Modal inversion using the acoustic field measured on a vertical array of hydrophones

Kyle M. Becker

Penn State Univ./Appl. Res. Lab.

kmbecker@psu.edu

Megan S. Ballard

Penn State Univ./Grad. Prog. Acoustics

Subramaniam D. Rajan

Scientific Solutions, Inc

Acoustics '08 Paris 1 July 2008

Measurement & Analysis Objectives

- Collect data appropriate for testing and validating geo-acoustic inversion schemes based on normal modes
- Obtain geospatially dependent sediment parameter estimates - mapping
- Exploit Doppler shift in a waveguide to infer geo-acoustic parameters and interpret results
- Compare inversion results obtained for co-located assets but different measurements and algorithms
- Investigate impact of water column variability on geo-acoustic parameter estimation

kmbecker@psu.edu

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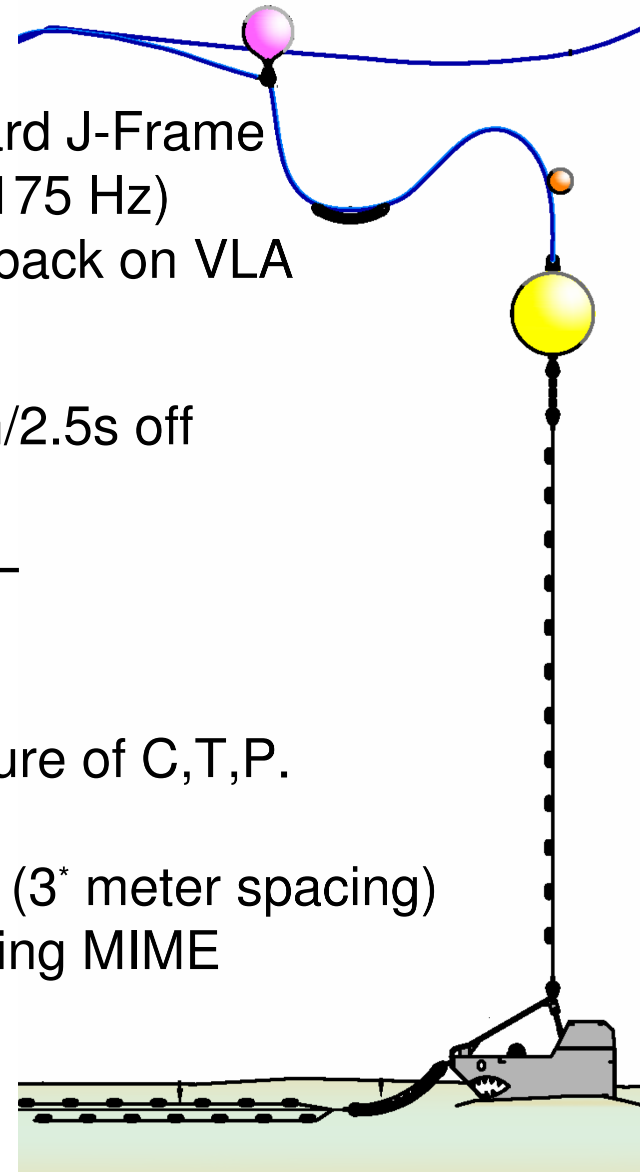
Primary Measurement Assets

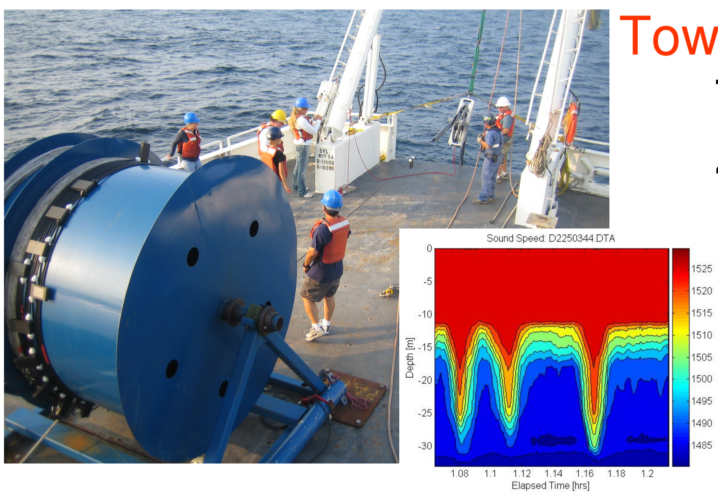


J-15-3 LF source – Starboard J-Frame

CW comb (50,75,125, 175 Hz)
Radial tows ~5 km out/back on VLA

FM sweep (40-290 Hz)
15 km standoff, 0.5s on/2.5s off

~173 dB re 1uPa/1m SL



Towed CTD chain

Temporal/spatial measure of C,T,P.
48 sensors
141 m vertical aperture (3* meter spacing)
NOT fully deployed during MIME

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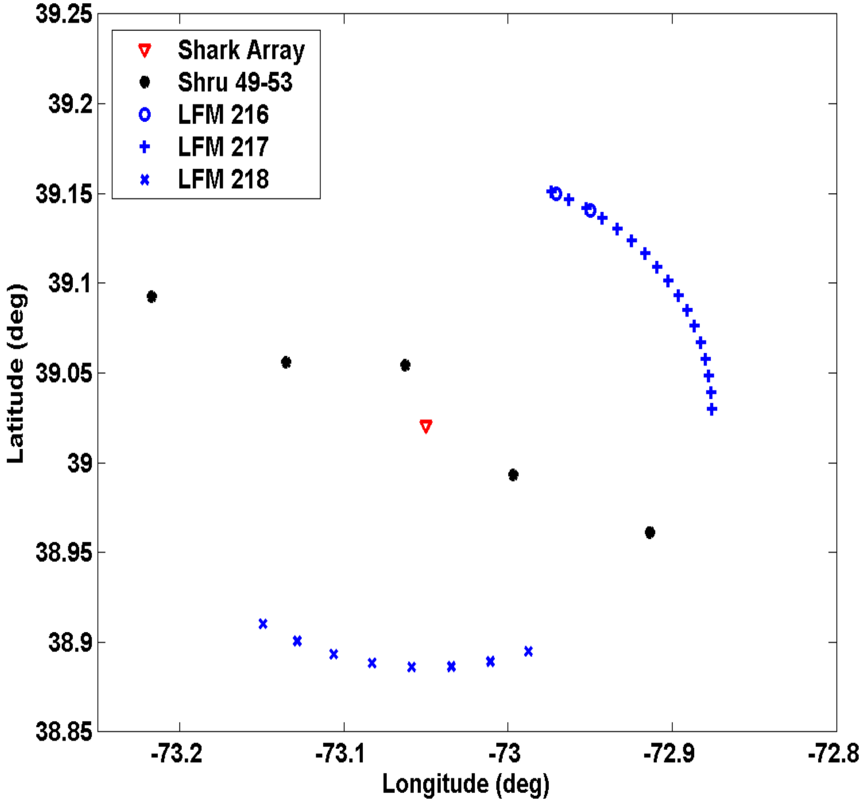
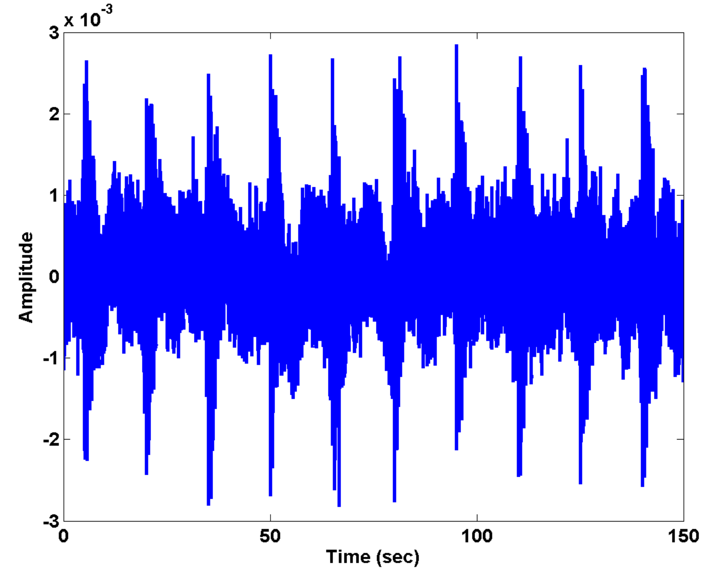
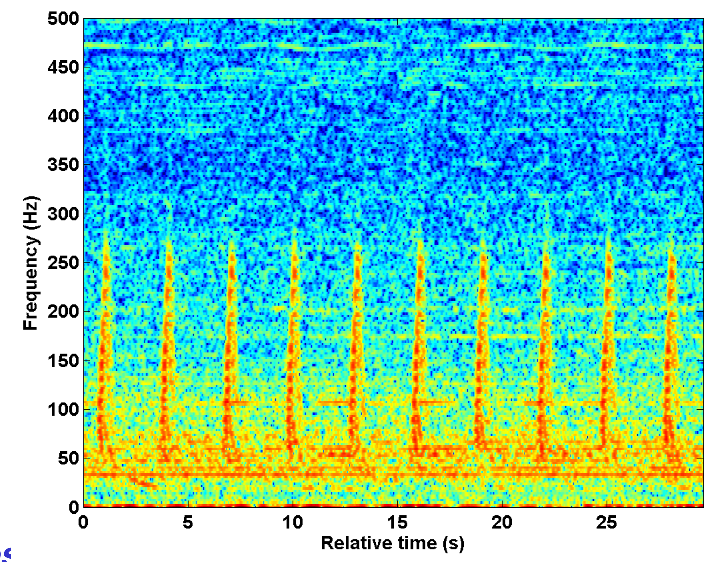
Broadband Data

