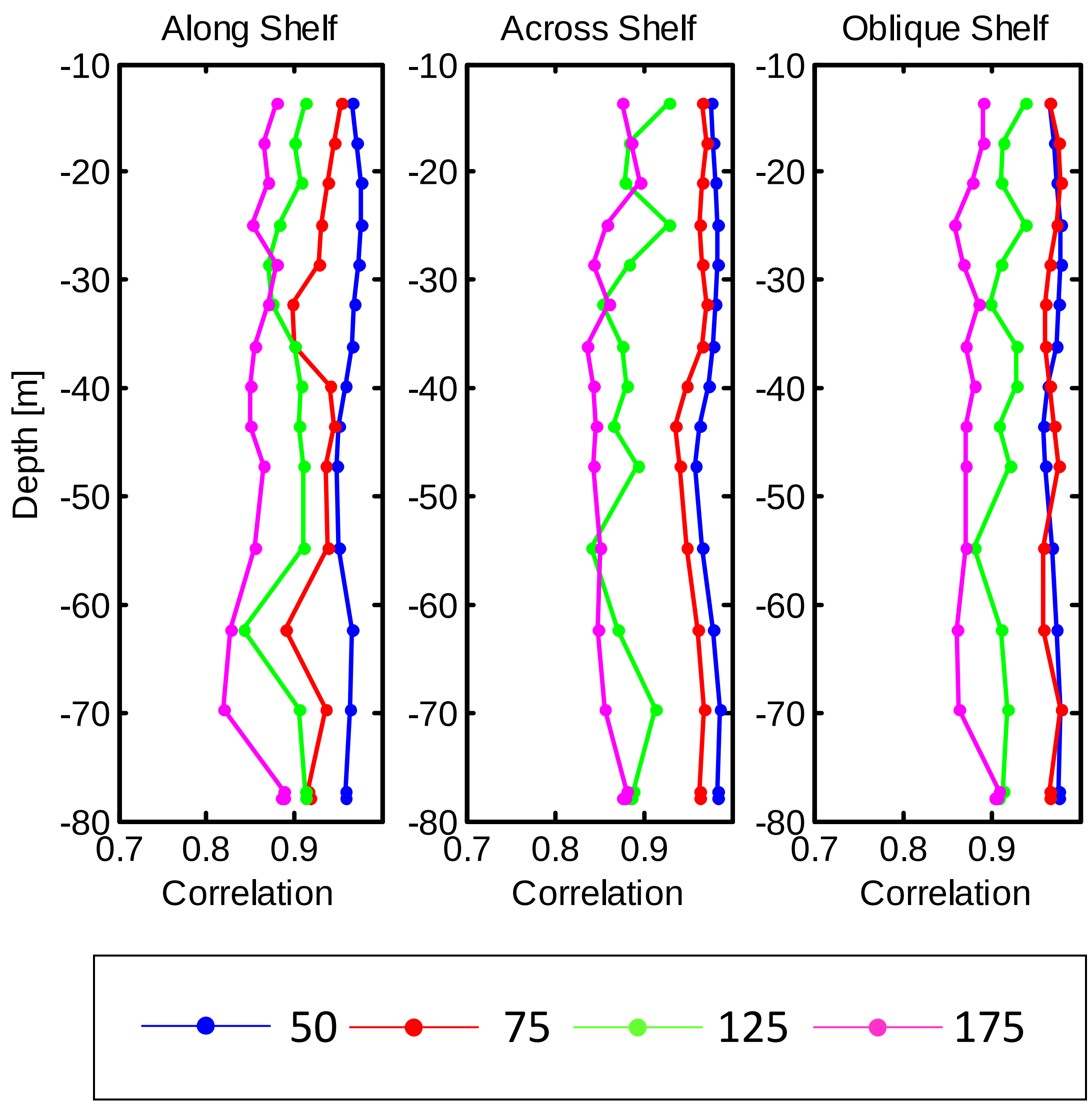
Incoherent Correlation of the pressure fields averaged over all tracks

