Applying the data nullspace projection method to a geoacoustic Bayesian inversion in a randomly fluctuating shallow-water ocean

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Introduction

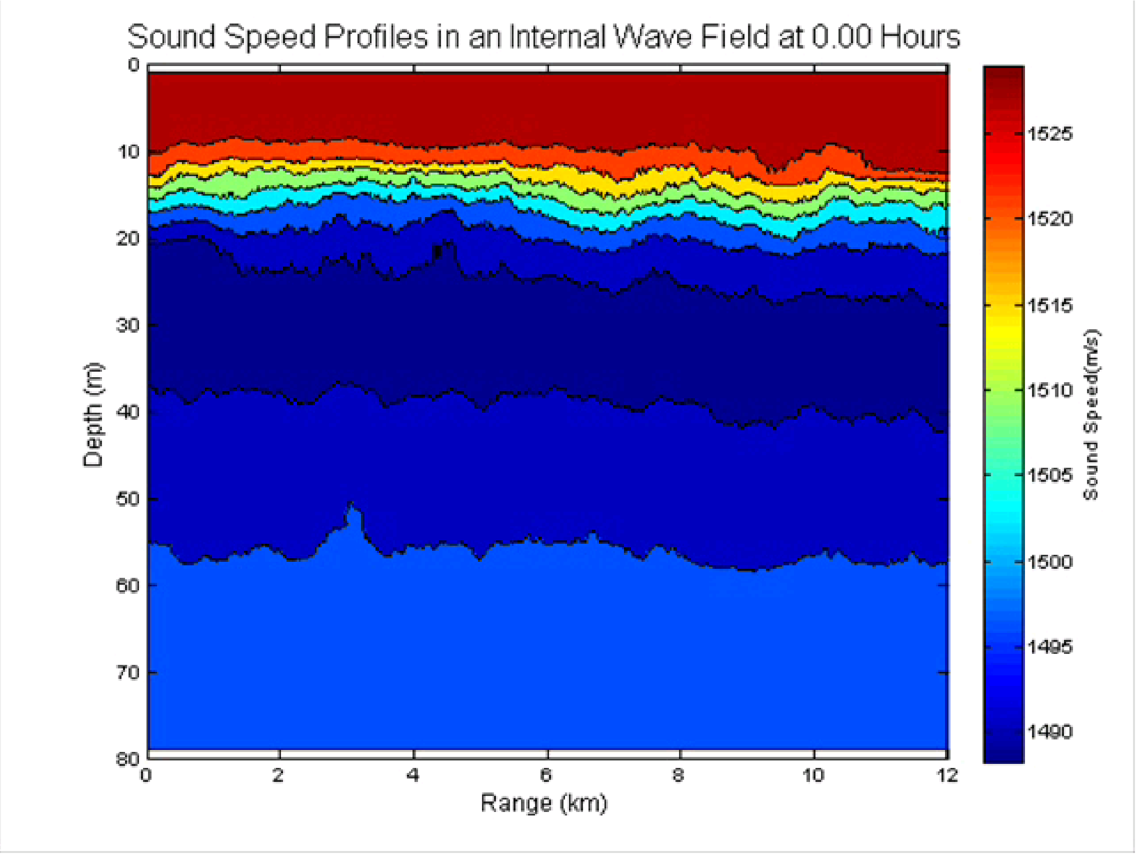
- Geoacoustic inversions can suffer the effects of uncertain water-column fluctuations.

- Inverting for the fluctuating water-column parameters increases the dimensions of parameter space so that the inversions may not be efficient, especially in the Bayesian inverse approach.

- With data nullspace projection, acoustic data are project onto a subspace that is insensitive to uncertain water-column fluctuations, and so one can directly invert for bottom properties from the projected data.

Random linear internal waves

- One of the sources causing water-column randomness is linear internal waves. (Can do non-linear as well).
- Sound speed variations in a linear internal wave field can be decomposed by a set of empirical orthogonal functions.