- 0.5 sec. sweep (250 Hz)
- ~15 km from VLA



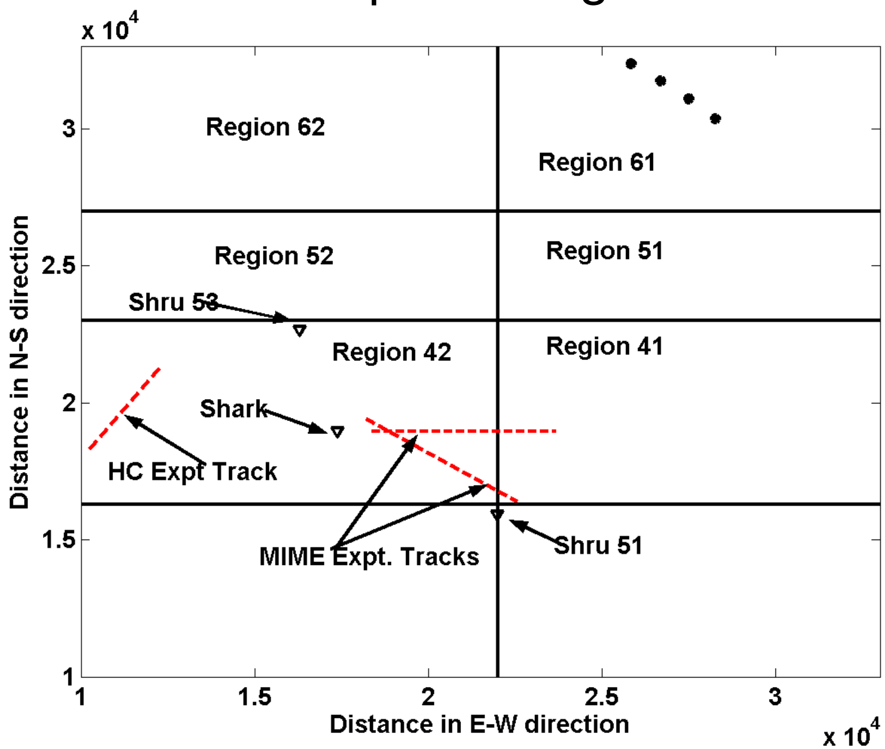




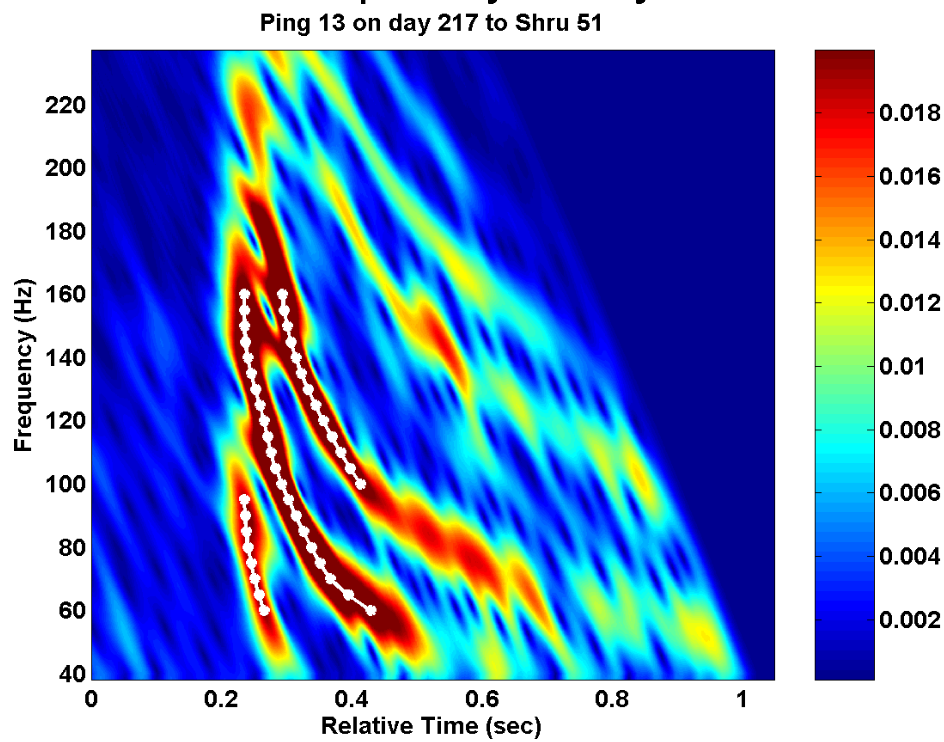
Perturbative Inverse - Time

$$dt_n = \frac{\partial \Delta\theta_n}{\partial \varpi} = \frac{\partial}{\partial \omega} \int_0^r \int_0^\infty \frac{1}{k_n(\omega)} \frac{\omega^2 \Delta c(s,z)}{c^3(s,z)\rho(s,z)} \left|\phi_n(s,z,\omega)\right|^2 ds dz$$

Geospatial Regions



Time-Frequency Analysis



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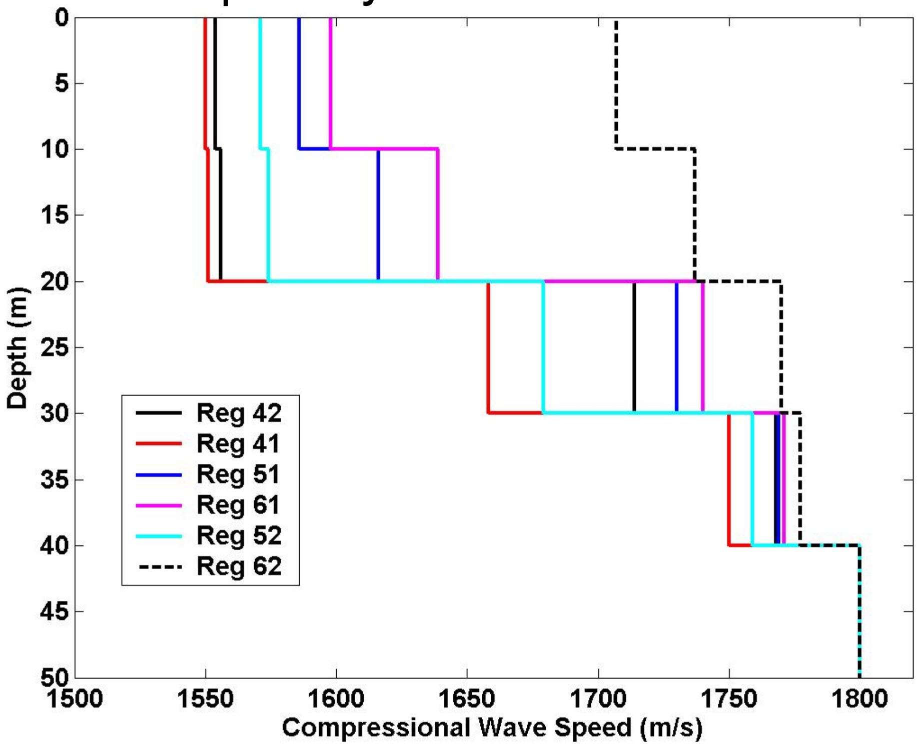
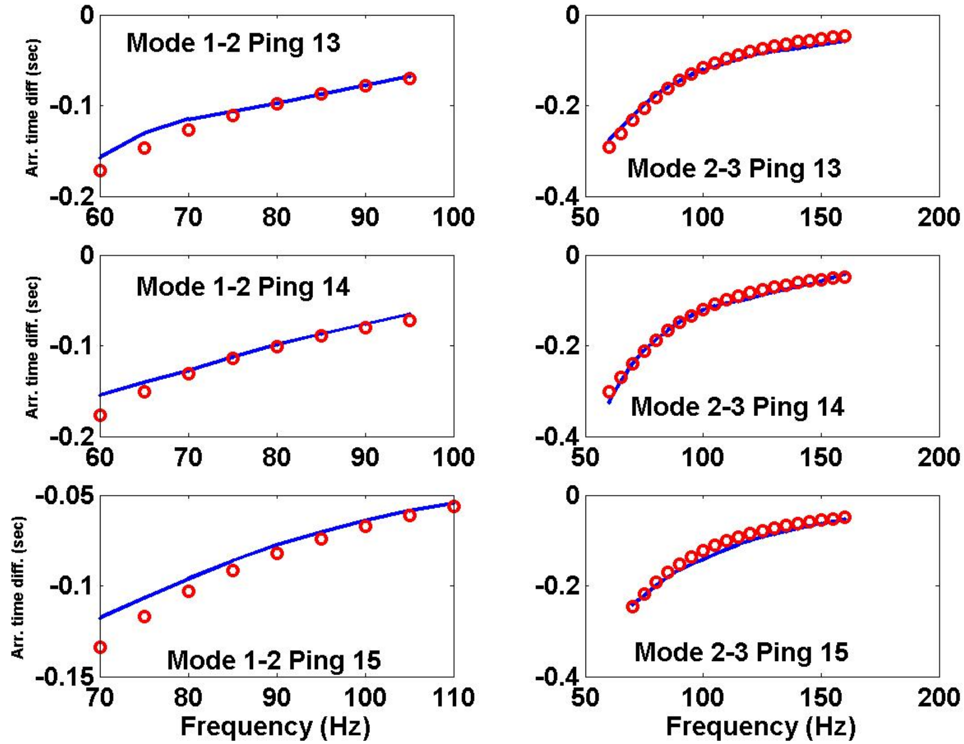
Perturbative Inverse - Time

Geospatially distributed inversion



Model validation



No low-speed layer – only 3 modes used in inversion

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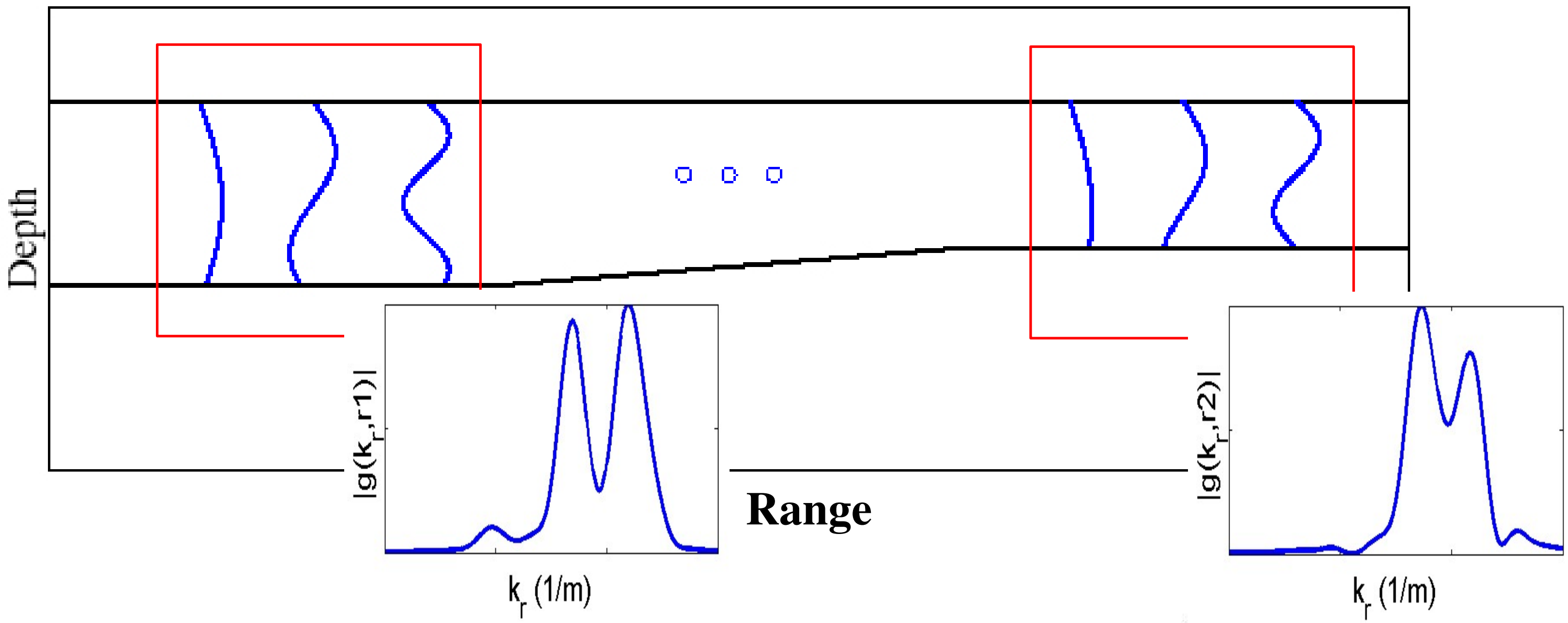
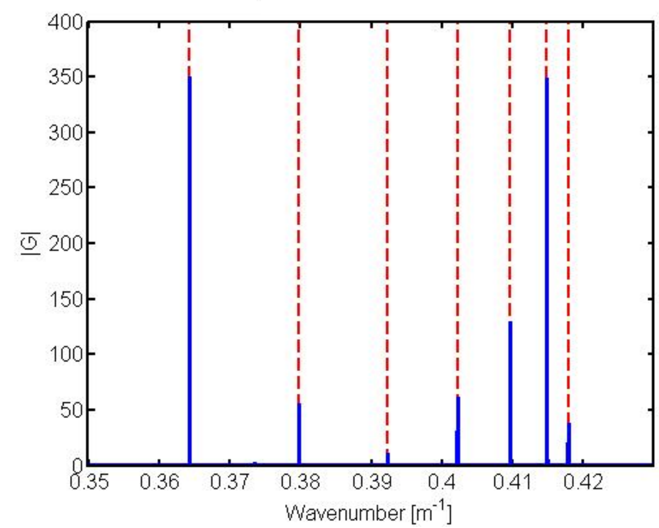
Synthetic Aperture Processing



$$g(k_r;\hat{r},z,z_0)=\frac{e^{i\pi/4}}{\sqrt{2\pi k_r}}\int_{-\infty}^{\infty} w_L(r;\hat{r})\,p(r;z,z_0)\sqrt{r}\,e^{-ik_r r}\,dr$$

$$x_n=\sum_{k=1}^{p} a_k x_{n-k} \longrightarrow P_{AR}=\frac{\sigma^2 T}{\left|1+\sum_{k=1}^{p} a_k e^{-i2\pi fkT}\right|^2}$$



