ARL
The University of Texas at Austin

Methods of characterization of seabed physics for a shallow water environment

D. P. Knobles

Colleagues: P. Wilson, J. Goff

S. Joshi, L. Collins, S. Cho

155th Meeting of the Acoustical Society of America
Paris, France June-July 2008
Work supported by ONR Code 321 OA

Outline

- Description of measurements and analysis approach
- Inversion and maximum entropy principle
- Application to low frequency data taken on the New Jersey continental shelf
    - ➢ Inversion for global minimum
    - ➢ Marginal probability distributions
        - ✓ Sensitivity to signal processing
    - ➢ Transmission loss uncertainty
    - ➢ Comparison to measured uncertainty

Description of measurements and analysis

From limited acoustic measurements in ocean water column

- Inverse problem solved for global minimum environmental solution
  - Better resolution of attenuation is inferred from long range propagation data
    - Inference of Biot parameter bounds
    - Scattering parameters inferred from reverberation measurements
    - Modeling of wind generated ambient noise

Inversion in a nutshell

Consider a space Γ with volume Ω that contains source-receiver positions, kinematical parameters, and ocean waveguide parameters

W is a vector in **Γ**

$$\mathbf{W} = (\mathbf{w}_1, \mathbf{w}_2, \ldots \mathbf{w}_N)$$

D - Measured data vector
M - Modeled data vector

An objective of cost function defined

$$C(\mathbf{W}) = C(\mathbf{D}, \mathbf{M}(\mathbf{W})) \sim 1 - \text{correlation}\ (\mathbf{M} \cdot \mathbf{D})$$

Inversion algorithm is used to explore C(**W**)

Simulated annealing is used to find the global minimum
$C_{min} = C(\mathbf{W}_{gm})$

Description of measurements and analysis

But

- Uncertainty of waveguide parameters leads to uncertainty in propagation
- How does one quantify the uncertainty?
- Under what circumstances does uncertainty obtained from models and inversion methods = true environmental uncertainty?
- How does this uncertainty affect inferences of seabed physics from basic measurements?

Therefore, one needs a mathematical framework from which to compute probability distributions

ARL

The University of Texas at Austin

Ideas for distribution

- Cost measures error relative to horizontal stratification
- Uncertainties arise from small fluctuations relative to horizontal stratification
- One does not know the distribution; thus derive the most conservative distribution that only predicts specific constraints

ARL

Uncertainty from Maximum Entropy Principle

What is the probability distribution for a specific parameter in W or transmission loss?
Following Jaynes (Phys. Rev. 106 1957)

$$S = -\int_\Omega d\mathbf{W}\ \rho(\mathbf{W}) \ln \frac{\rho(\mathbf{W})}{\rho^0(\mathbf{W})}$$

Gibbs or Shannon relative Entropy

The two constraints are

$$\int_\Omega d\mathbf{W}\ \rho(\mathbf{W}) = 1$$

Analogy with statistical mechanics for a closed system in thermodynamic equilibrium with heat reservoir

$$< C >= \int_\Omega d\mathbf{W}\ \ \rho(\mathbf{W})\ C(\mathbf{W})\ = \frac{1}{2}(C_{min} + \bar{C})$$

C_{min} is global minimum determined from simulated annealing

$\bar{C}$ average value of cost function space $= 1/N \sum C(W_i)$

ARL

Maximum entropy principle and canonical ensemble

$$\delta\left(\int_\Omega d\mathbf{W}\ \left[A1\ \rho(\mathbf{W}) + A2\ C(\mathbf{W})\rho(\mathbf{W}) - \frac{\ln([\rho(\mathbf{W})]}{\rho^0(\mathbf{W}))}\right]\right) = 0$$

$$\rho(\mathbf{W}) = \frac{\rho^0(\mathbf{W})\exp\left(-C(\mathbf{W})/T\right)}{Z}$$

canonical ensemble

$$Z = \int_\Omega d\mathbf{W}\ \rho^0(\mathbf{W})\exp\left(-C(\mathbf{W})/T\right)$$

partition function

Average <C> constraint determines T

$$S = \ln Z + \frac{<C>}{T}$$

Entropy in terms of Z, T, and <C>

★ *ARL* Mean, standard deviations, and marginals

$$< Y >= \frac{\int_\Omega d\mathbf{W} \mathbf{Y}(\mathbf{W}) \exp(-\mathbf{C}(\mathbf{W})/\mathbf{T})}{Z}$$

$$\sigma_Y = \sqrt{\frac{\int_\Omega d\mathbf{W} (\mathbf{Y}(\mathbf{W}) - < \mathbf{Y} >)^2 \exp(-\mathbf{C}(\mathbf{W})/\mathbf{T})}{Z}}$$

Reduced or marginal distribution

$$P(\mathbf{w_i}) = \frac{\int_\Omega \mathbf{dW}' \delta(\mathbf{w_i'} - \mathbf{w_i})(\exp(-\mathbf{C}(\mathbf{W}')/\mathbf{T})}{\mathbf{Z}}$$

Evaluation of volume integrals

$$\mathbf{W_j} = (\mathbf{{}_j w_1, {}_j w_2, \ldots {}_j w_N})$$ A point in Γ

$$< Y >= \frac{(\Omega/N)\sum_i^N Y(\mathbf{W_i})\exp(-\mathbf{C}(\mathbf{W_i})/\mathbf{T})}{(\Omega/N)\sum_i^N \exp(-C(\mathbf{W_i})/\mathbf{T})}$$

$$< Y >= \frac{\sum_i^N Y(\mathbf{W_i})\exp(-\mathbf{C}(\mathbf{W_i})/\mathbf{T})}{\sum_i^N \exp(-C(\mathbf{W}_i)/T)}$$

$$P(\mathbf{w_i}) = \frac{\sum_{\mathbf{j}}^{\mathbf{N}} \delta(\mathbf{w_i} -_{\mathbf{j}} \mathbf{w_i})(\exp(-\mathbf{C}(\mathbf{W})_{\mathbf{j}}/\mathbf{T})}{\mathbf{Z}}$$

Volume integrations

- Sampling of Γ by random walks in limit that N becomes large ~ Monte Carlo sampling
- Convergence criteria: Marginal distributions remain unchanged when number of samples increased
- ~ 2×10^6 samples appears sufficient for problem considered

ARL

Hybrid Cost Function

Center Frequency Averaged Source Level

Center Frequency Model Cross Spectrum

Center Frequency Averaged Cross–Spectral Data

$$C = \sum_{\substack{\text{center} \\ \text{frequencies}}} \frac{\sum_{\substack{\text{element pairs,} \\ \text{sequences}}} \left| < D_i D_j^* > - |S_f| M_i^* M_j \right|^2}{N_{CEN} \sum_{\substack{\text{element pairs,} \\ \text{sequences}}} \left| < D_i D_j^* > \right|^2}$$

$$< D_i D_j^* > = |D_{RL}|^2 \sum_{\text{bins}} D_i D_j^* / |D_{REF}|^2$$

Cross Spectrum Normalization for Center Frequency Averaging

$$|D_{REF}|^2 = N_{BINS} \sum_{\text{elts}} |D_i|^2 / N_{ELTS}$$

Cross Spectrum Normalization for Center Frequency Average RL

$$|D_{RL}|^2 = \sum_{\text{bins,elts}} |D_i|^2 / (N_{ELTS} N_{BINS})$$

Minimization of Cost Gives SLs

$$|S_f| = \frac{\sum_{\substack{\text{element pairs,} \\ \text{sequences}}} < D_i D_j^* > M_i^* M_j + \text{c.c.}}{2 \sum_{\substack{\text{element pairs,} \\ \text{sequences}}} |M_i^* M_j|^2}$$

ARL Hybrid Cost Function, cont.

Substituting for $|s_f|$ gives correlation form of cost function: $0 \le C \le 1$

$$C = 1 - \underbrace{\sum_{\substack{\text{center}\\\text{frequencies}}}}_{\text{incoherent sum over center frequency}} \frac{\left| \underbrace{\sum_{\substack{\text{element pairs,}\\\text{sequences}}}}_{\text{coherent sum over pairs and sequences}} \frac{\langle D_i D_j^* \rangle M_i^* M_j + \text{c.c.}}{2} \right|^2}{N_{CEN} \sum_{\substack{\text{element pairs,}\\\text{sequences}}} \left| \langle D_i D_j^* \rangle \right|^2 \sum_{\substack{\text{element pairs,}\\\text{sequences}}} |M_i^* M_j|^2}$$

Includes gain in the coherent sum over pairs and sequences to fit multipath arrivals and source track dependence.

Includes amplitude information to fit TL shape.

Greater weight for higher RL data.

Increases number of unknowns.