Simulated linear internal wave field
(model inputs derived from the SW06 experimental data)

Bayesian approach to geoacoustic inversion

- Inherited from Bayes' theorem

$$P(\mathbf{m} \mid \mathbf{d}_{obs}) = \frac{P(\mathbf{d}_{obs} \mid \mathbf{m}) \cdot P(\mathbf{m})}{\int P(\mathbf{d}_{obs} \mid \mathbf{m}) \cdot P(\mathbf{m}) \, d\mathbf{m}},$$

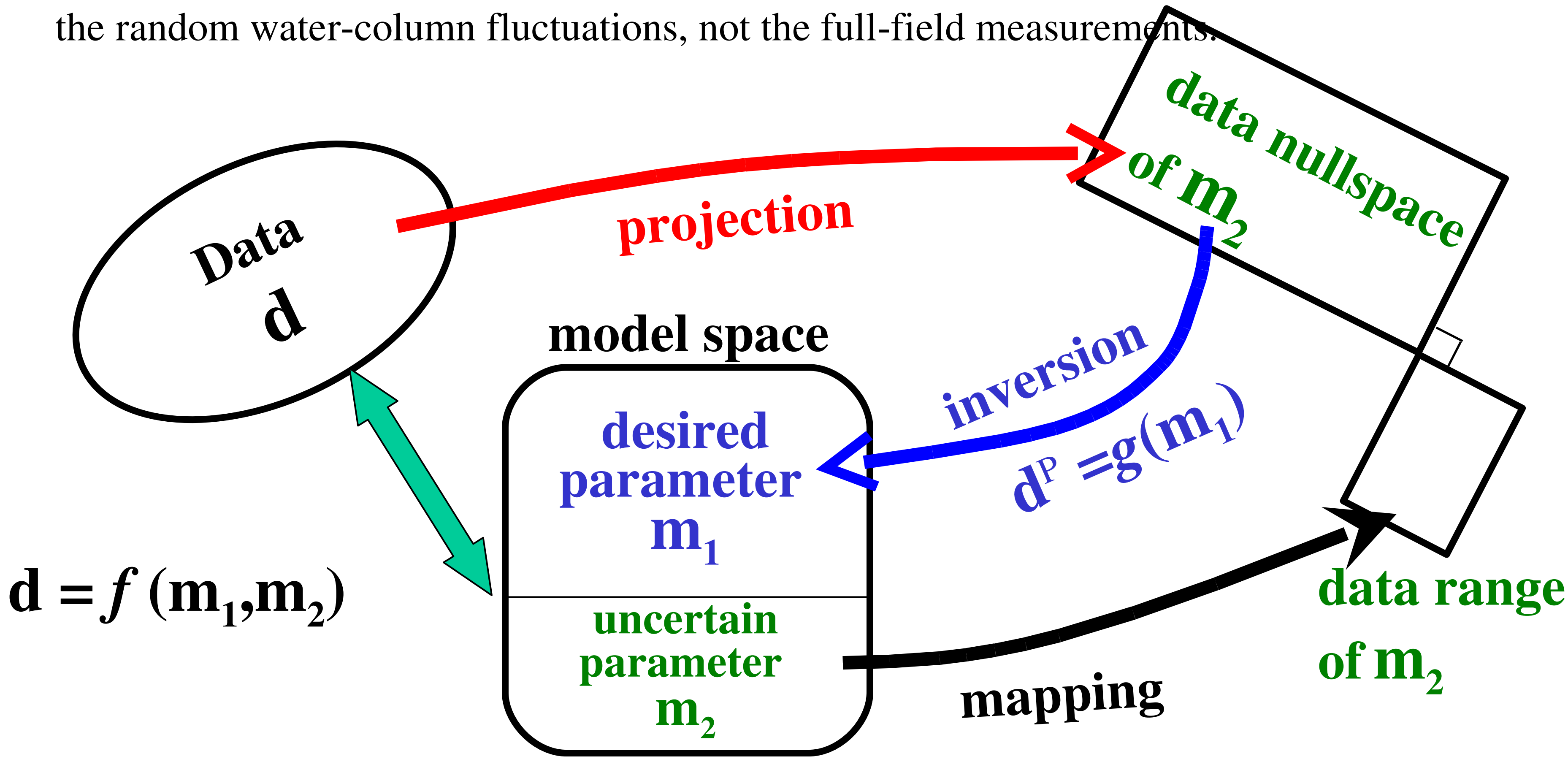
  where $\mathbf{d}_{\mathrm{obs}}$ and **m** are acoustic data measurements and environmental model parameters, respectively.