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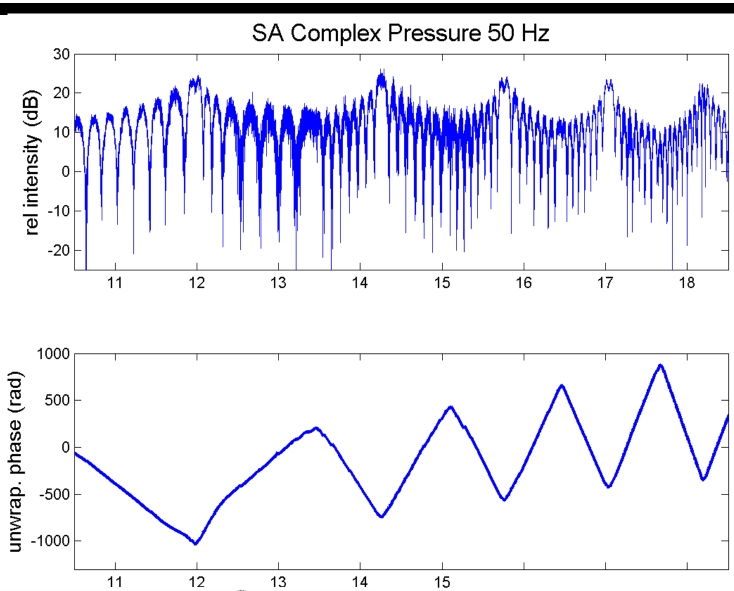
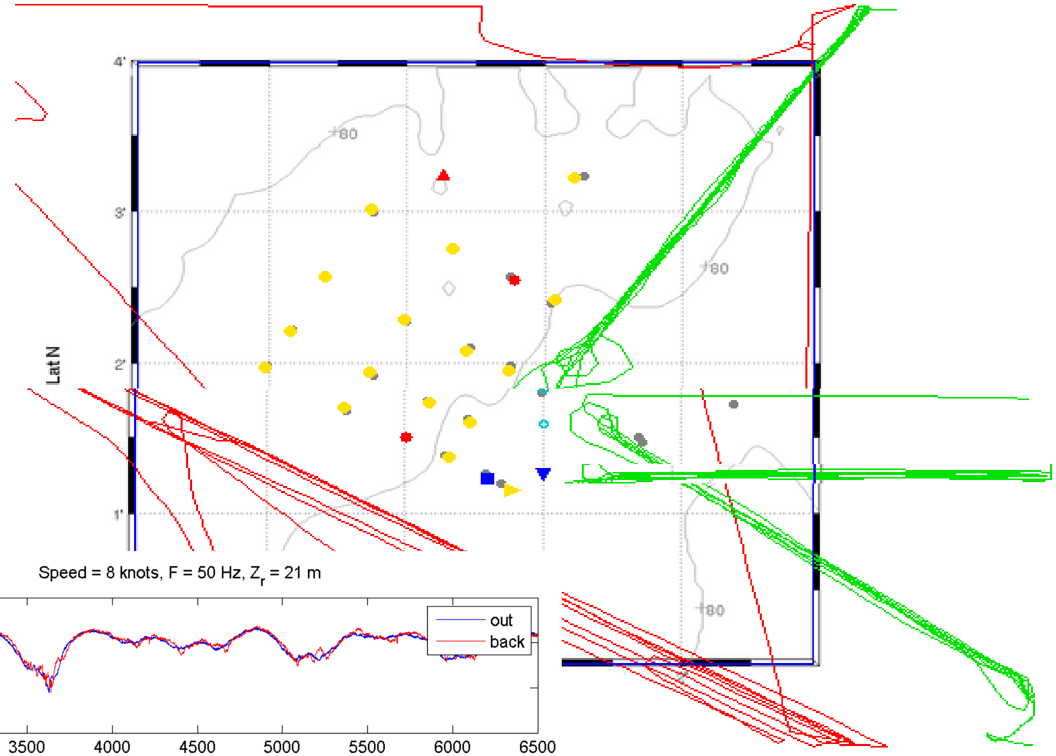
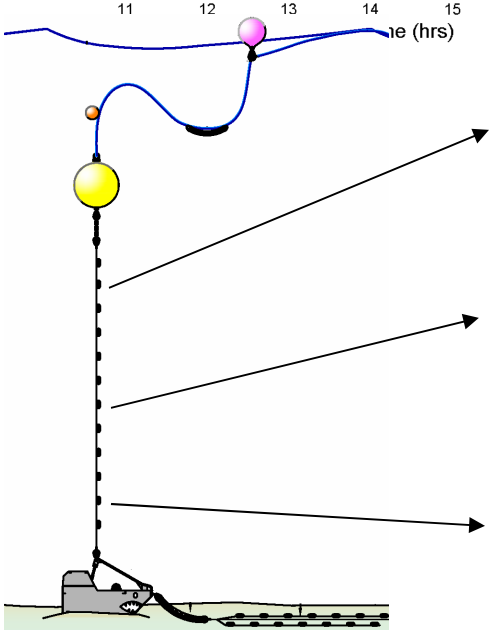
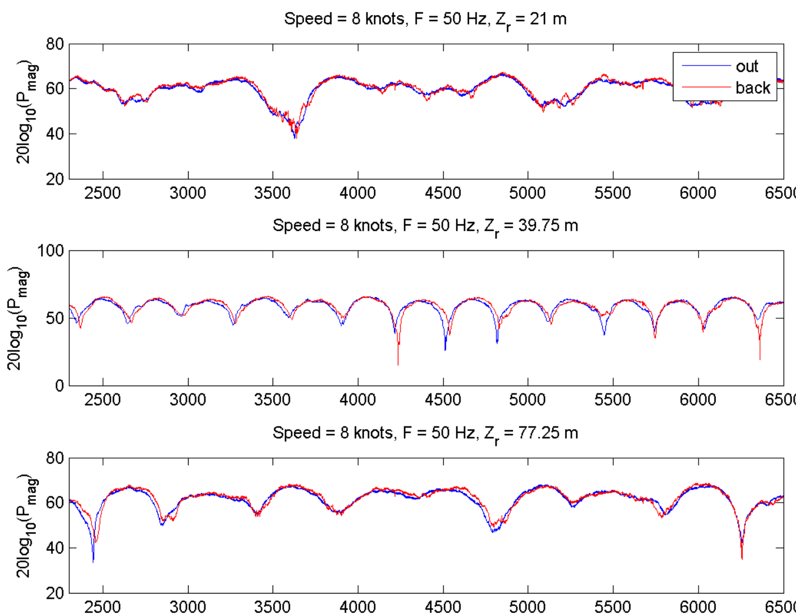
Synthetic Aperture Data









- 16 depths
- out and back
- 5 speeds

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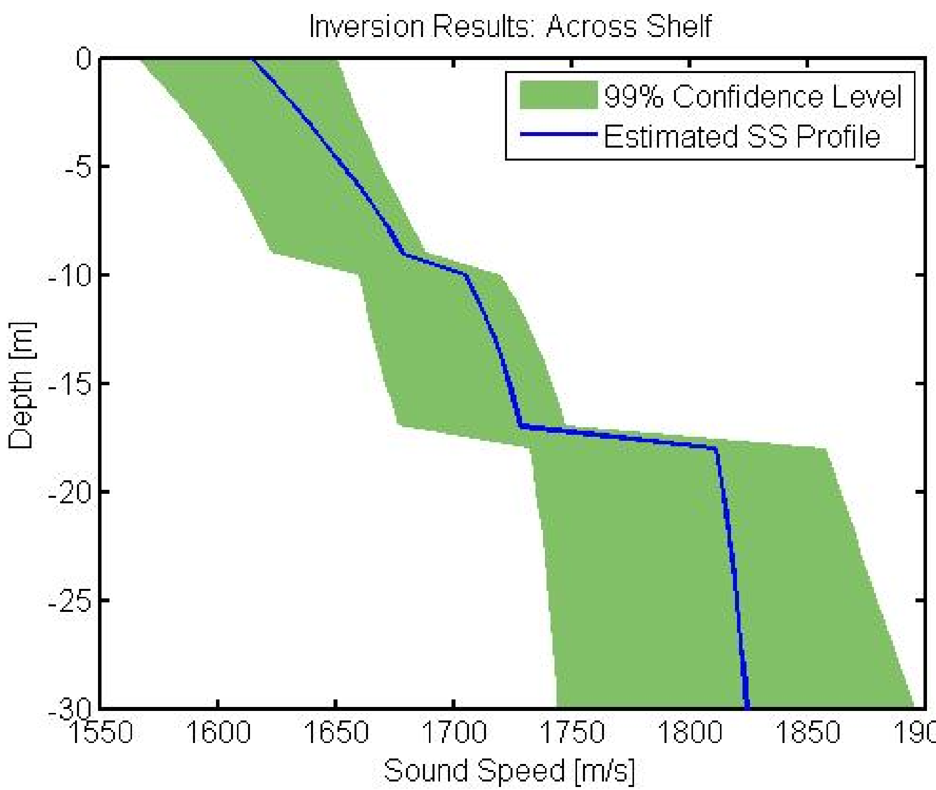
Perturbative Inverse – Wavenumber Partial Data

Wavenumber Perturbation Equation

$$\Delta k_n = \frac{1}{k_n} \int_0^{\infty} \rho^{-1}(z) Z_n^2(z) k^2(z) \frac{\Delta c(z)}{c_0(z)} dz$$