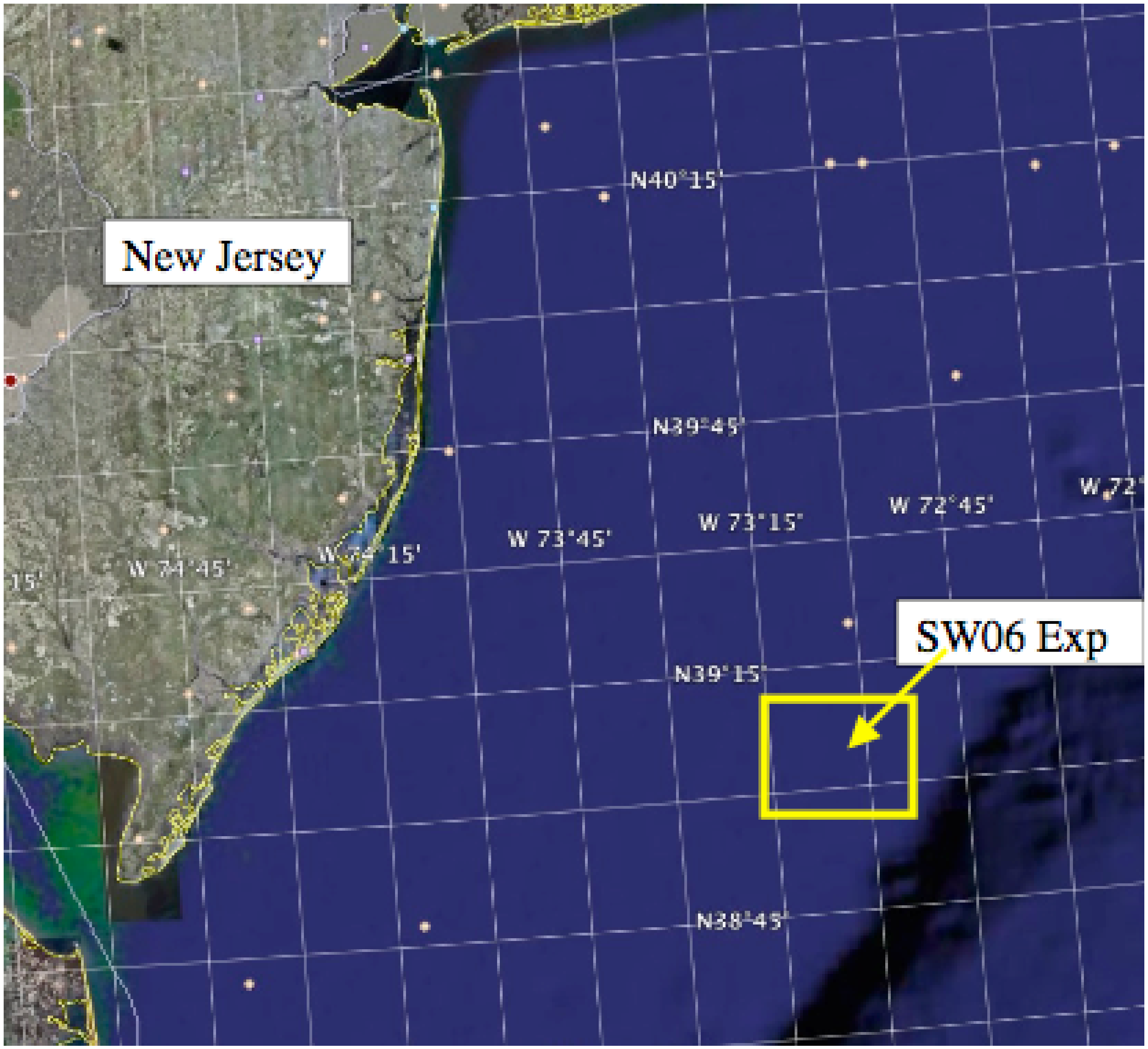
Outline

- Description of measurements and analysis approach
- Inversion and maximum entropy principle
- Application to low frequency data taken on the New Jersey continental shelf
  - Inversion for global minimum
  - Marginal probability distributions
    - Sensitivity to signal processing
  - Transmission loss uncertainty
  - Comparison to measured uncertainty

Experimental goals

- Infer frequency dispersion of seabed attenuation
- Test various theories of seabed physics that predict attenuation
- Effects of seabed variability on propagation
- Sensitivity of ambient noise and reverberation on seabed physics