- The conditional probability function $P(\mathbf{d}_{\mathrm{obs}}|\mathbf{m})$ defines a likelihood function $L(\mathbf{m})$ for the model parameters with fixed acoustic data measurements.

$$\underbrace{P(\mathbf{m} \mid \mathbf{d}_{obs})}_{\text{posterior probability density of } \mathbf{m}} \propto L(\mathbf{m}) \cdot \underbrace{P(\mathbf{m})}_{\text{prior information of } \mathbf{m}}$$

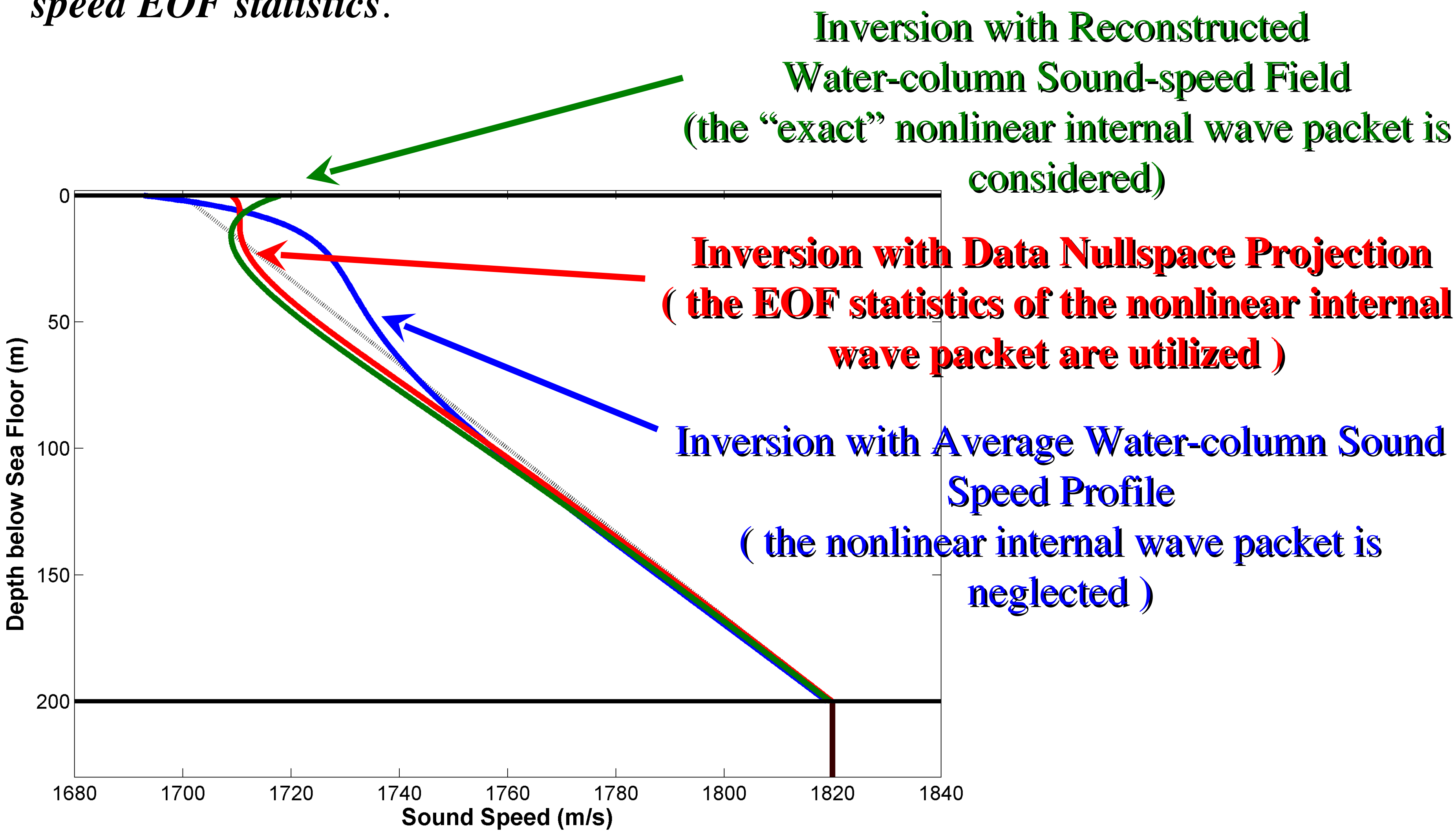
Nullspace pre-processor

- Uncertain/random water-column fluctuations can cause errors in acoustic inversions, and ***the data nullspace projection method*** has been developed to reduce the errors.
- This method is designed to expose desired information (bottom geoacoustic parameters or acoustic source location) by projecting the acoustic signal in an uncertain water-column channel onto its data nullspace.
- This projection method requires the knowledge ***the mean and the second-order statistics*** of the random water-column fluctuations, not the full-field measurements.