$P(r)$ Calculated Field

$\hat{P}(r)$ Measured Field

$$cor(R) = \frac{\left\langle P(r)\hat{P}^{*}(r)\right\rangle}{\left[\left\langle P(r)P^{*}(r)\right\rangle\left\langle \hat{P}(r)\hat{P}^{*}(r)\right\rangle\right]^{1/2}}$$

$$\left\langle PQ\right\rangle = \frac{1}{R-r_{\min}}\int_{r_{\min}}^{R} P(r)Q(r)dr$$



Conclusions

- Inversion Results
  - Range Dependent Inversion Results for three distinct tracks
  - Creation of a 3-D model by determining for sound speed for each layer
- Validation of Results
  - Comparison shows agreement between inversion result and core data
  - Ability to predict the acoustic field at all depths and frequencies

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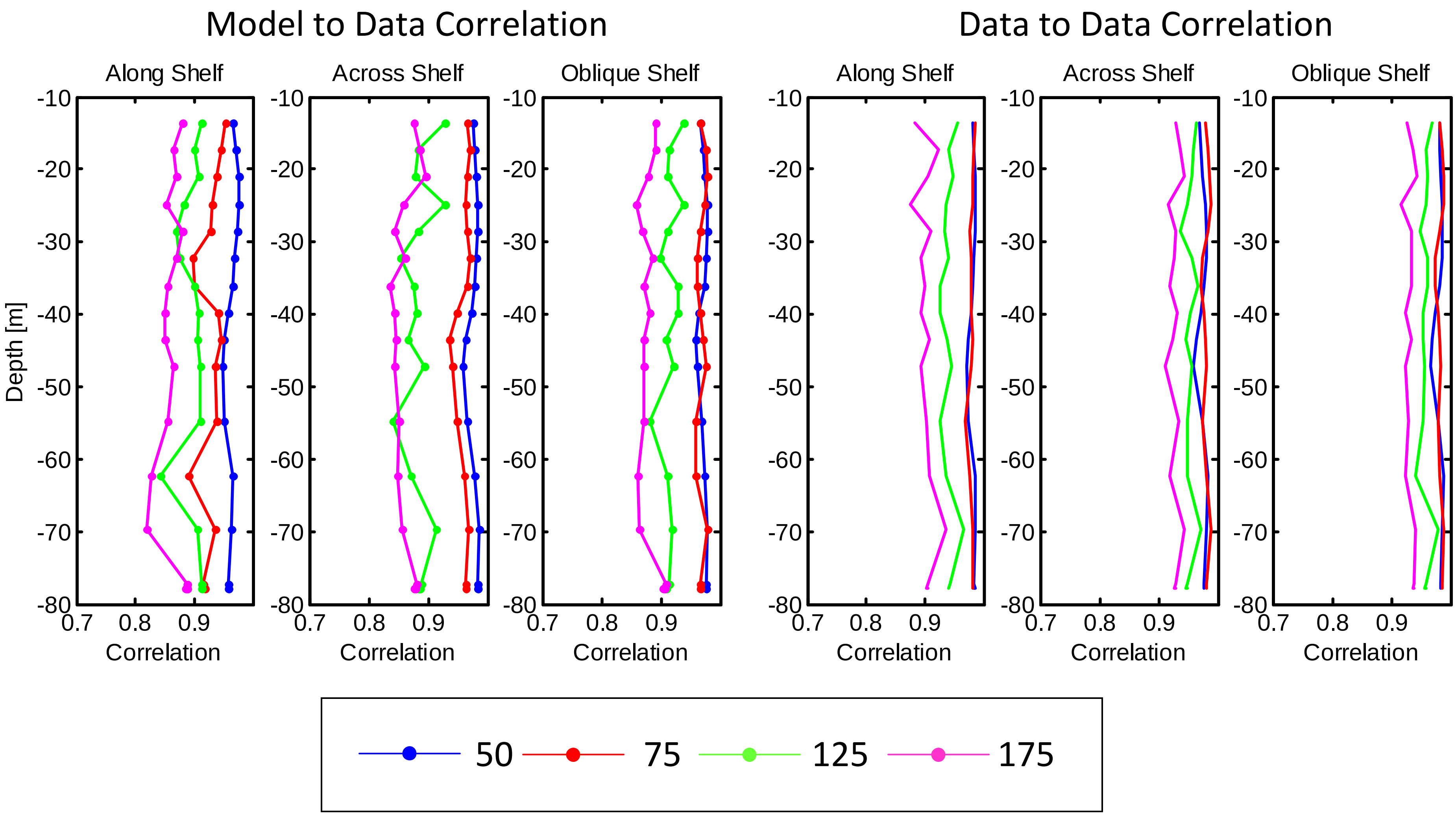
Evaluation of Results: Correlation