Experimental area



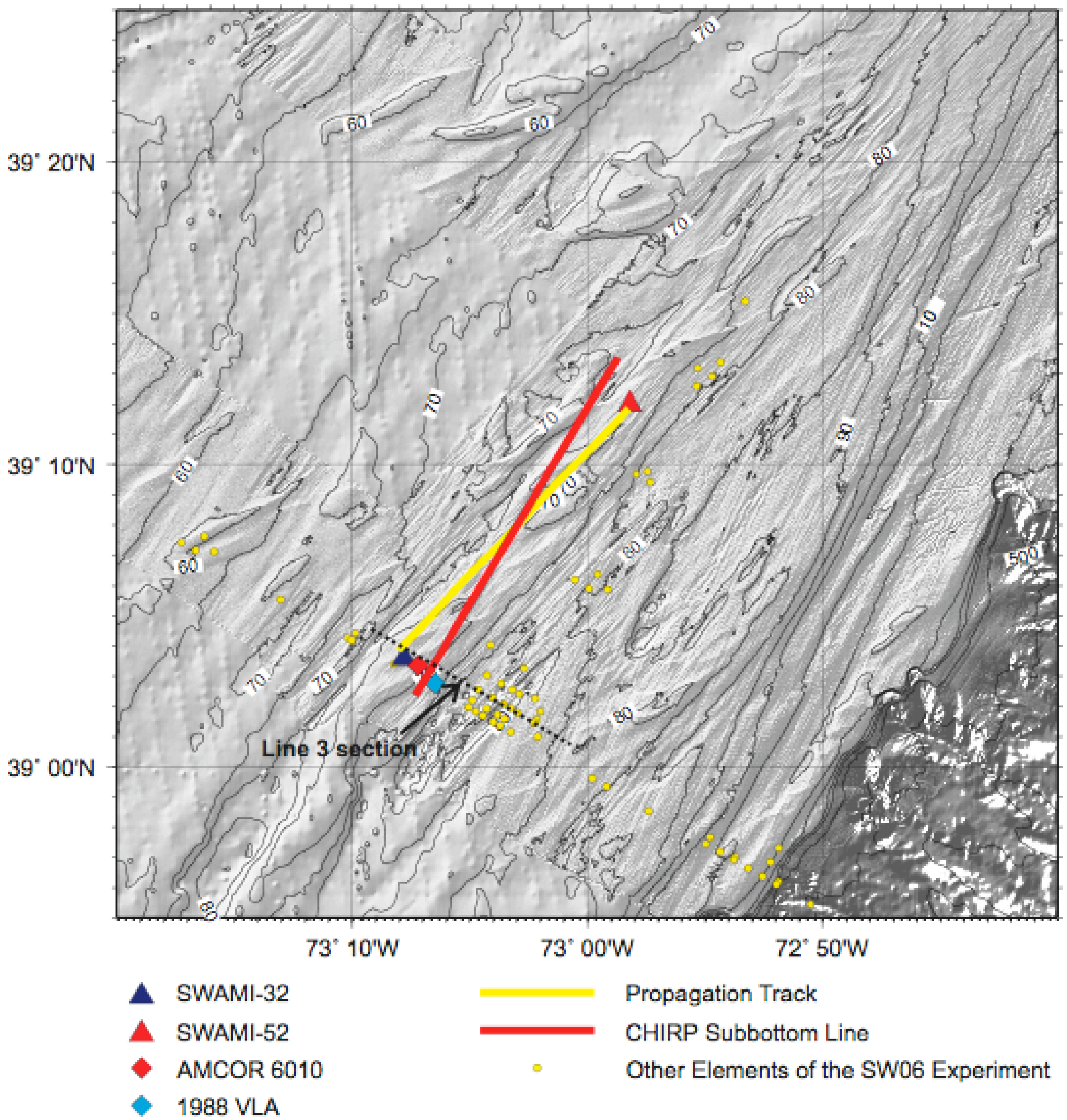
August-September 2006 SW06

BTEC measurement September 2003

ARL

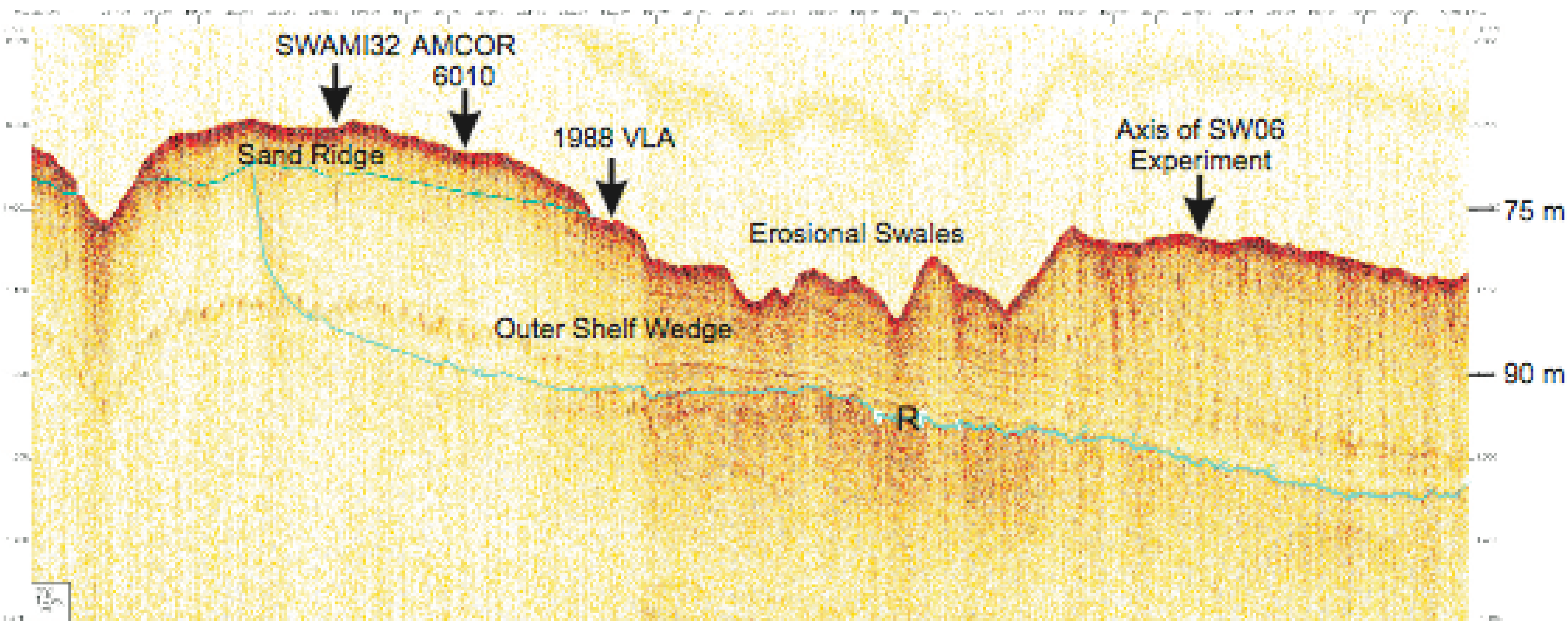
The University of Texas at Austin

Array locations during SW06



ARL Sub-bottom layering along dip-line

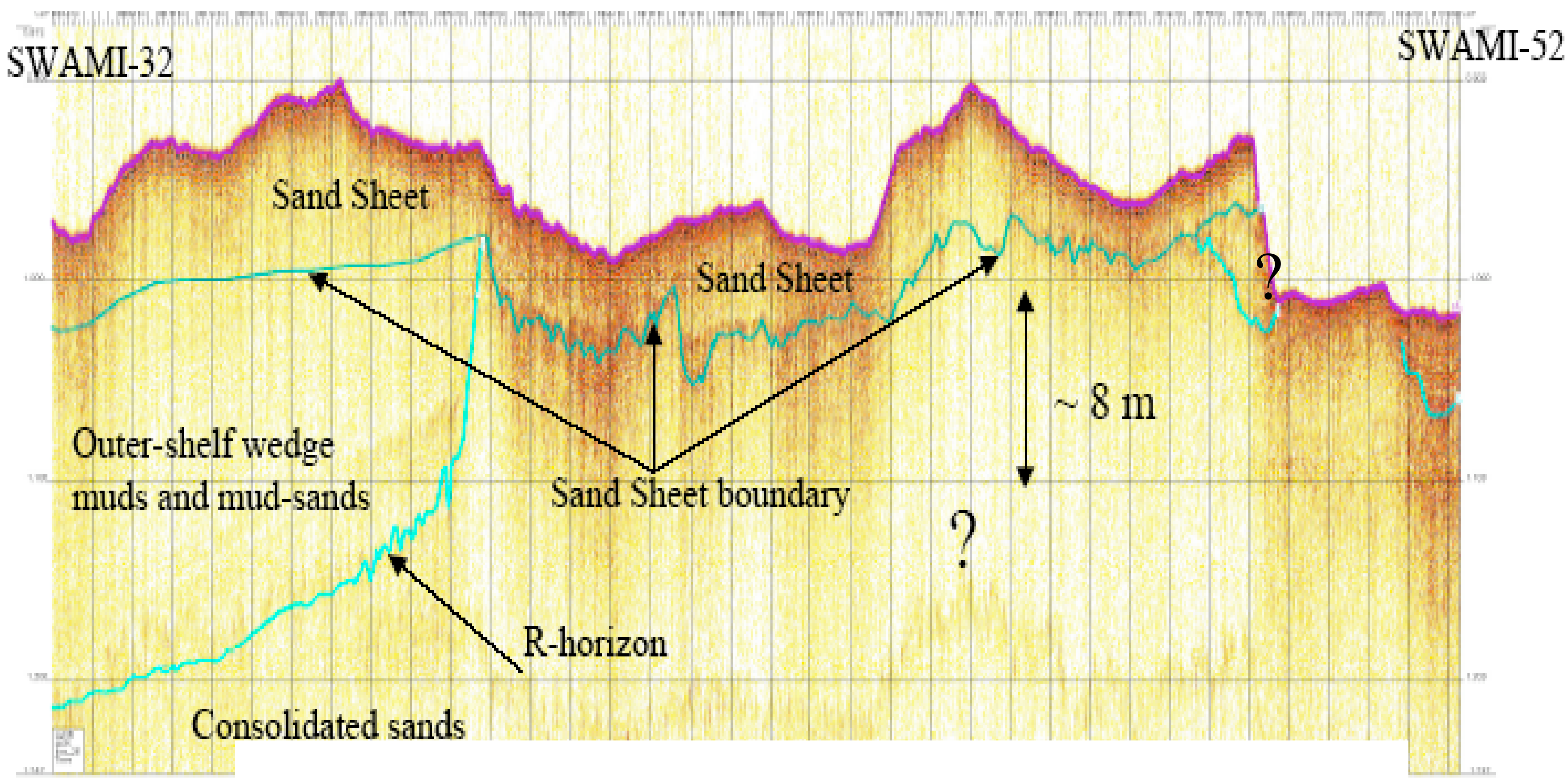
Design of experiment was to place L-array on uniform sand sheet



Chirp reflection image provided by John Goff

ARL

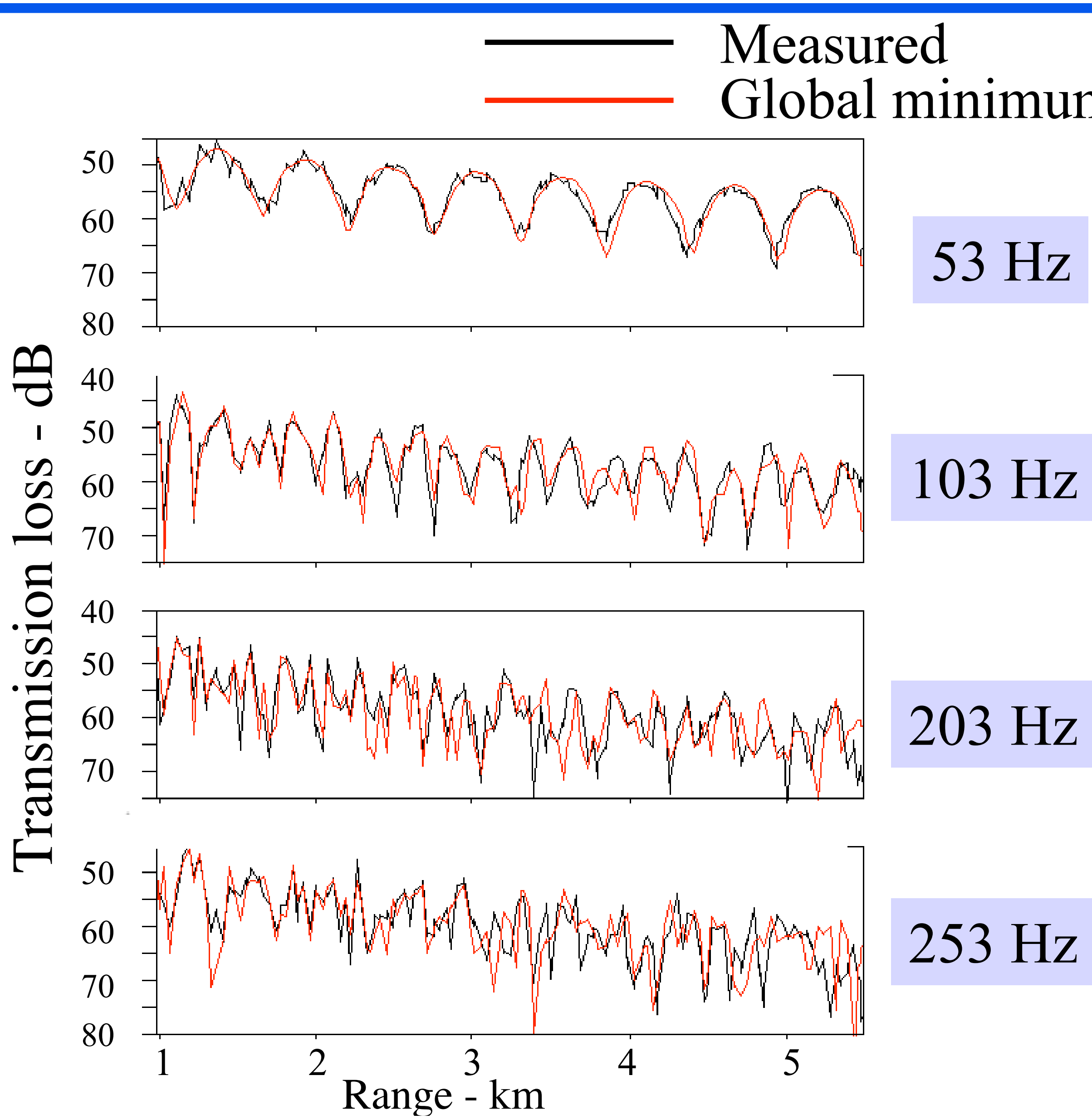
Sub-bottom between two L-arrays



Chirp reflection image provided by John Goff

ARL
The University of Texas at Austin

Comparison of TL at global minimum of hybrid cost function; Array 2



Cost function used complex beam data for two subapertures with full HLA divided into two equal lengths

HLA aperture = 230 m

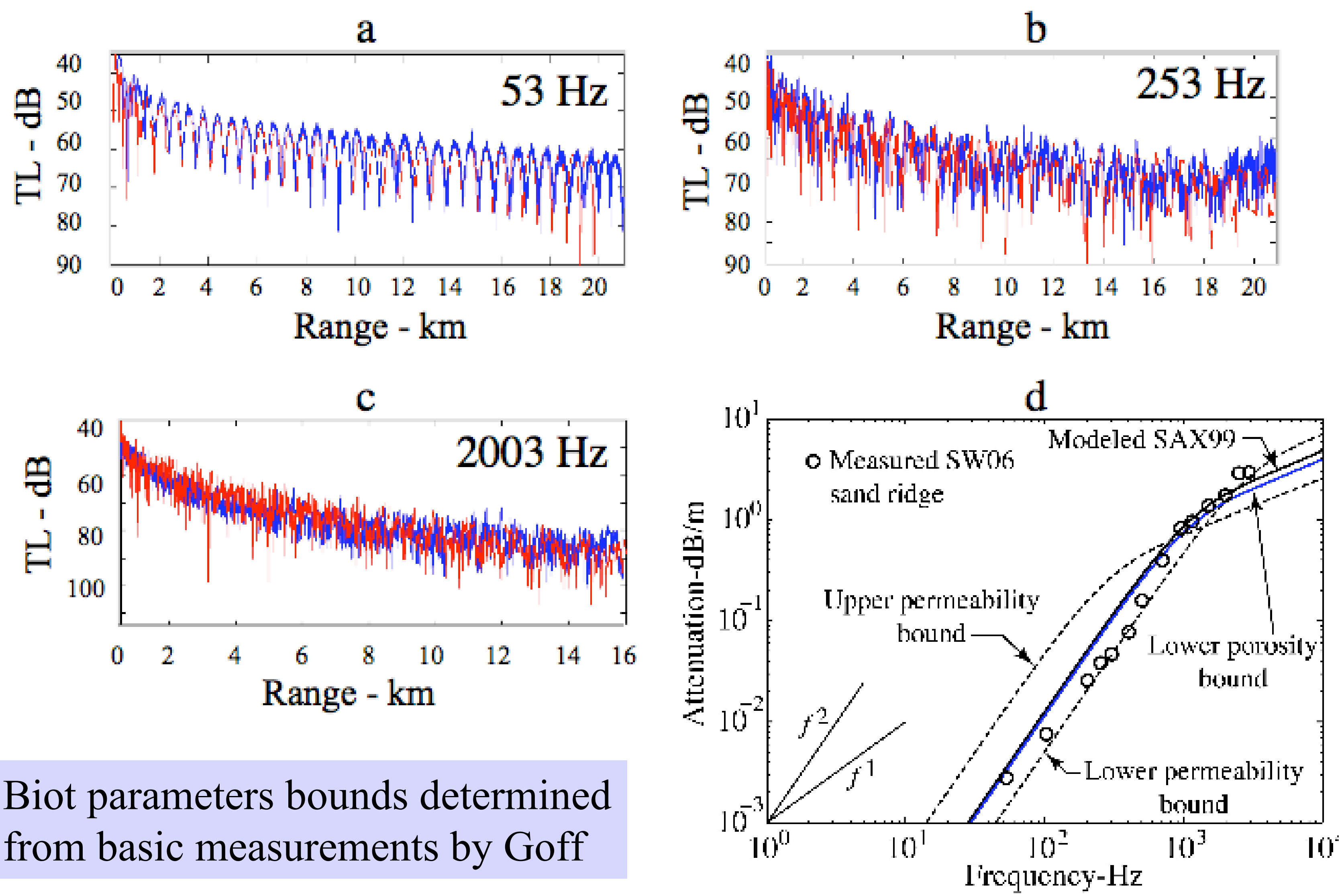
ARL

Methodology to extract frequency dependence

Horizontal variability is small enough on range scales of 20 km to extract attenuation structure over large bandwidth