Range-Averaged Bottom Sound Speed Inversion using Modal Group Velocity

In using the data nullspace projection method, we determine ***the acoustic data nullspace of water column fluctuations*** from perturbation theory with ***sound speed EOF statistics***.



Bayesian inversion with data nullspace projection

- With the projection method, the data observation and replica are projected onto the data nullspace prior to calculating likelihood function.
  - Original form of Gaussian likelihood function

$$L(\mathbf{m}) \propto \exp\left(-\frac{1}{2}\left(G(\mathbf{m}) - d_{obs}\right)^{\mathrm{T}} C_d^{-1}\left(G(\mathbf{m}) - d_{obs}\right)\right)$$

$$\propto \exp\left(-\frac{1}{2}\Delta d(\mathbf{m})^{\mathrm{T}} C_d^{-1} \Delta d(\mathbf{m})\right)$$

  - After projection,

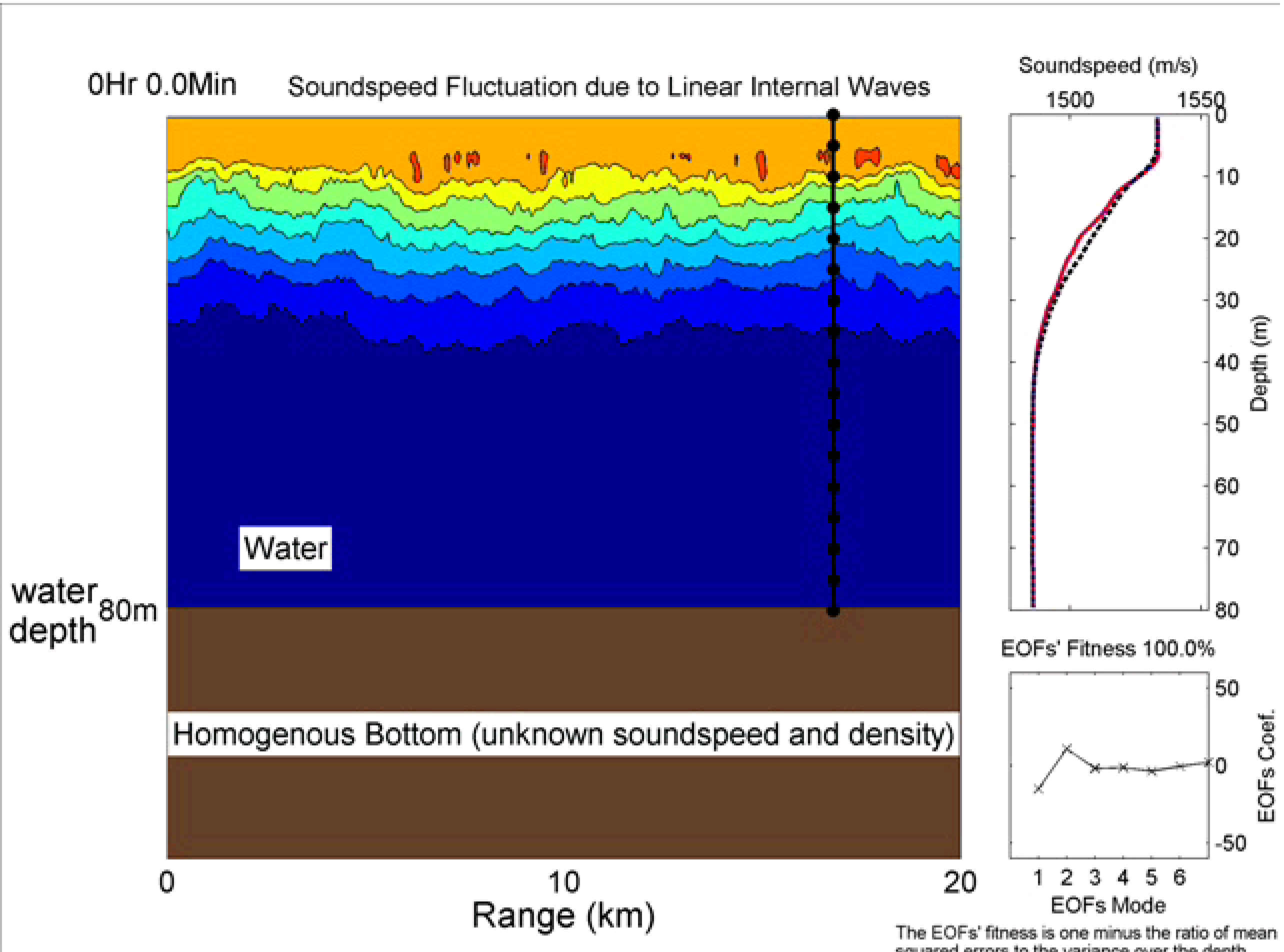
$$L(\mathbf{m}) \propto \exp\left(-\frac{1}{2}\left(N \cdot \Delta d(\mathbf{m})\right)^{\mathrm{T}}\left(N \cdot C_d \cdot N^{\mathrm{T}}\right)\left(N \cdot \Delta d(\mathbf{m})\right)\right)$$