Monte Carlo Error Estimates

The complete solution of the inverse problem requires not only the estimates of the model parameter values, but also a measure of the uncertainty of the estimates.

Monte Carlo Error Propagation:

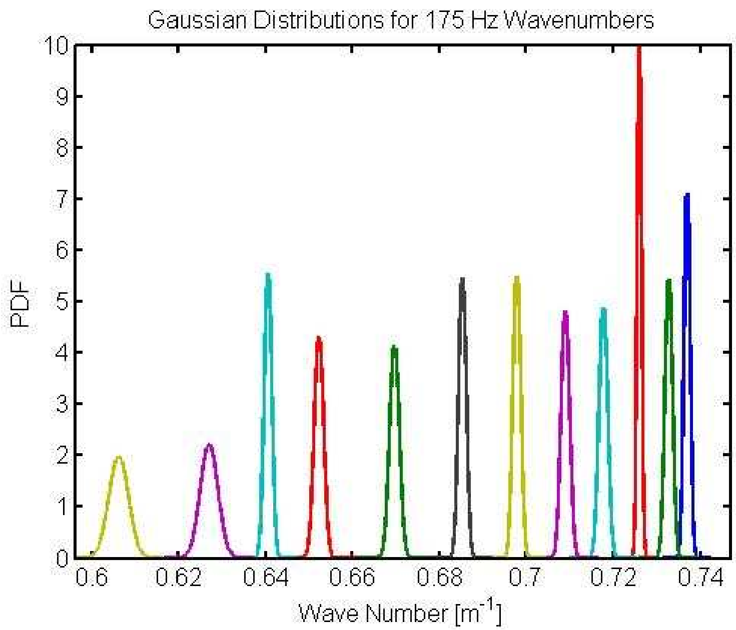
$$\mathbf{y} + \mathbf{n}_i = \mathbf{A}\mathbf{x}_i \qquad i = 1, 2, ..., N$$

The empirical estimate of the covariance matrix

$$\mathbf{Cov} = \frac{\mathbf{D}^T\mathbf{D}}{N} \quad \text{where} \quad \mathbf{D} = \mathbf{x}_i^{\,T} - \overline{\mathbf{x}}_i^{\,T}$$