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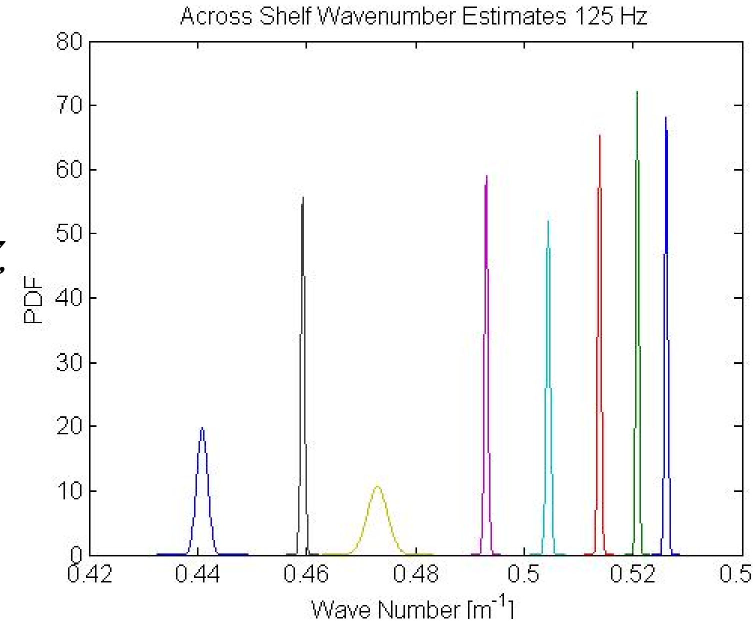
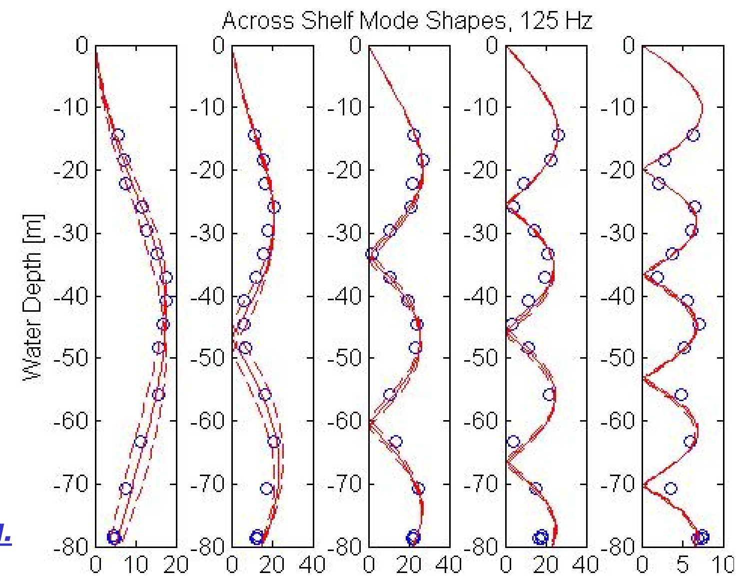
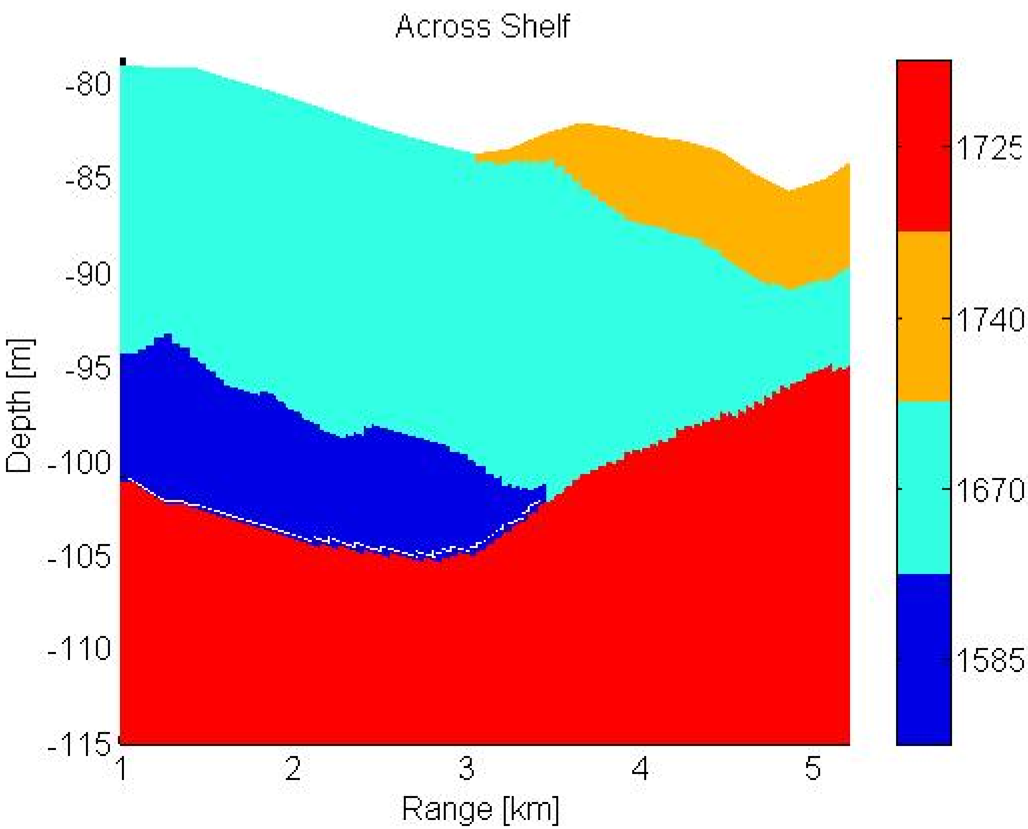
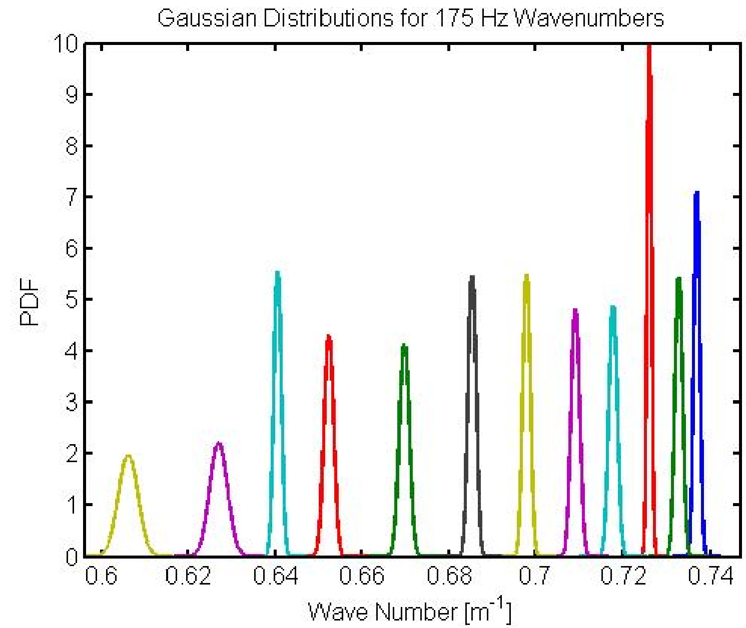


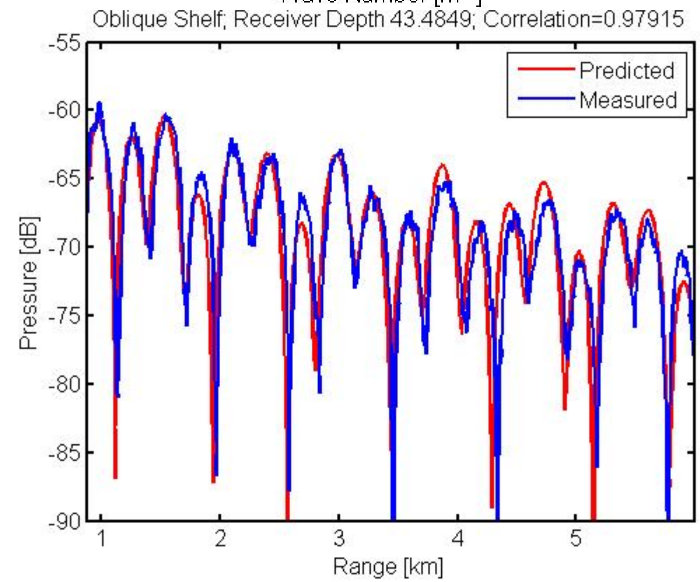


Perturbative Inverse - Wavenumber

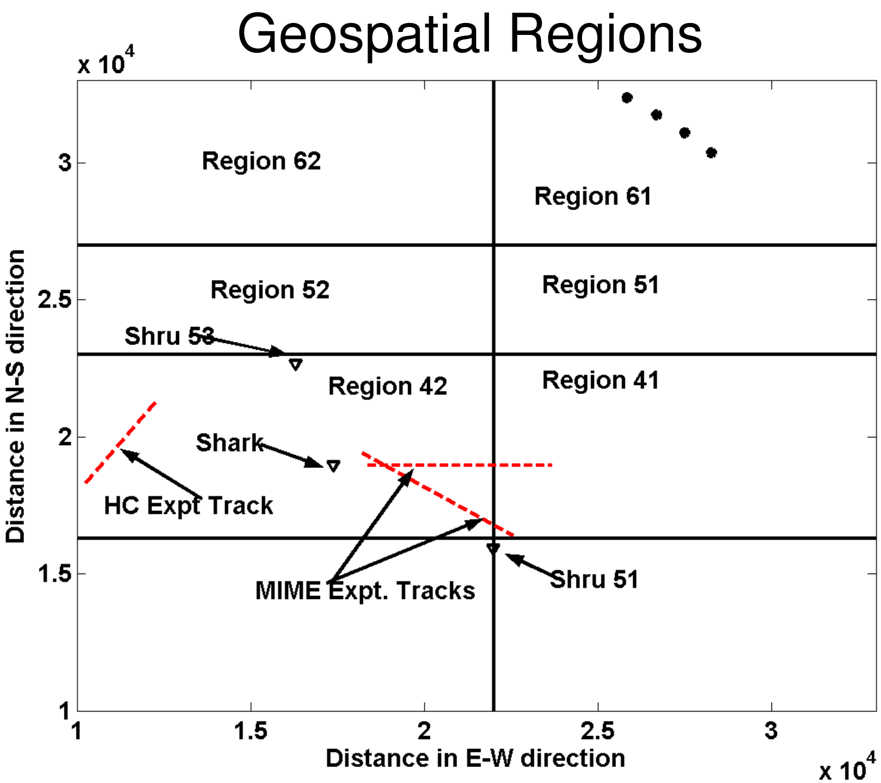
- All modes used
- Low speed layer resolved

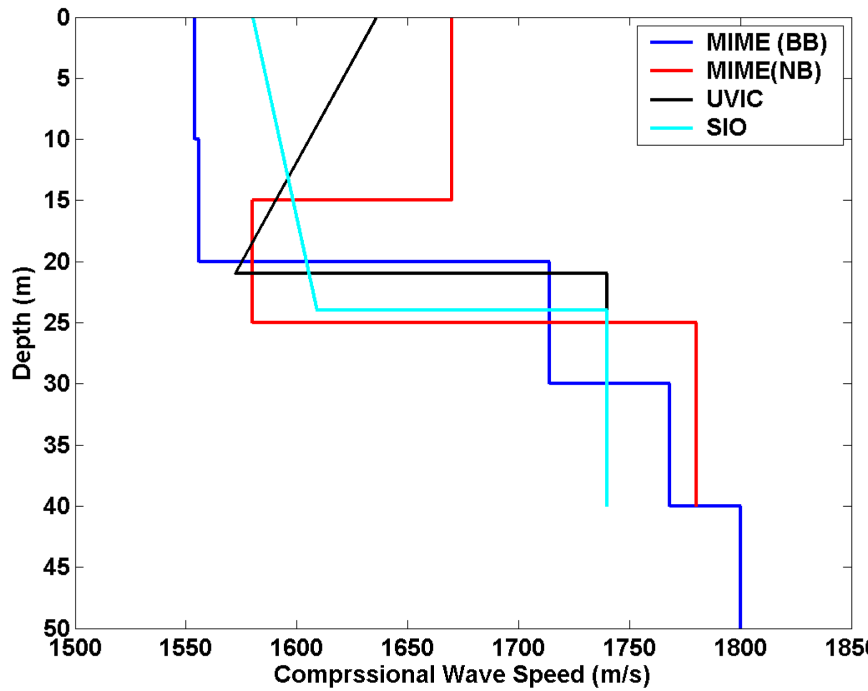






Comparison of profiles obtained by other investigators





Presence of Low-speed layer is suggested in 2 of 4 results

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Doppler Shift in a Waveguide