where C_d and N are data covariance matrix and data nullspace matrix, respectively.

Geoacoustic inversion in the presence of internal waves
— Bayesian approach using modal phase speeds

(numerical simulation)

Linear internal wave model



multiple frequencies and modes

50Hz (2 modes),

75Hz (3 modes),

125Hz (4modes)

175Hz (4modes)

total 13 mode data

unknown parameters

first <u>3 watercolumn soundspeed EOF coefficients</u>

<u>soundspeed</u> and <u>density</u> in the homogeneous bottom

total 5 unknown parameters

Geoacoustic inversion in the presence of internal waves

— Bayesian approach using modal phase speeds

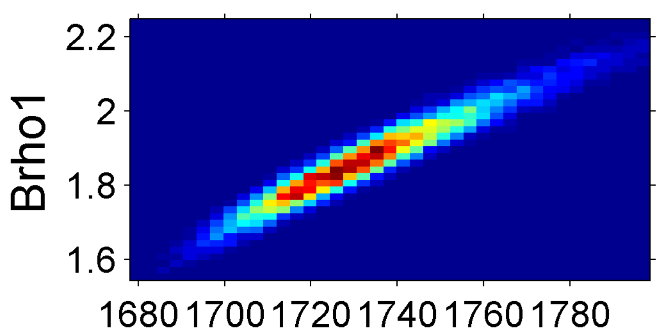
- **Two inversions are compared**
  - ***without data nullspace projection***
    - 2 bottom parameters and 3 water-column soundspeed EOF coefficients
  - ***with data nullspace projection***
    - 2 bottom parameters

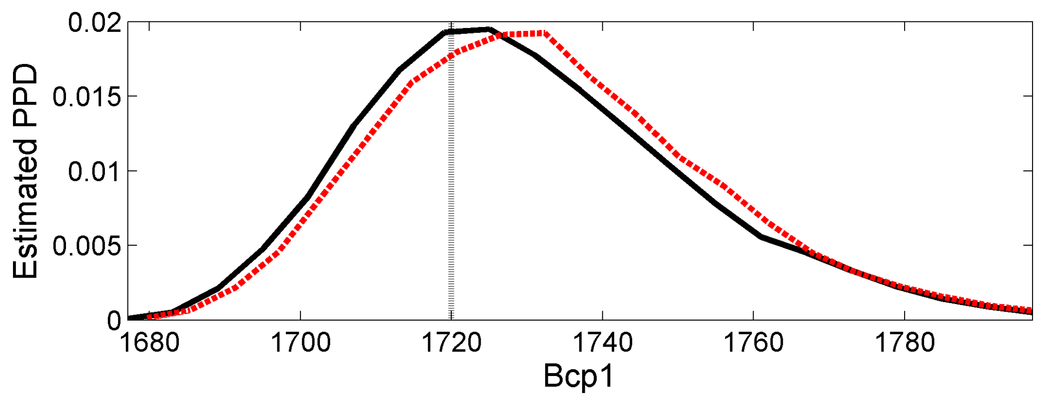
- The Metropolis sampling algorithm is used to calculate the posterior probability density of model parameters.