- Use coherent Full Field Inversion (FFI) technique on low-frequency tow data and impulsive sources at two array locations to invert for
  - Sound speed structure in sediment
- Include range-variability with PE RAM to extract attenuation
- Extend to higher frequencies at Array 1 location

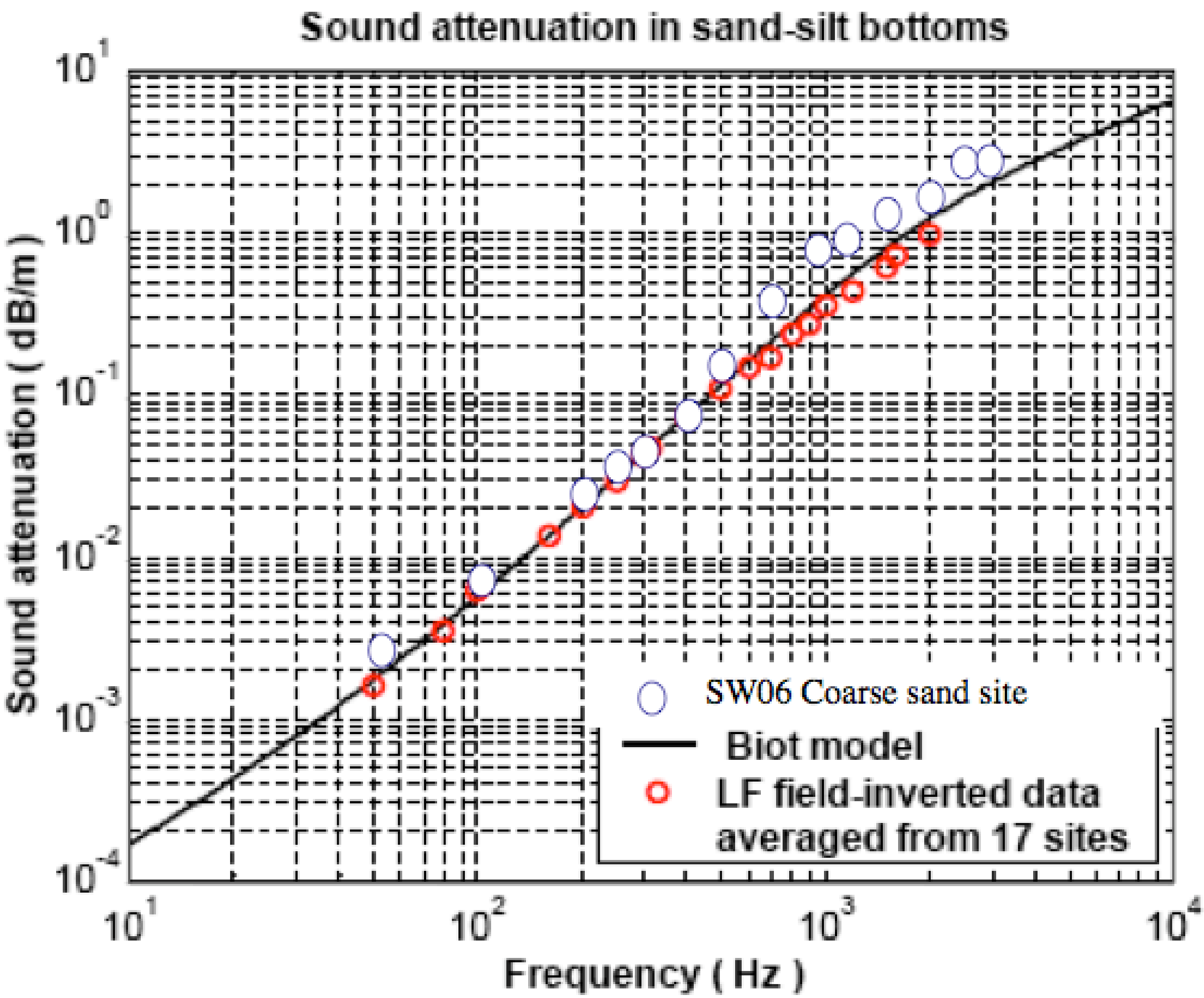
Inferred attenuation and comparison to Biot model



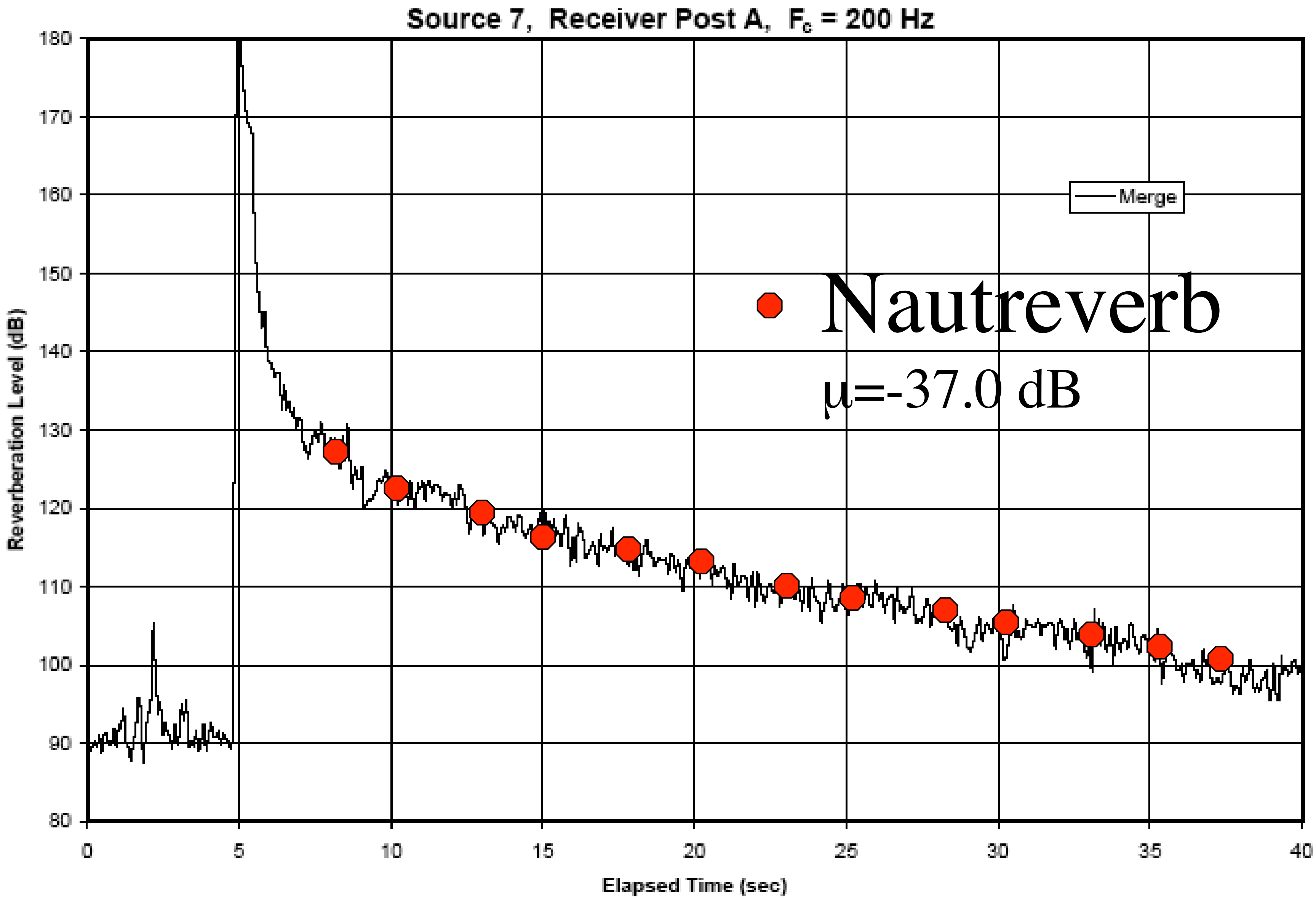
Biot parameters bounds determined from basic measurements by Goff