Objective: Measure frequency and wavenumber Doppler shifts on synthetic aperture created by moving source and stationary receiver to infer model group velocity

$$p(r,z,\omega) \approx \frac{1}{2}\int S(\Omega_k)\,g(k_r,z,\omega)H_0^{(1)}(k_r r)k_r dk_r$$

$$\Omega_k = \omega\left(1-\frac{v_s}{c}\right)$$

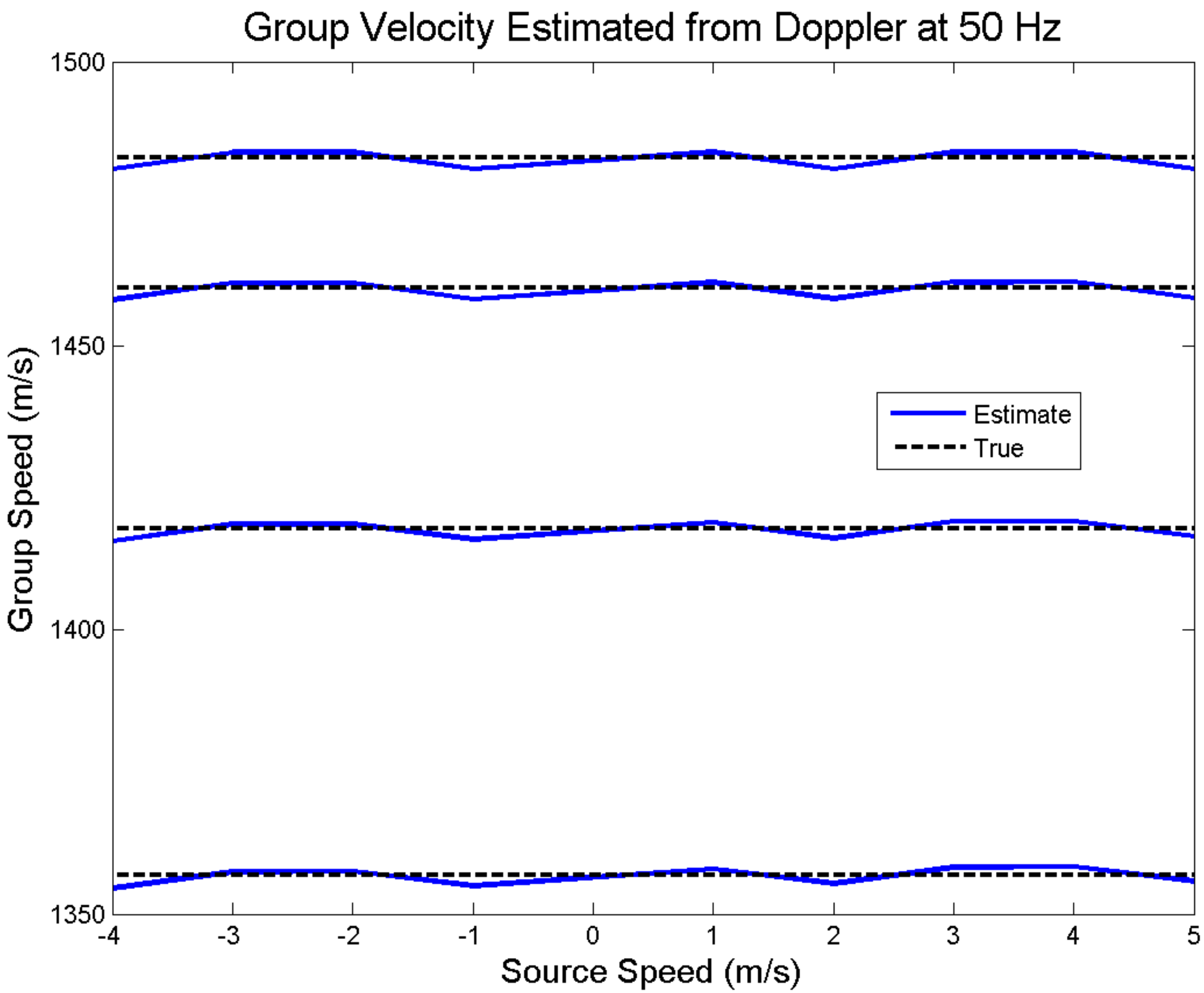
Modal group velocity is given by: $v_{gn} = \dfrac{d\Omega_k}{dk_n}$

Evaluating the modal pressure field at the Doppler shifted frequencies, the differential can be evaluated to estimate group velocity

[Ref: H Schmidt and W.A. Kuperman, JASA **96**(1) pp386-395 (1994)]

kmbecker@psu.edu

Doppler Shift in a Waveguide – $d\Omega/dk_n$



Field generated at 10 different freqs. Corresponding to Doppler shift for Source moving Plus/minus 5 m/s.

Horizontal wave-Numbers estimated And differential Determined.

Estimates compared To group velocity At 50 Hz (KRAKEN)

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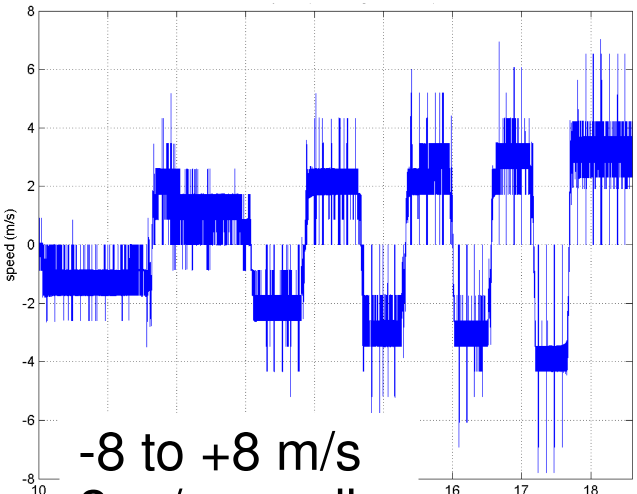
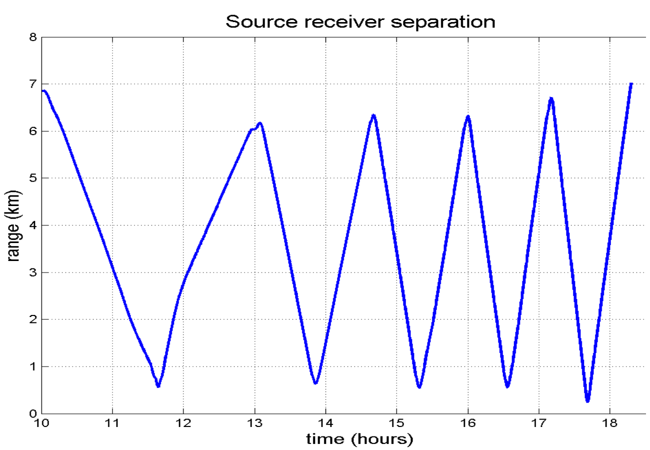
Doppler Shift in a Waveguide - $d\Omega$

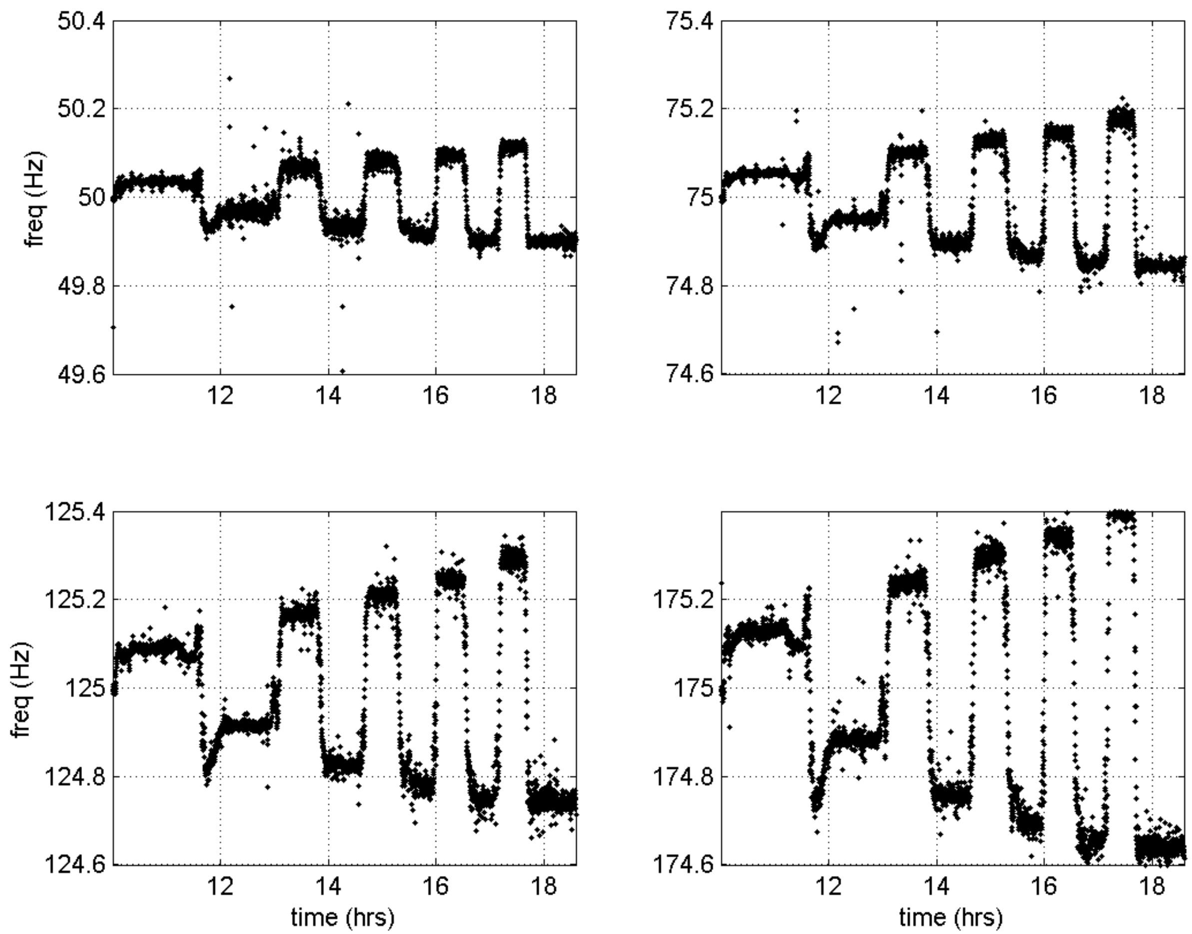
Tow Speed



-8 to +8 m/s
2 m/s per div.