Geoacoustic inversion in the presence of internal waves
— Bayesian approach using modal phase speeds

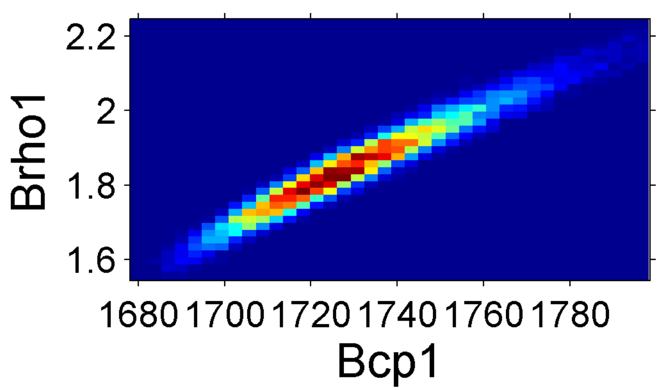
- Inversions comparison

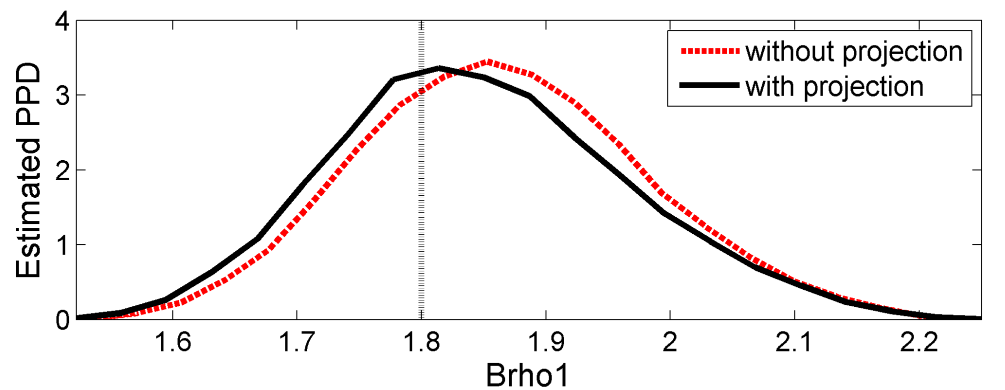
without data nullspace projection





with data nullspace projection





Conclusion

- Determine the ***acoustic data nullspace*** of water-column fluctuations from the ***EOF statistics***, and expose the bottom information contained in the acoustic signals propagating in the random ocean.
- The numerical simulation shows that the inversion with data nullspace projection produces better solutions than the inversion without projection, even when solving for both bottom and water-column parameters.

Future Work

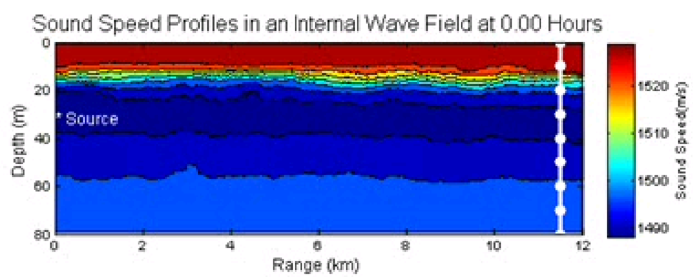
• Applying this projection method to the acoustic data collected in the SW06 experiment for bottom inversion, e.g. range-dependent modal wavenumbers (K. Becker, OSU, and G. Frisk, FAU).