★ *ARL*

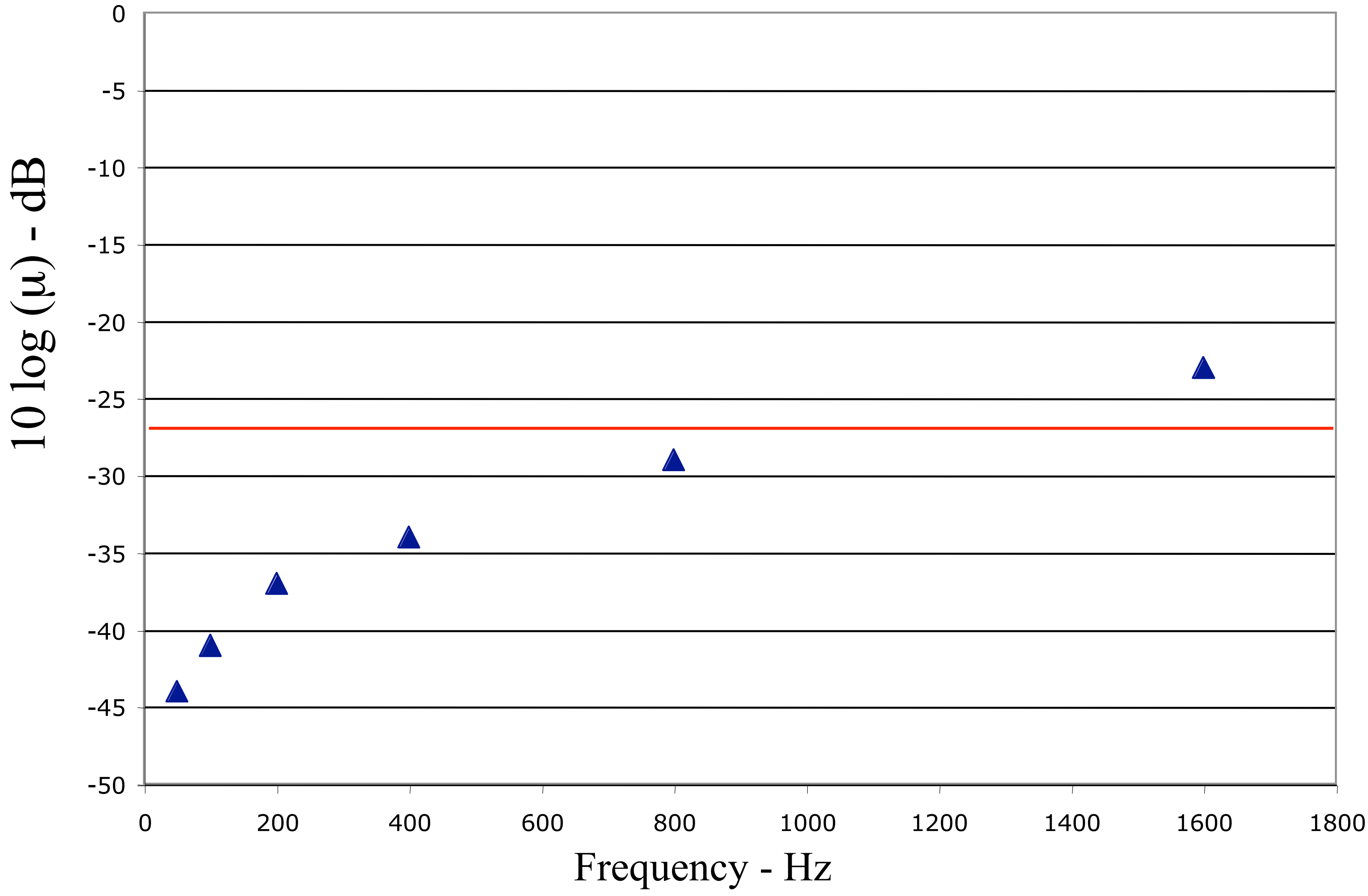
Comparison to Zhou study



Example of use of global minimum solution: Modeling measured reverberation time series with Lamberts law

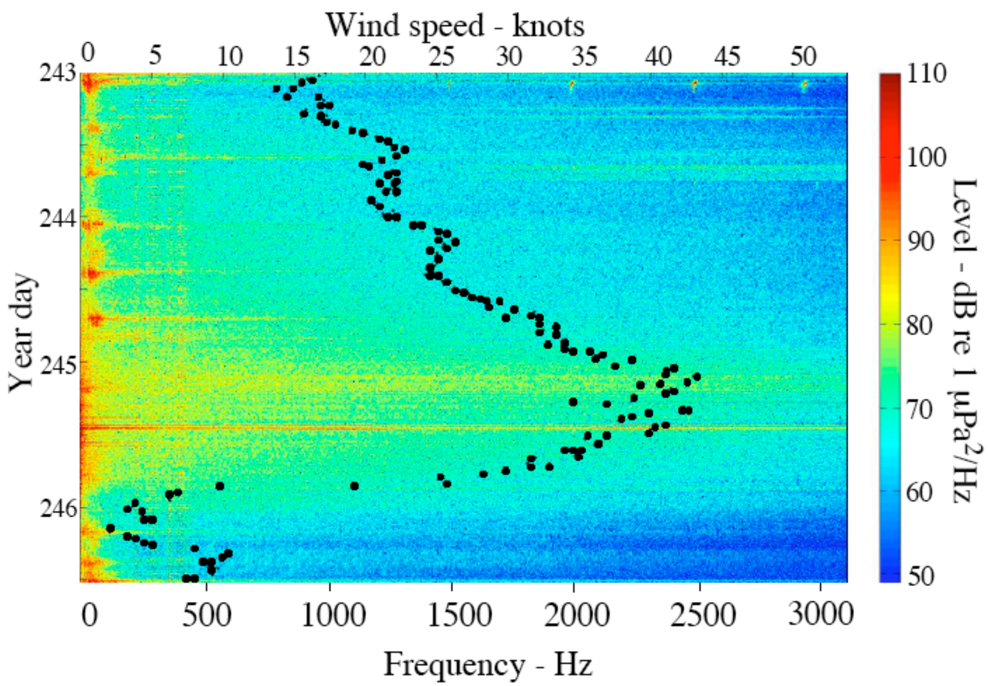


Example of use of global minimum solution: Extracted μ values assuming Lamberts law