Frequency Observed at VLA



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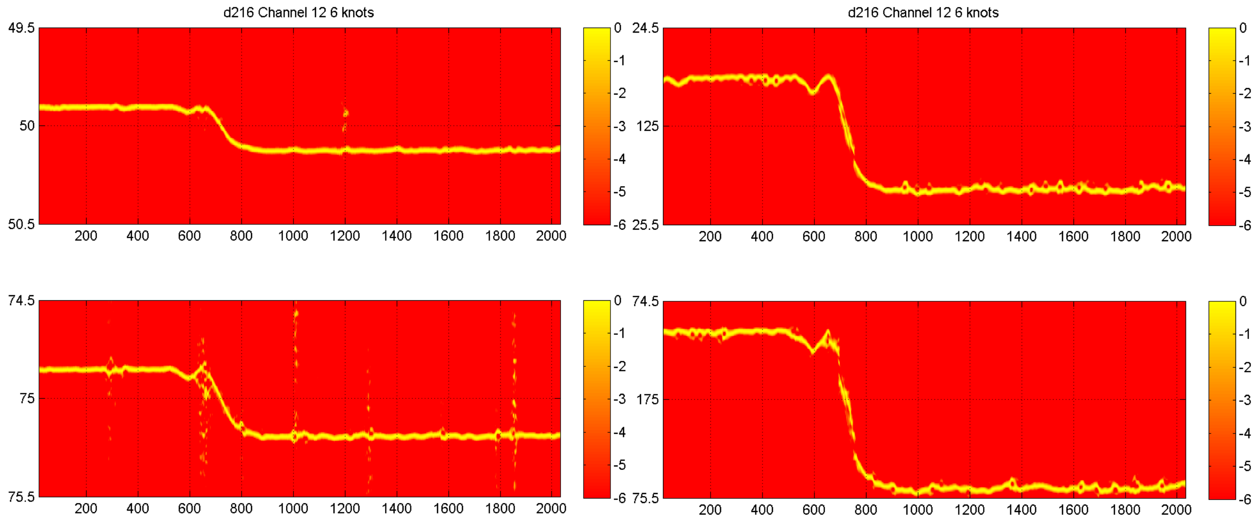
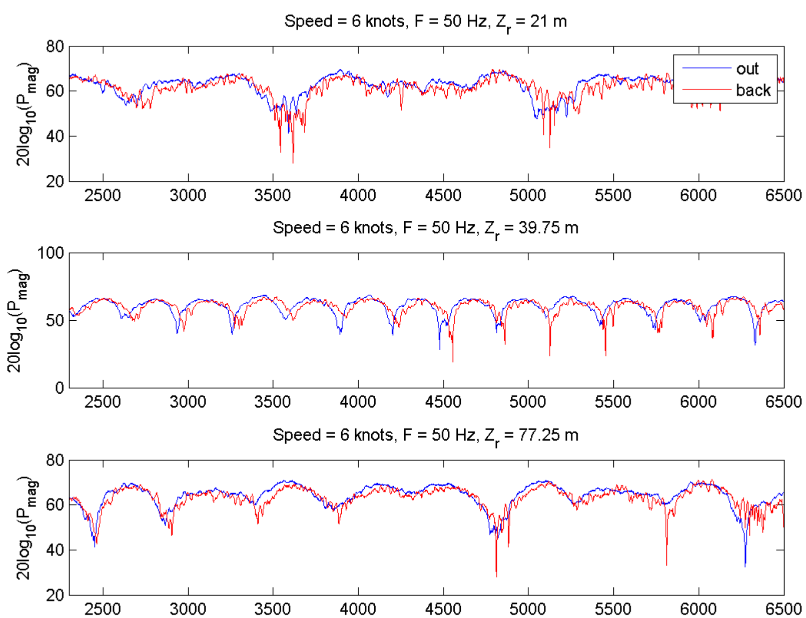
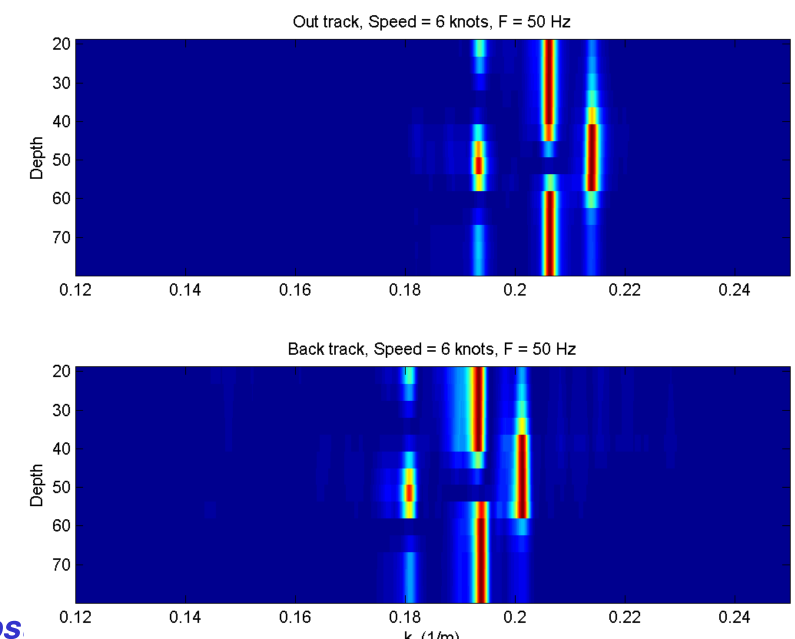
Doppler Shift in a Waveguide – $d\Omega/dk_n$



- Doppler prop. Speed & symmetric
- BW prop to time app.
- Modal content equiv.
- k_n shift symmetry, but incorrect

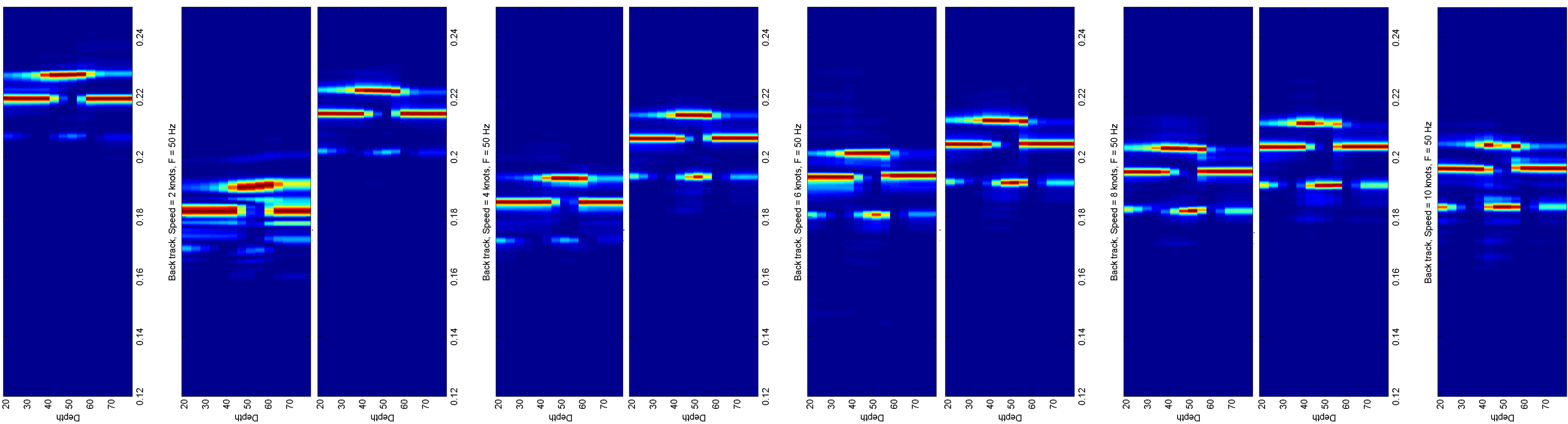




Doppler Shift in a Waveguide – $d\Omega/dk_n$

- Wavenumber shift should increase with increasing Doppler
- Measurements indicate decreasing shift in wrong direction

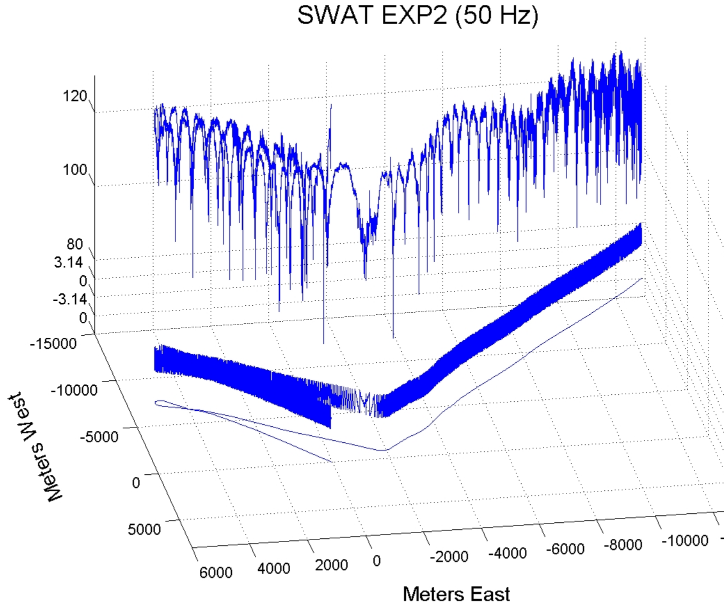
/ 1 m/s \/ 2 m/s \/ 3 m/s \/ 4 m/s \/ 5 m/s \

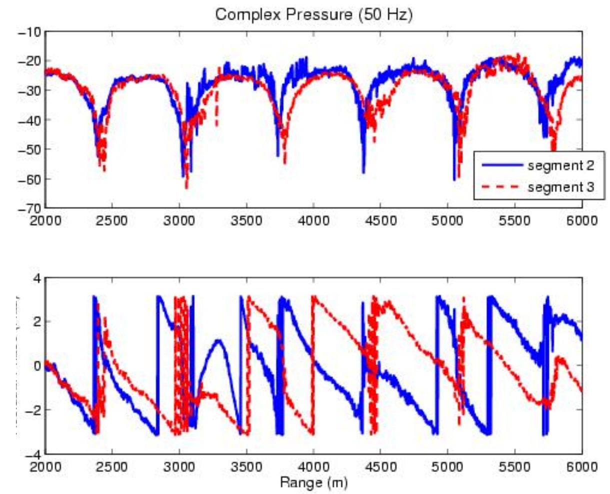


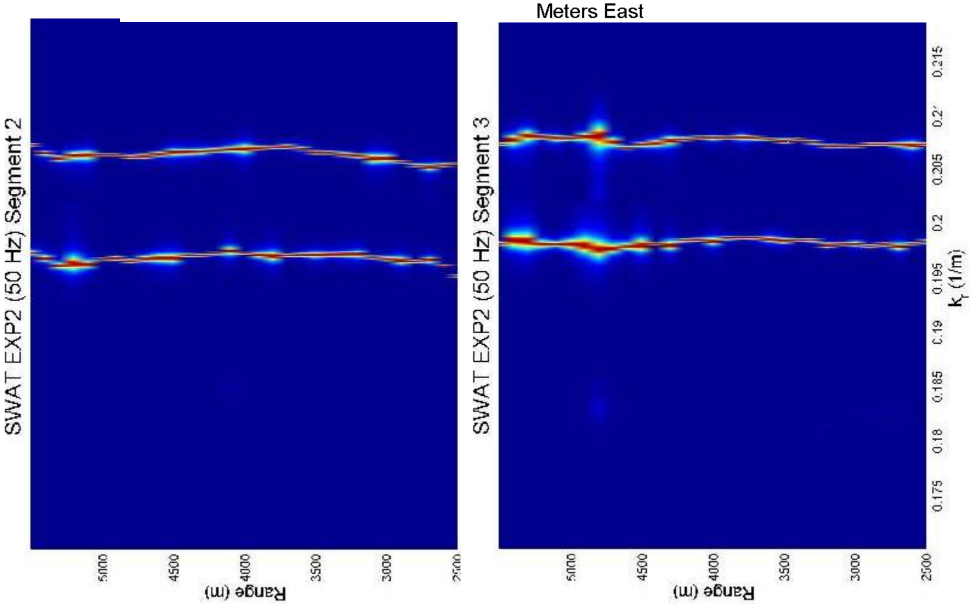
- Unresolved cause of shift – not observed in previous data sets
- Asynchronous data collection may be cause