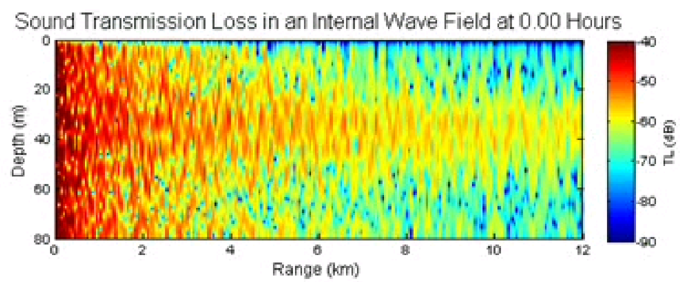
•Applying this projection method to other inverse problems and acoustic signal processing in the dynamic ocean.

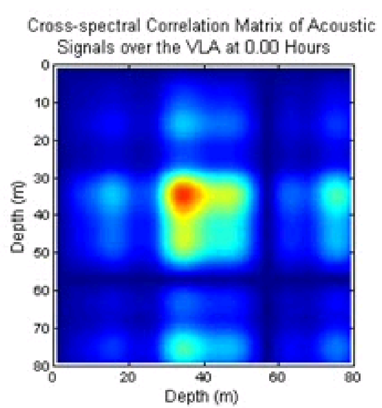
Matched-Field Source Localization

Simulation Study

- In this simulation study, a random linear internal wave field is generated, where the mean profiles follow the measurements in the **SW06**.
- A 25-elements VLA is placed at 11.5km far from the source, which is at depth 32m and transmits 225Hz monotone signal. The signal to noise ratio is set to be 20dB.
- In this test case, the data nullspace is directly estimated from the data-data covariance over the VLA, which is the best estimate. A perturbation approach, incorporating sound speed EOF's, is a work in progress.







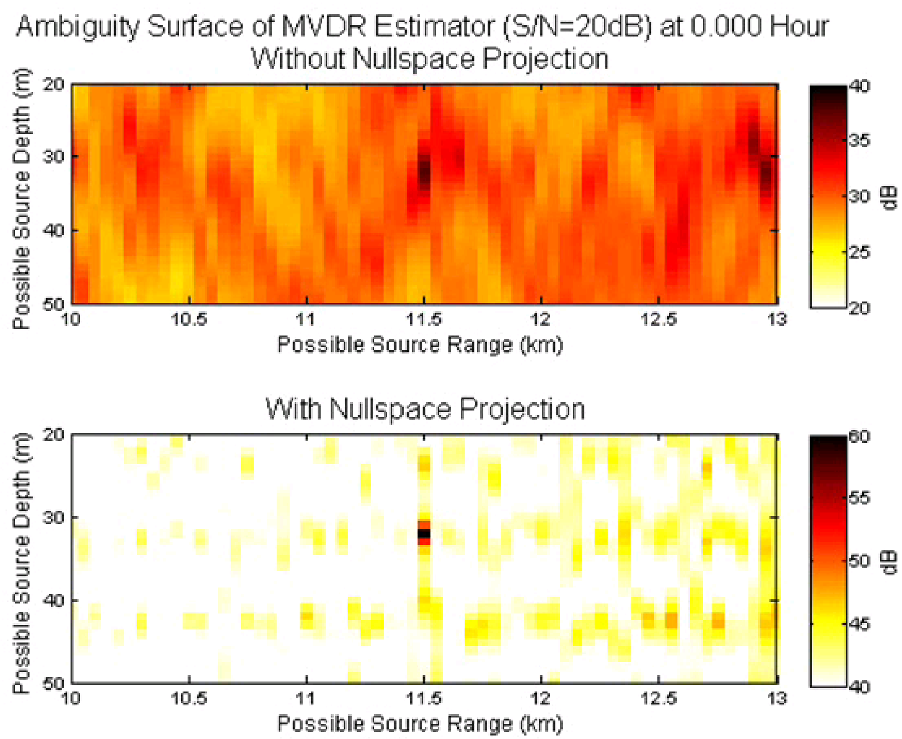
Matched-Field Source Localization

- To add complexity to the simulation, the synthetic received signals on the VLA at three successive time steps are coherently averaged. The **mean soundspeed field** is utilized to calculate the acoustic signal replica.

- The Capon's MVDR operator is applied. With the projection method, the received signal and replica are projected onto the data nullspace prior to implementing the MVDR operator.

- The result from using the data nullspace projection method is far better than not using the method.

True source location (range 11.5km, depth 32m)



Questions??