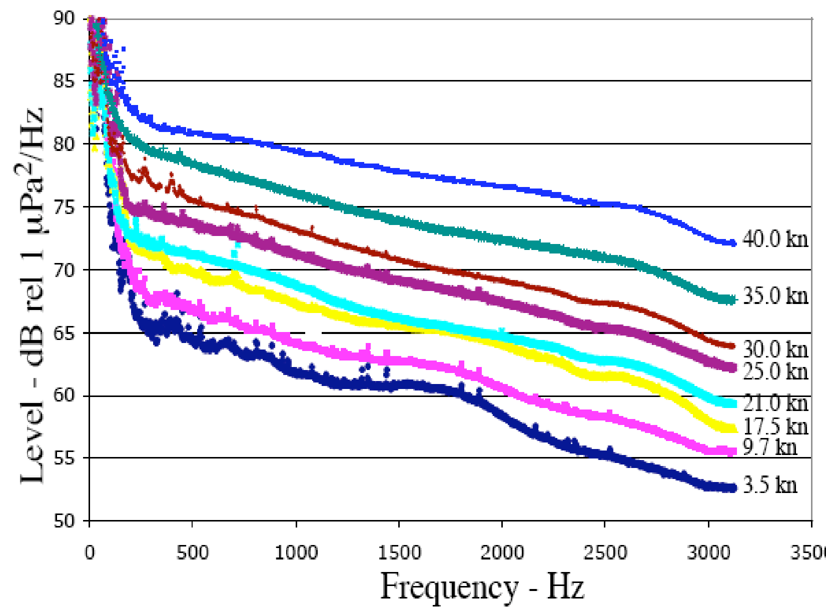


Measured wind noise during TS Ernesto

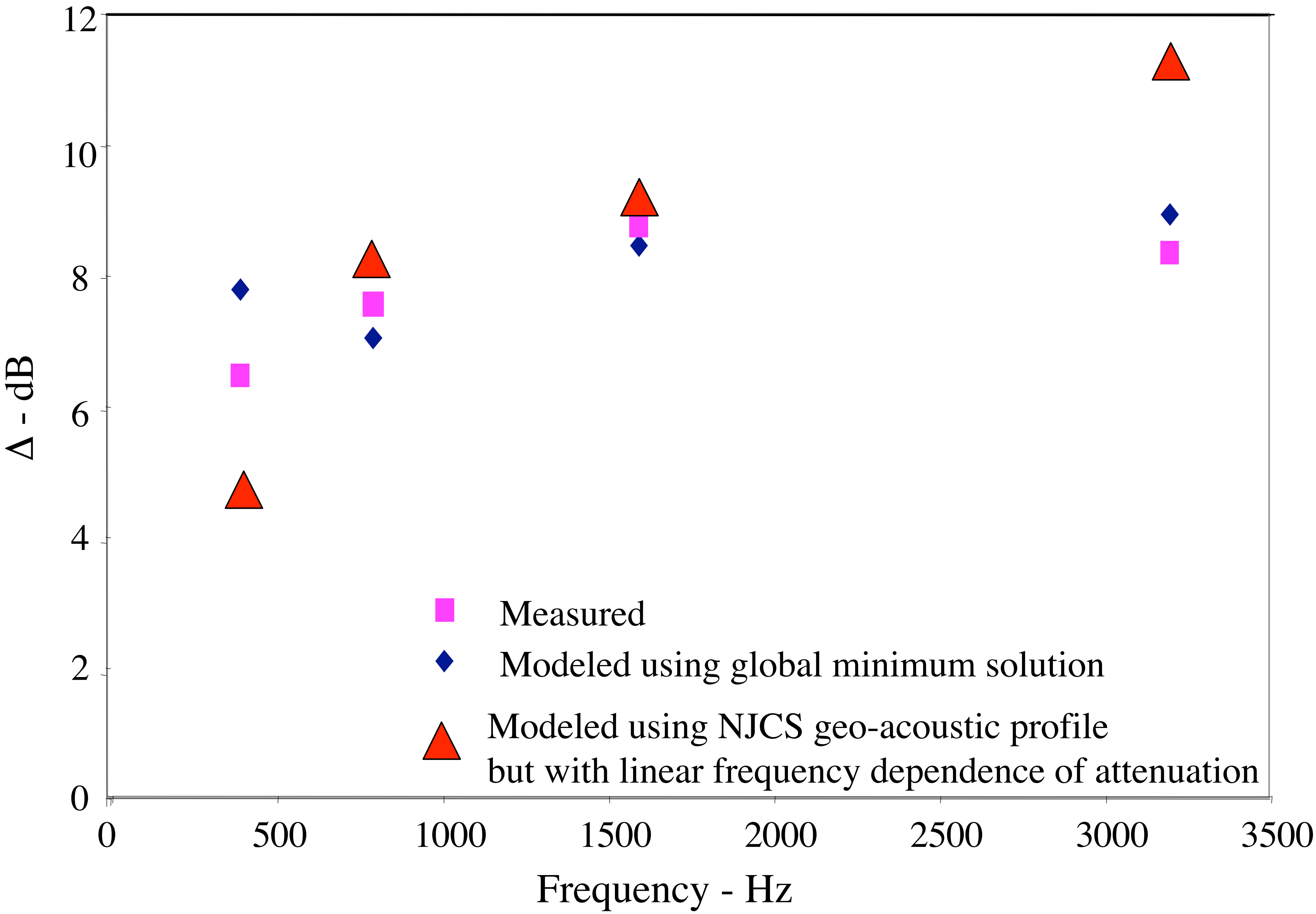
a



b

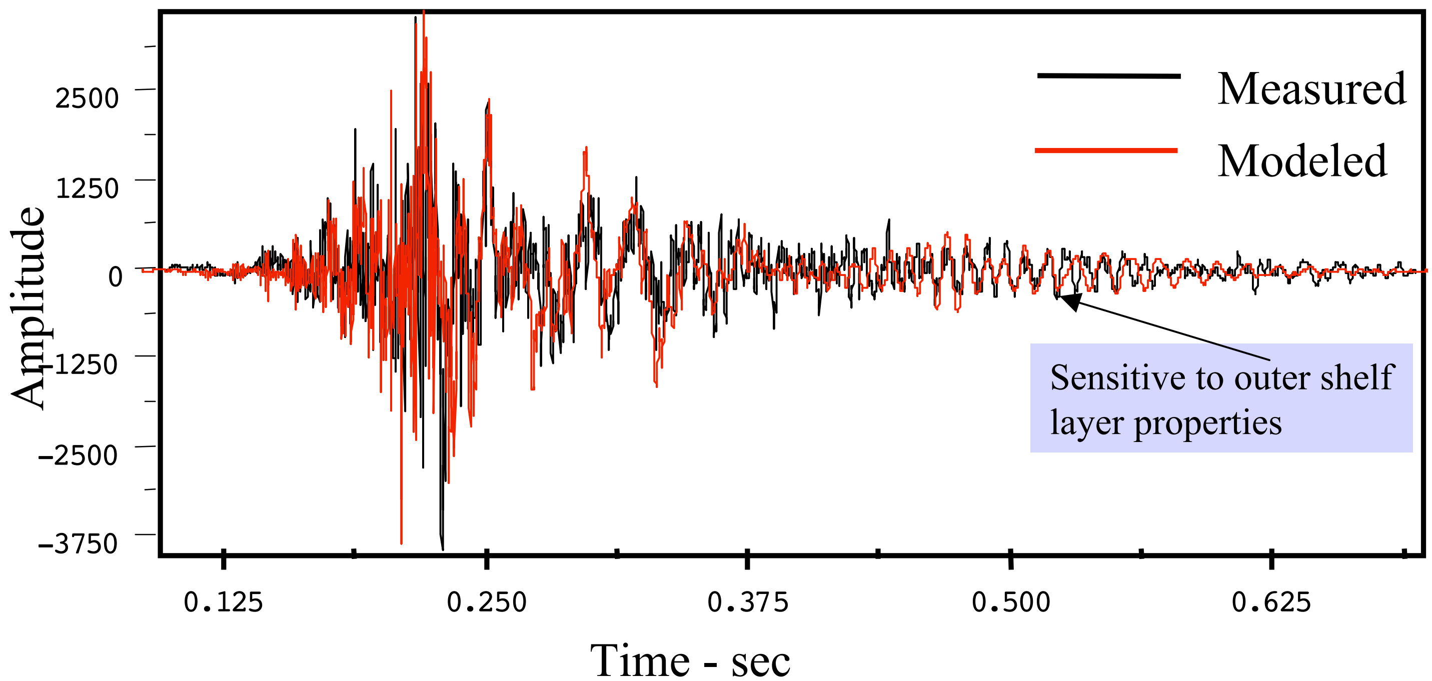


Wind noise relative to deep water location

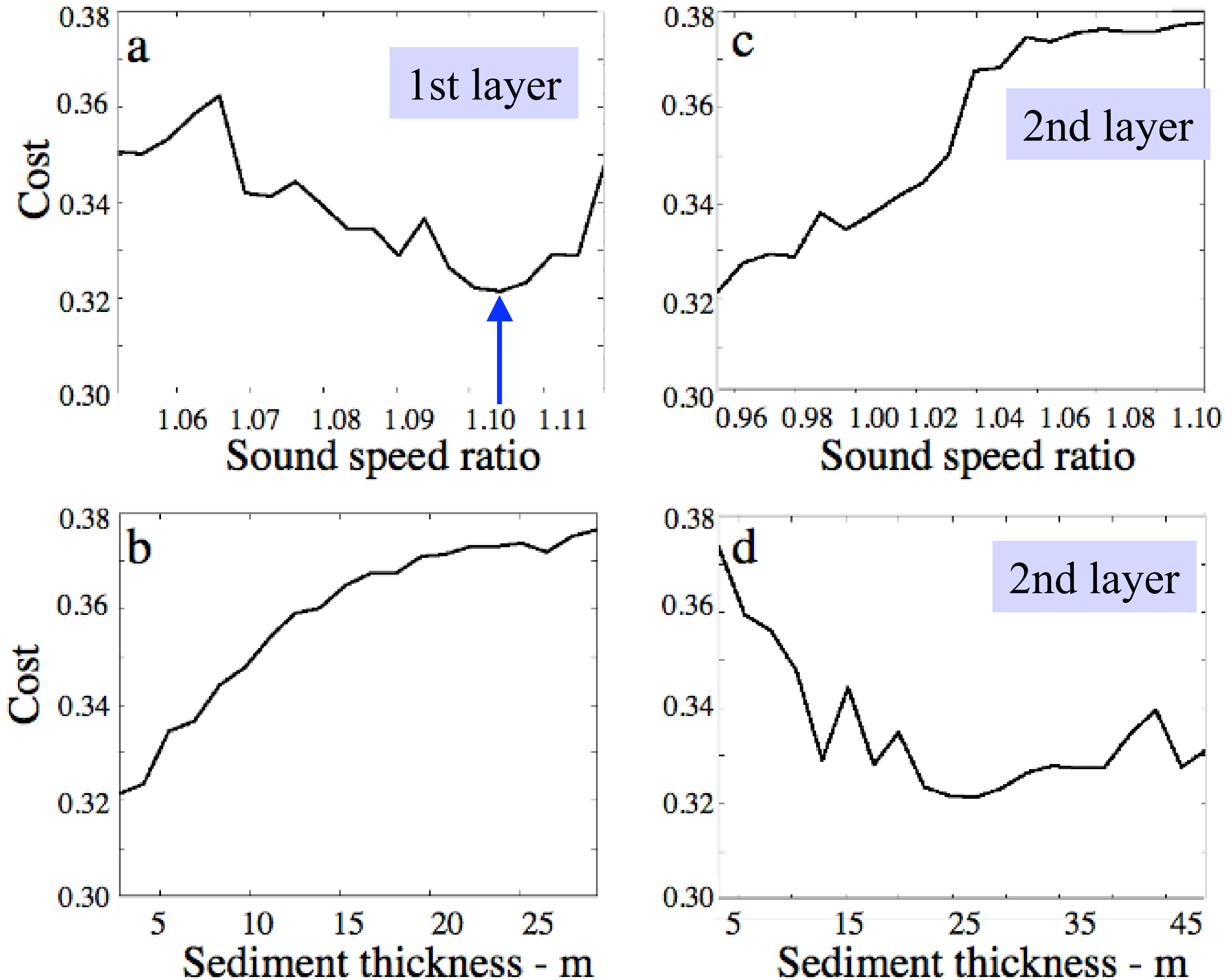


ARL CSS time series comparison

Range = 4.7 km
SD=26.2 m
RD=69.5 m
BW=10-3000 Hz



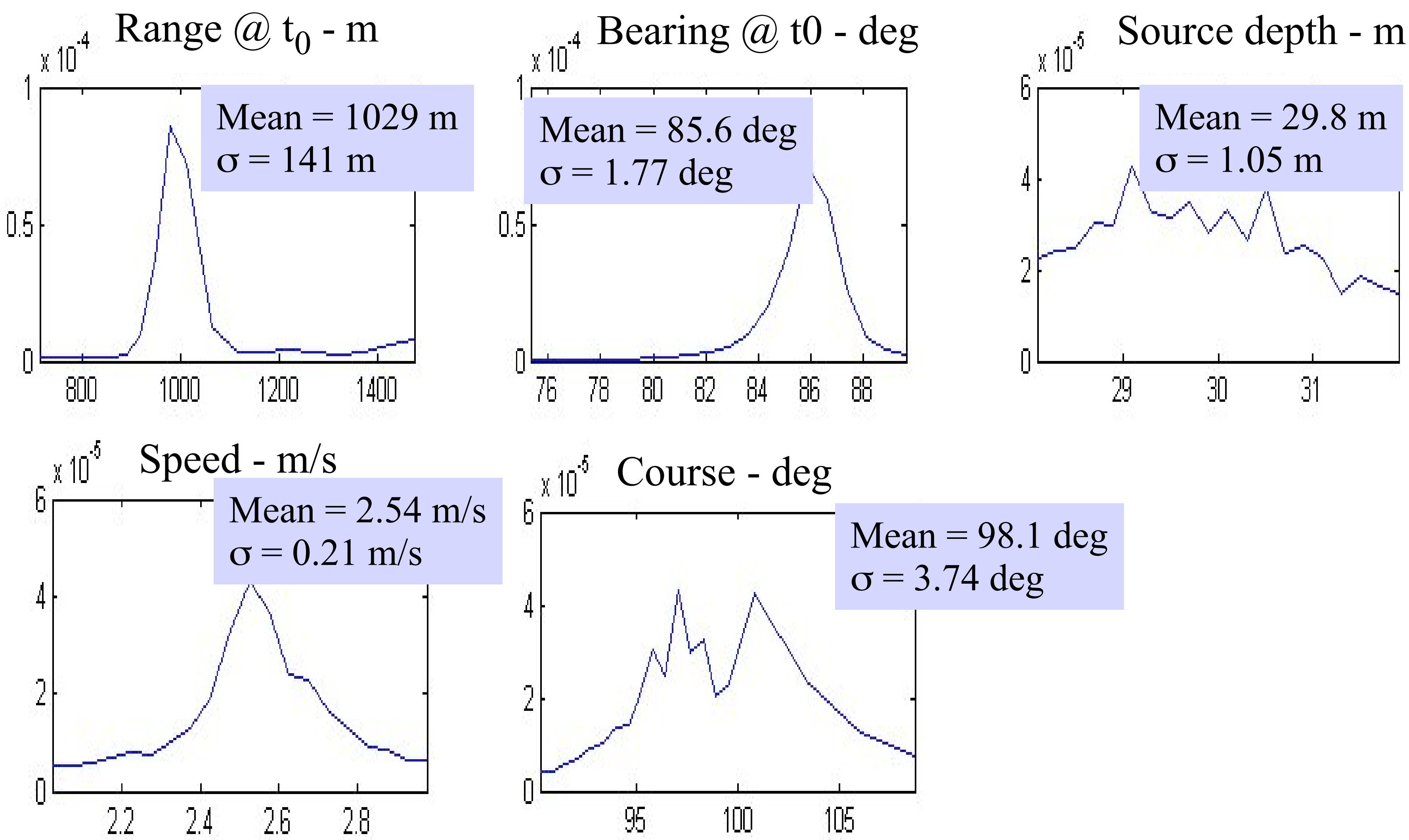
Cost envelopes for CSS inversions near Array 1