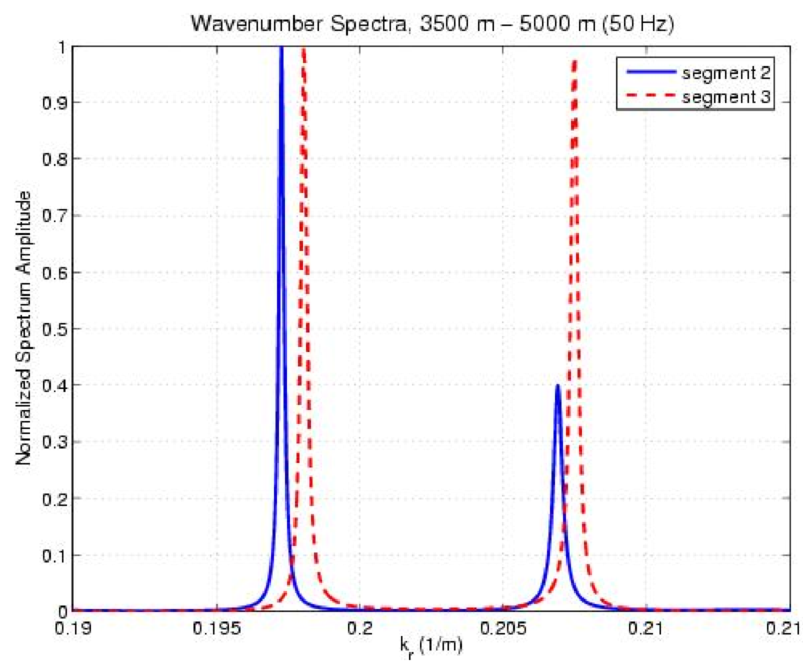
kmbecker@psu.edu

Doppler Shift in a Waveguide – dk_n



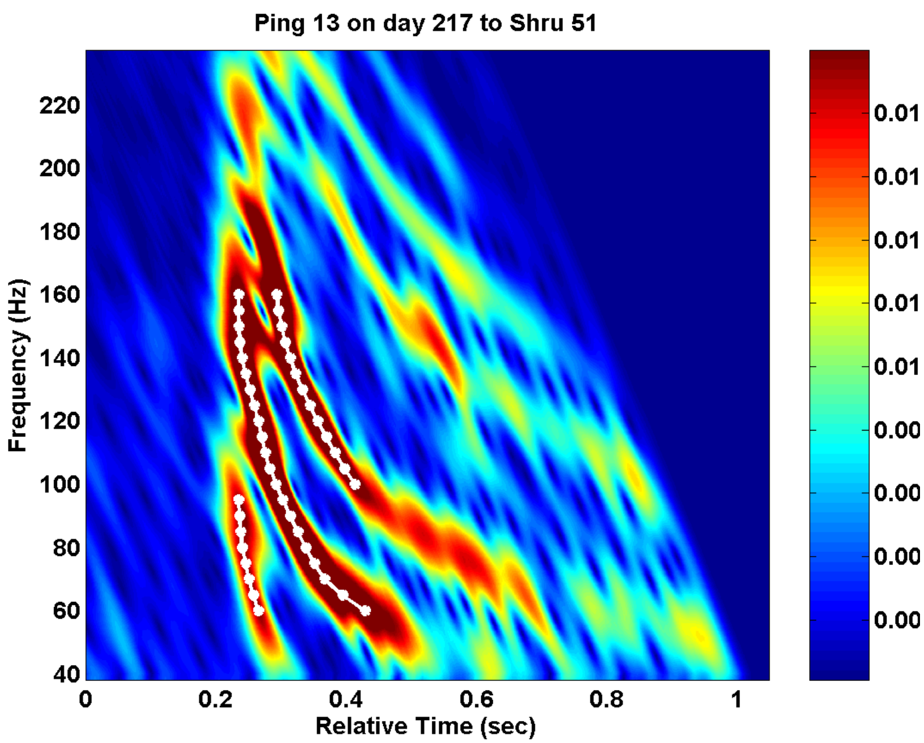


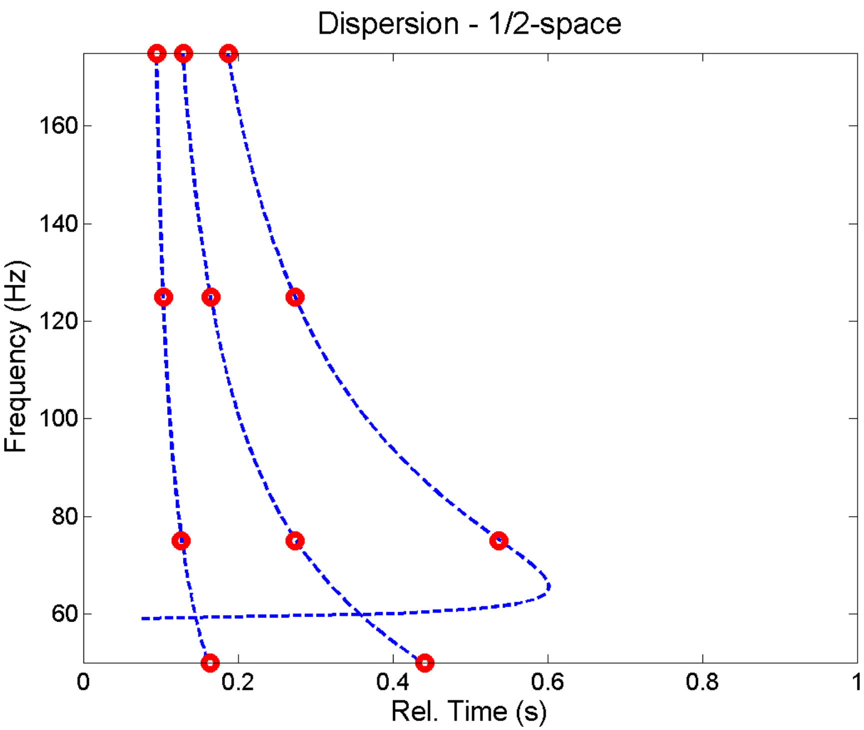




Doppler Shift – Dispersion Analysis

Possibility to obtain full dispersion representation from Doppler shift about small number of discrete transmit frequencies?





For half-space – simple dispersion can be approximated

kmbecker@psu.edu

Doppler Shift – Dispersion Analysis

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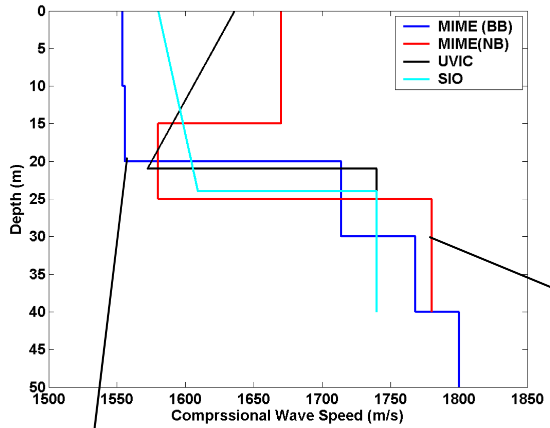
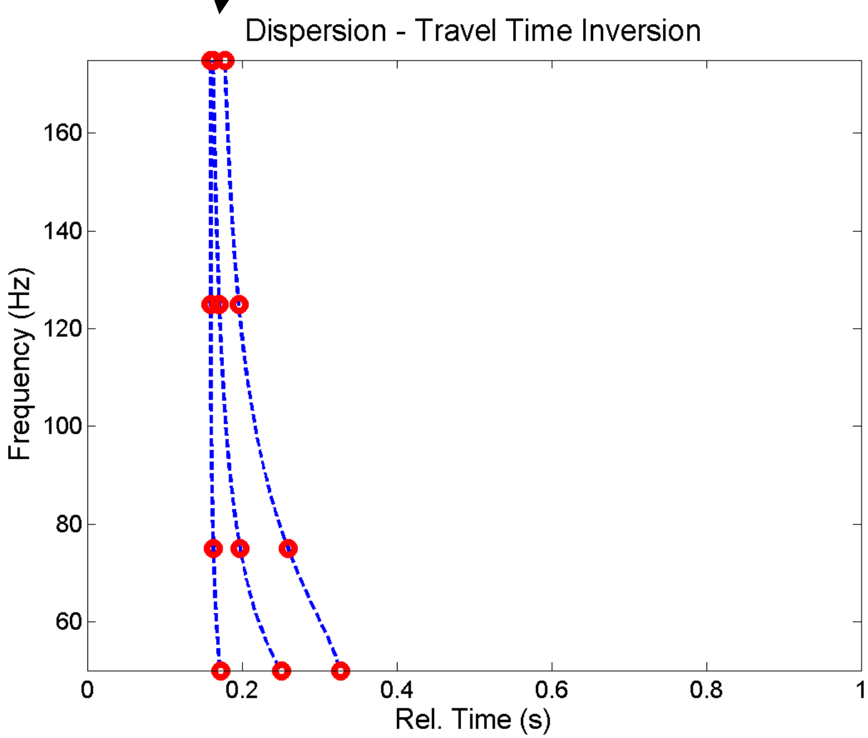
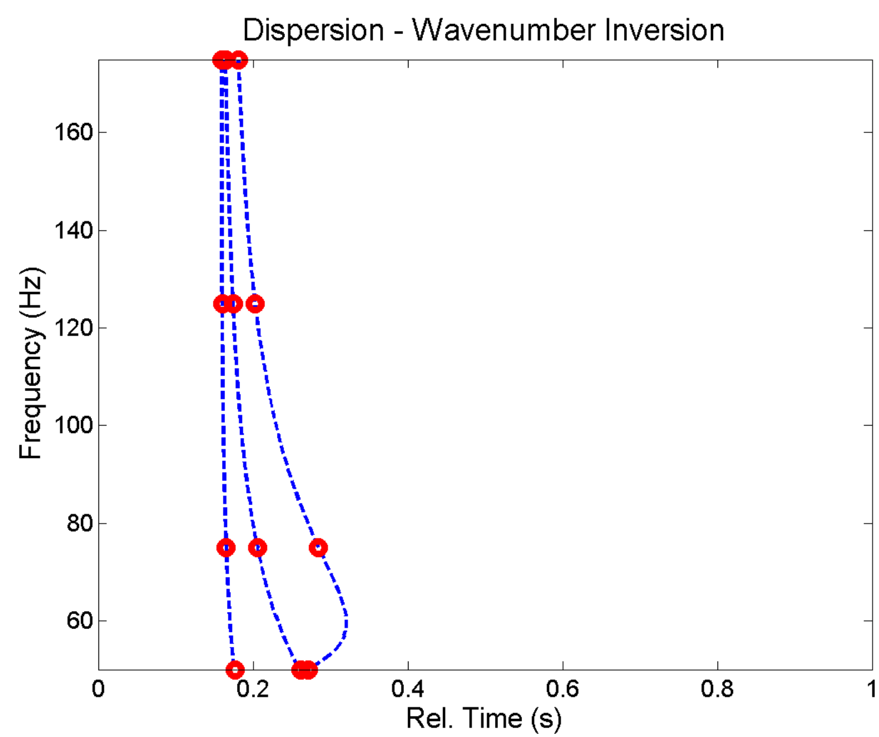