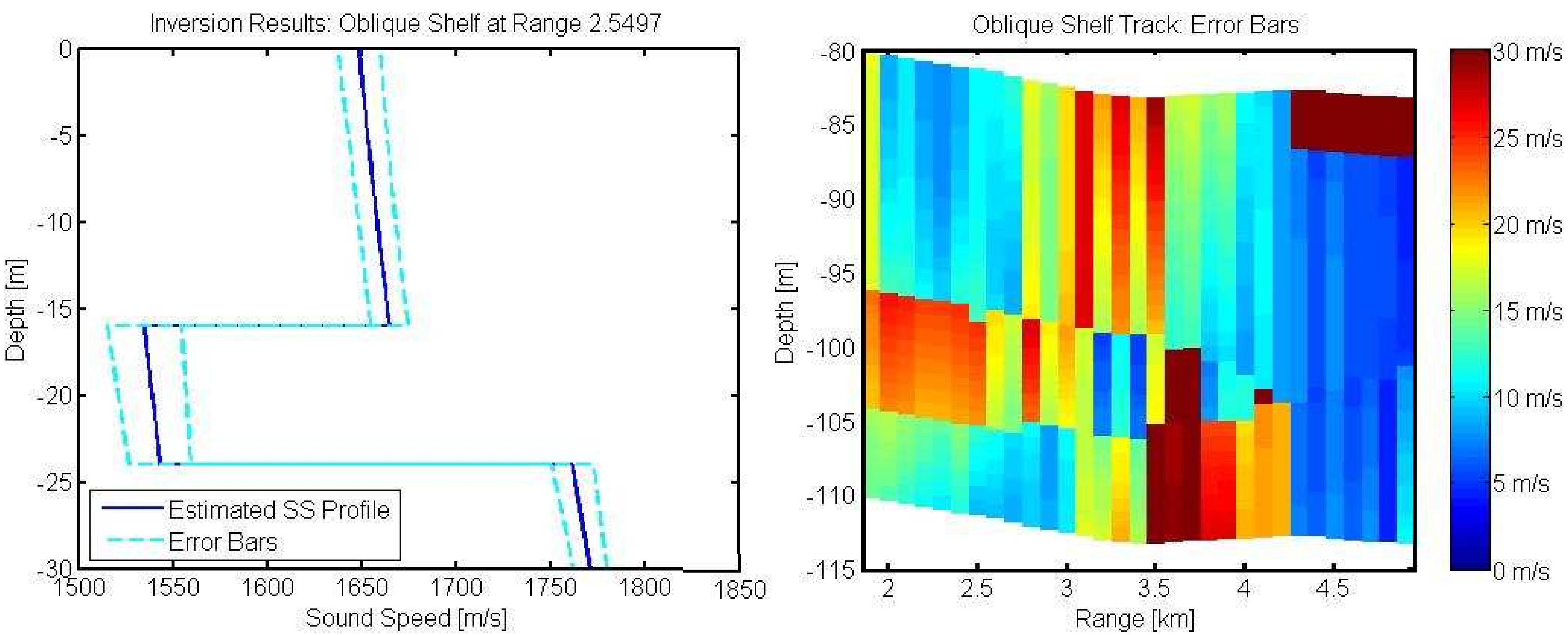
Error bars: standard deviation of the model estimates

$$\sigma_m(z) = \sqrt{diag(\mathbf{Cov})}$$



Inversion Results: Error Bars

For the Oblique Shelf Track



Calculated Error Bars

For the nonlinear problem examined here, the solution is arrived at iteratively. Assuming the final solution to the problem is linear, it is valid to use linear theory to obtain the resolution and covariance matrices.

Beginning with the linear problem: $\mathbf{d} + \mathbf{v} = \mathbf{Gm}$

$\mathbf{C_v}$ Data covariance matrix; $\mathbf{C_m}$ Model covariance matrix

The resolution matrix is given by: $\mathbf{R_m} = (\mathbf{G}^T\mathbf{C_v^{-1}G} + \mathbf{C_m})^{-1}\mathbf{G}^T\mathbf{C_v G}$

Resolution length is given by: $$rl(i) = \frac{\sum_{j=1}^{M} R_{ij}^2}{R_{ii}^2}$$

The posterior model covariance matrix is given by: $\widehat{\mathbf{C}}_\mathbf{m} = (\mathbf{G}^T\mathbf{C_v^{-1}G} + \mathbf{C_m^{-1}})^{-1}$

Calculated Error Bars

For the Oblique Shelf Track