Thin hard sand over thicker softer layer

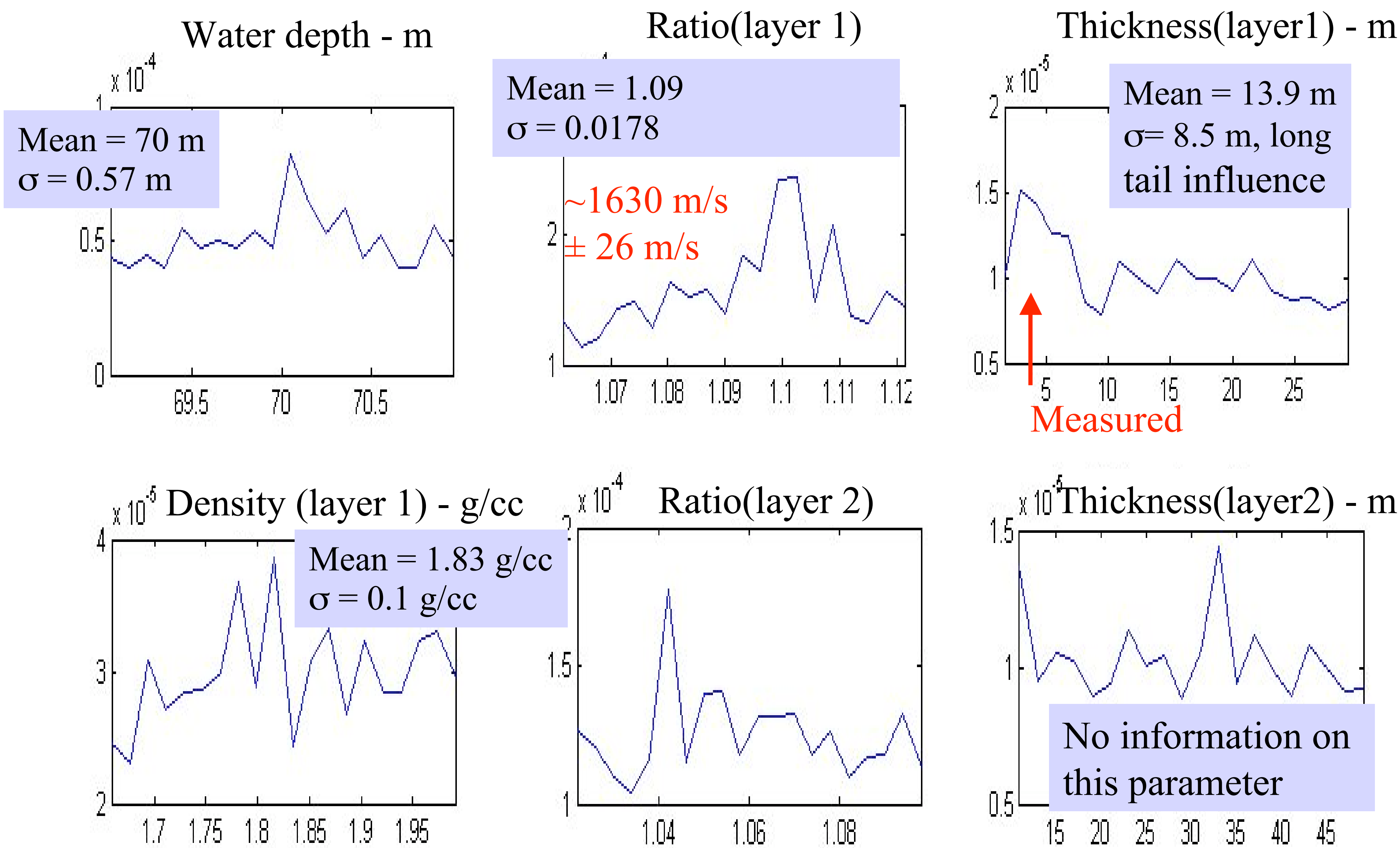
ARL
The University of Texas at Austin

Marginal probability distributions of position and kinematical parameters at Array 2