Dispersion for Wavenumber and Travel Time Inversions

3 modes





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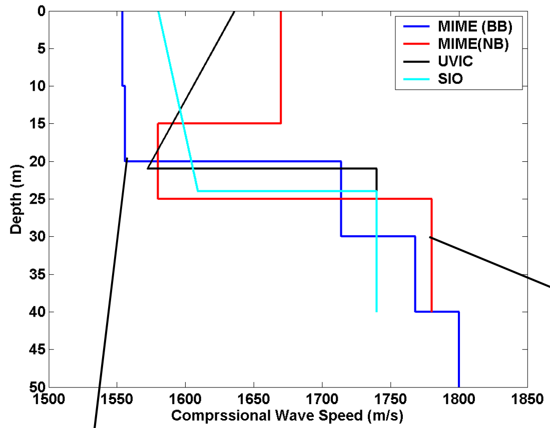
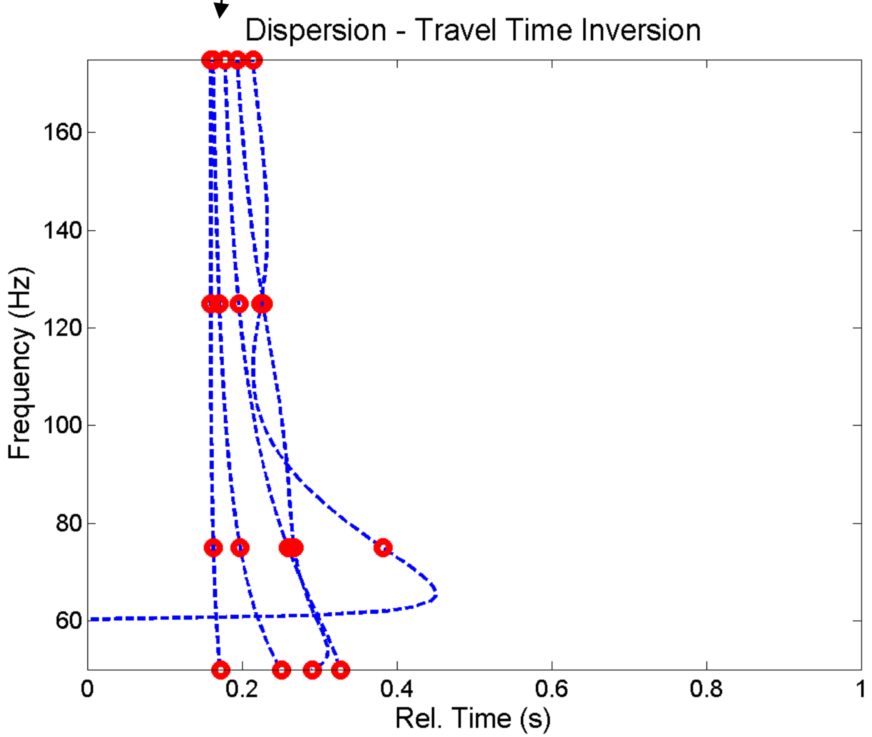
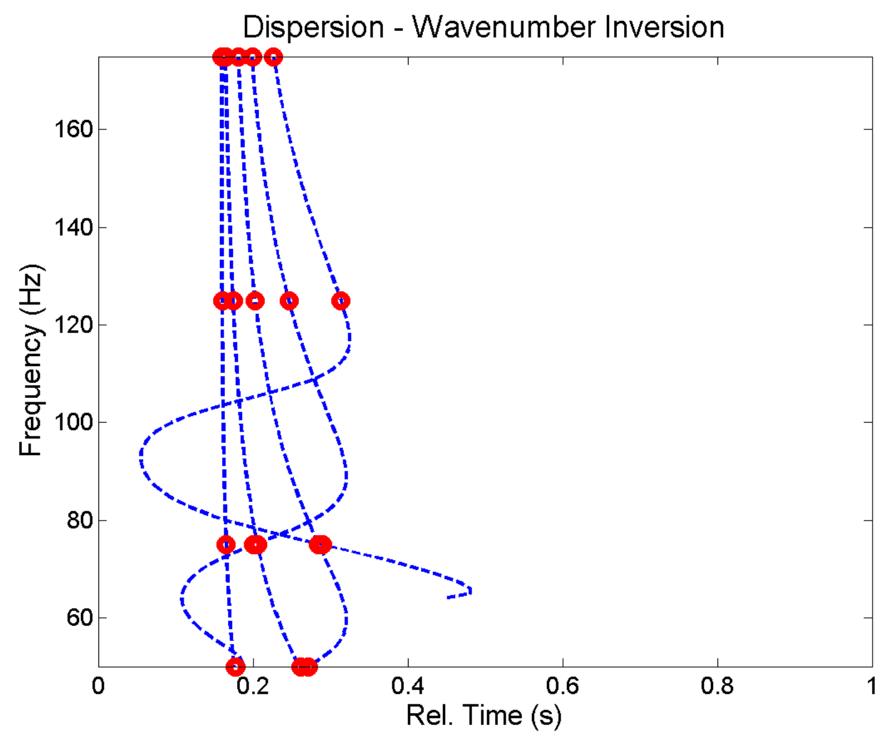
Doppler Shift – Dispersion Analysis



Dispersion for Wavenumber and Travel Time Inversions

5 modes

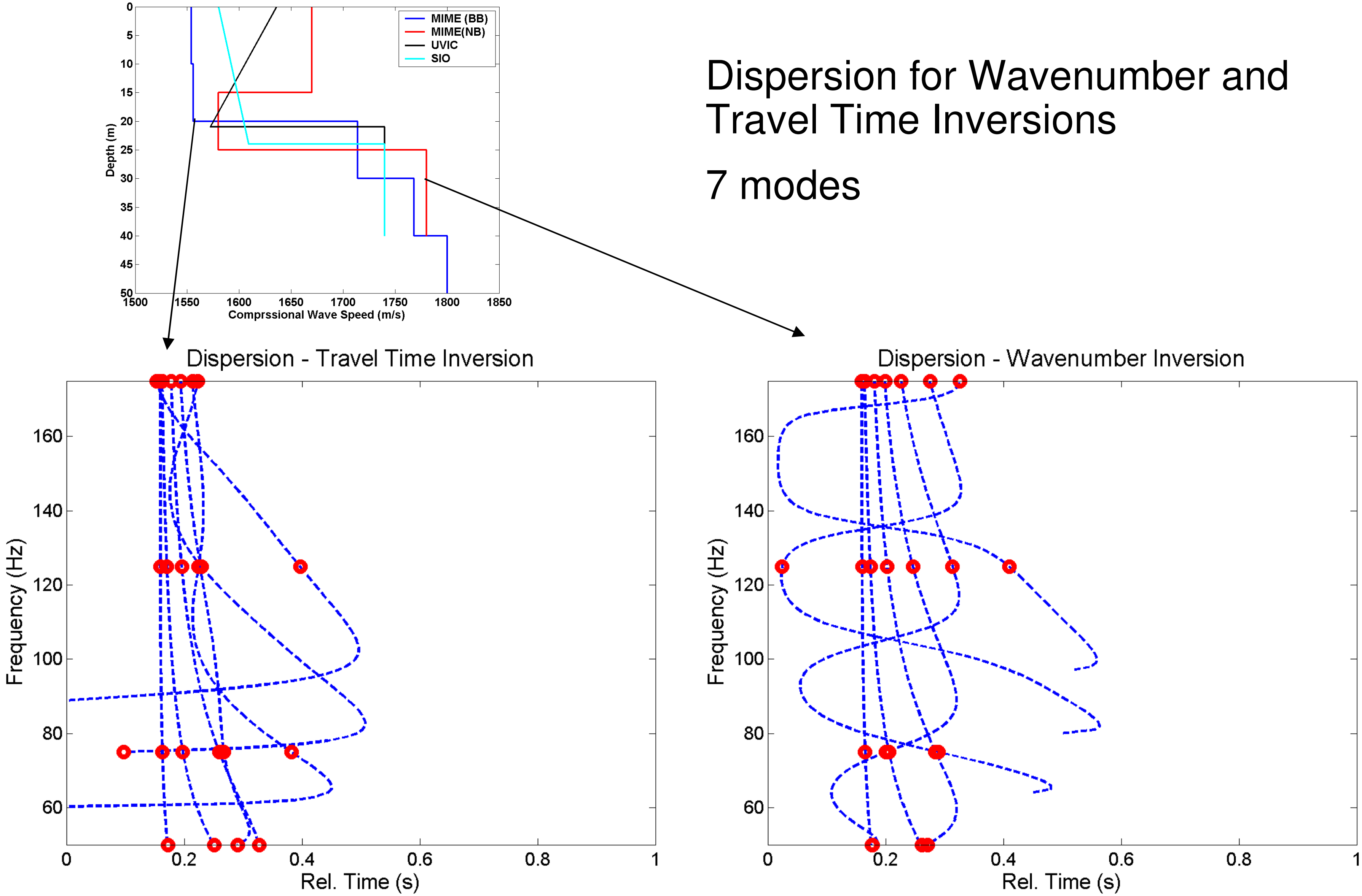




Doppler Shift – Dispersion Analysis

Dispersion for Wavenumber and Travel Time Inversions

7 modes



Doppler Shift – Dispersion Analysis

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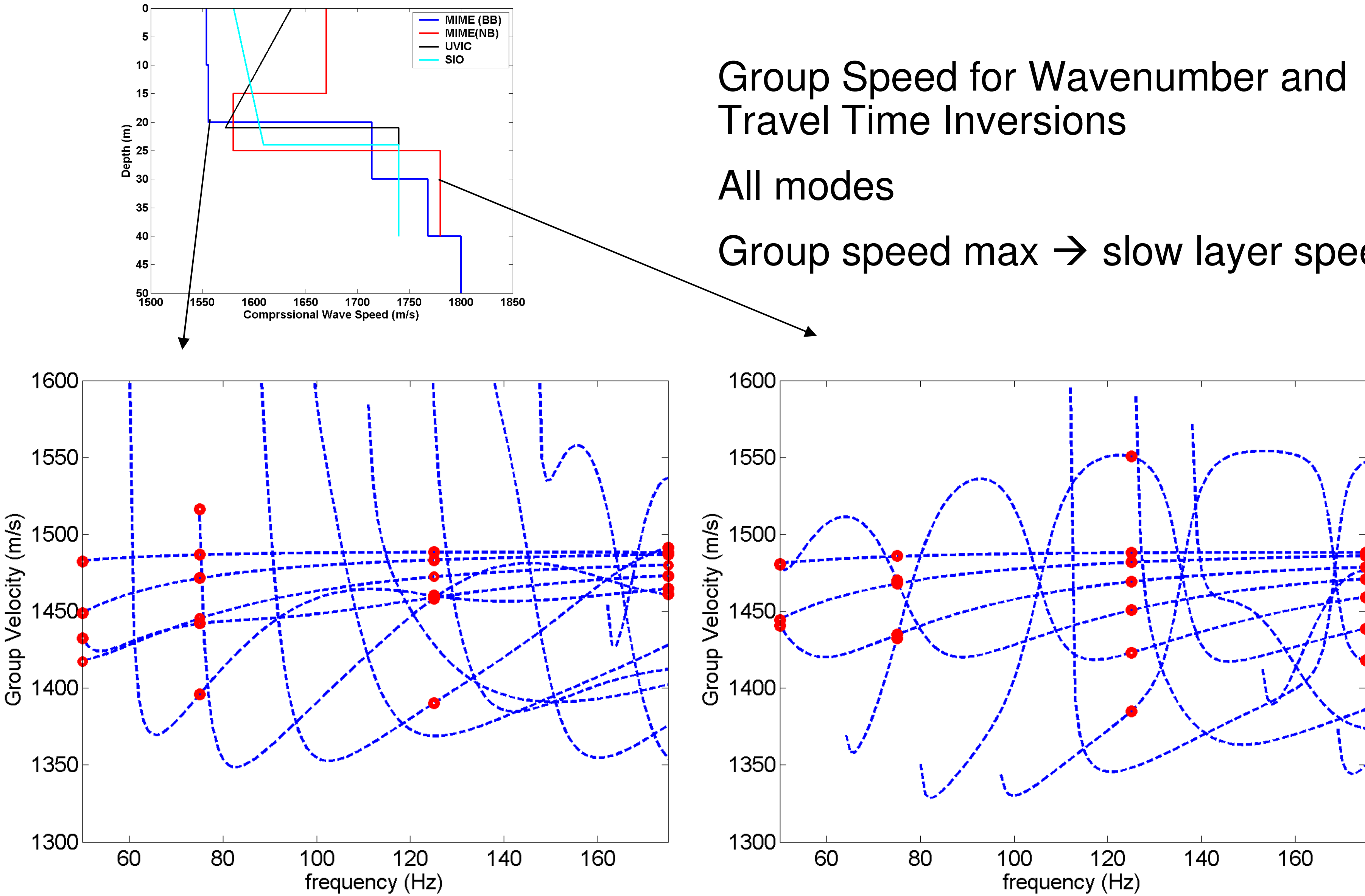

Group Speed for Wavenumber and Travel Time Inversions

All modes

Group speed max → slow layer speed

Summary

- Wavenumber and travel time inversions obtained for low-frequency acoustic measurements on VLA
- High Freq./Wavenumber analysis indicates slow-speed layer
- Dispersion characteristics for low-order modes similar
- Dispersion relationship for high-order modes has potential to validate presence of layer
- Doppler frequency shifts measurable for slow tow speeds
- Doppler wavenumber shift not consistent with physics - undetermined range registration (asynch. clock) issue

kmbecker@psu.edu