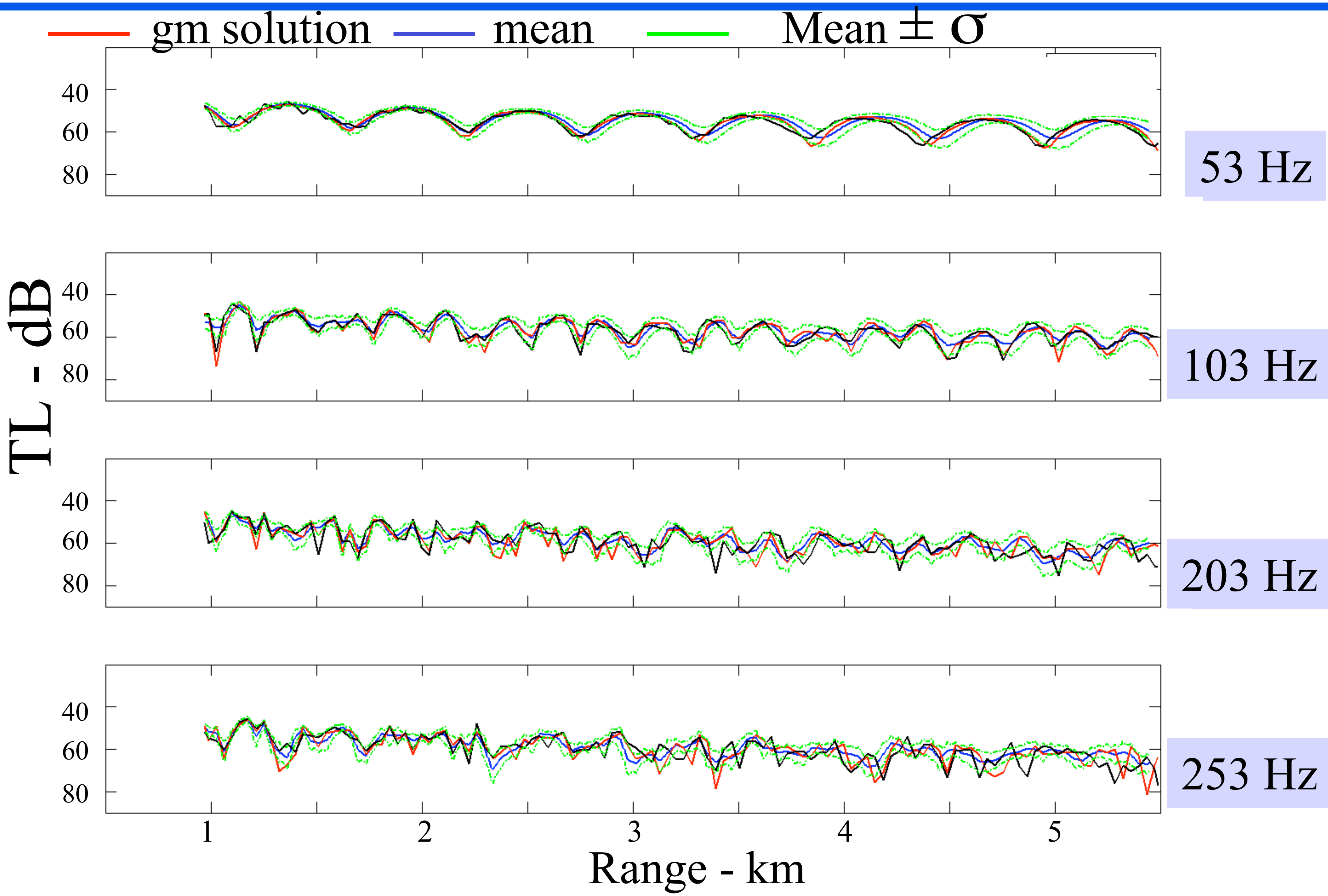


ARL
The University of Texas at Austin

Marginal probability distributions of selected environmental parameters

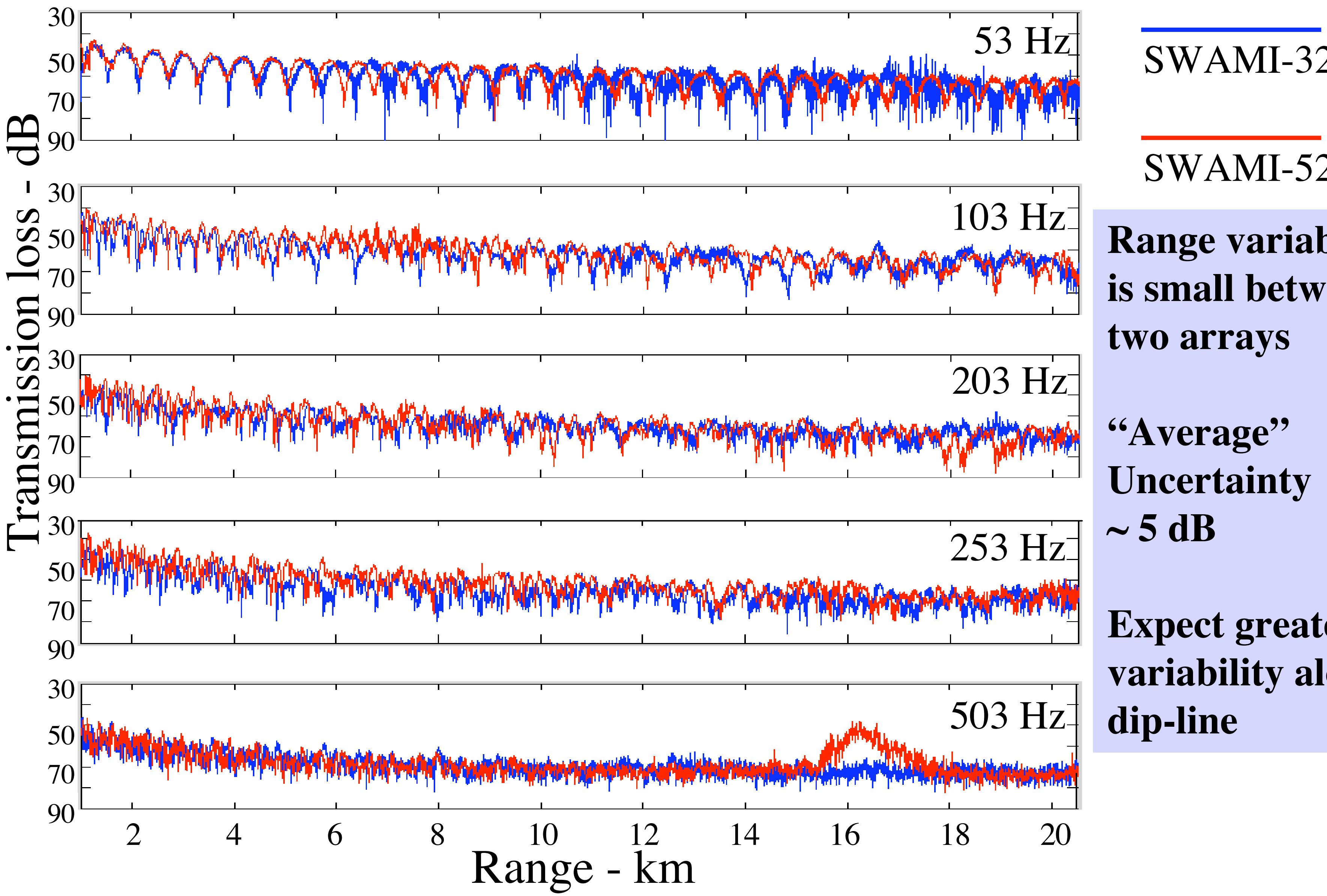


Global minimum solution and <TL>± σ solutions compared to measured TL



Measurement of variability along propagation track: Single J-15-1 tow

ARL The University of Texas at Austin



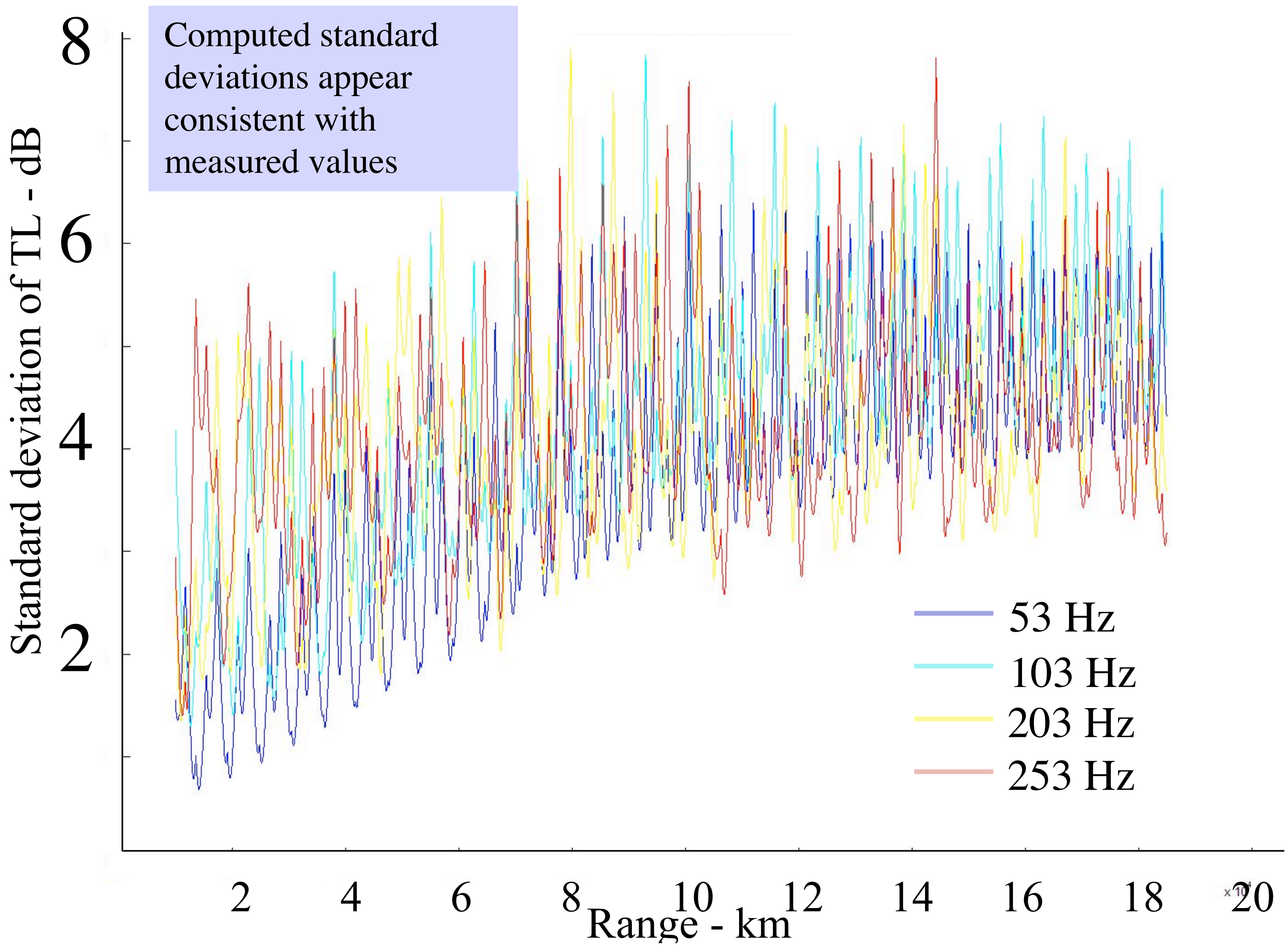
Range variability is small between two arrays

"Average" Uncertainty ~ 5 dB

Expect greater variability along dip-line

Computed TL s from maximum entropy principle versus range

ARL
The University of Texas at Austin

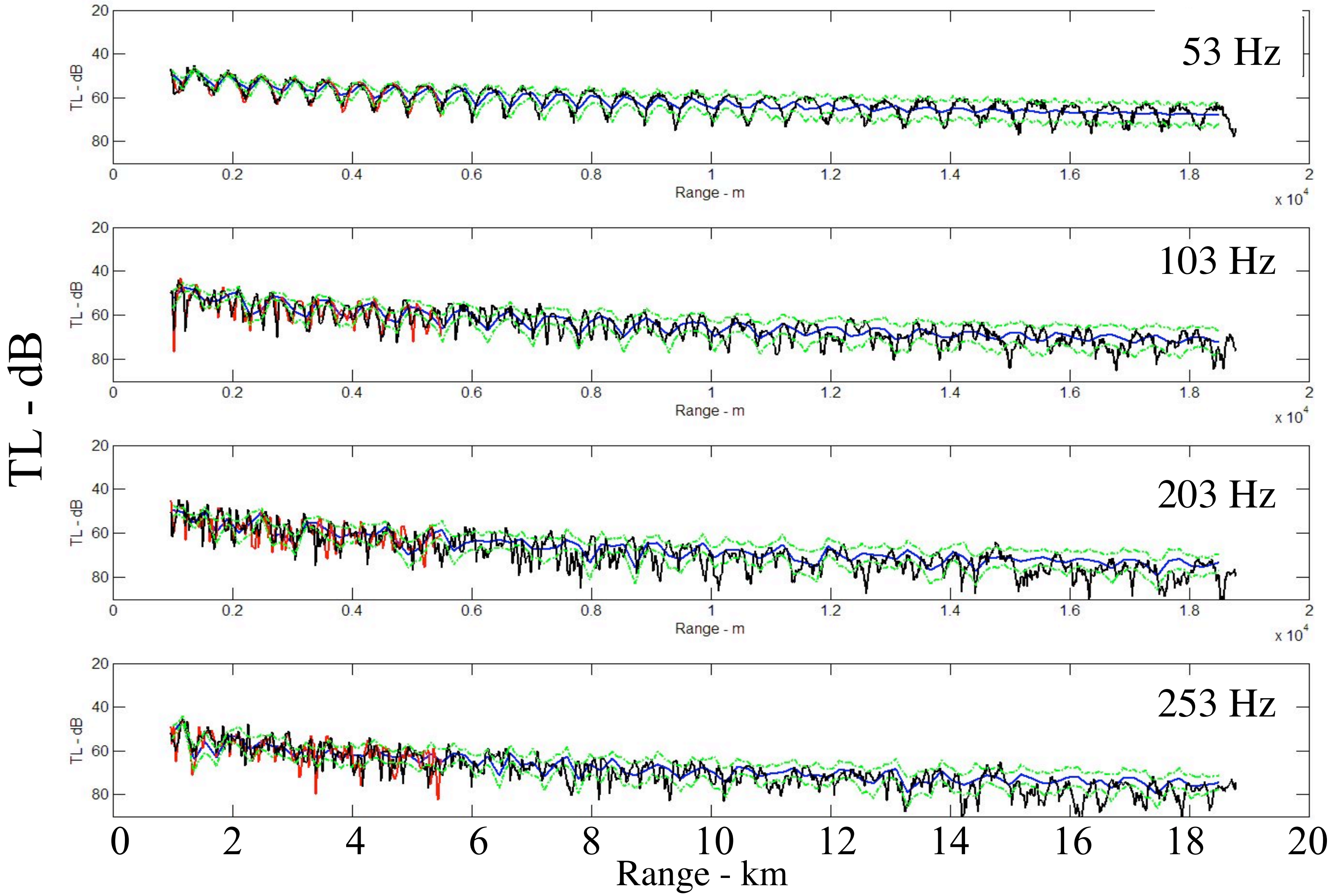


ARL

The University of Texas at Austin

Long range TL uncertainty

gm solution short range — mean — Mean $\pm\ \sigma$



ARL

Summary

- Current inferences of attenuation based on global minimum solutions
- Maximum entropy principle applied to quantify statistics of environmental parameters and propagation
  - ➢ Computed and measured TL uncertainty consistent
- Environmental parameter marginals
  - Sensitive to signal processing
  - Dependent on volume and sensitivity
- Uncertainty of Biot parameters is ongoing research