Spatial and temporal sound field fluctuations due to propagating internal waves in shallow water

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Abstract

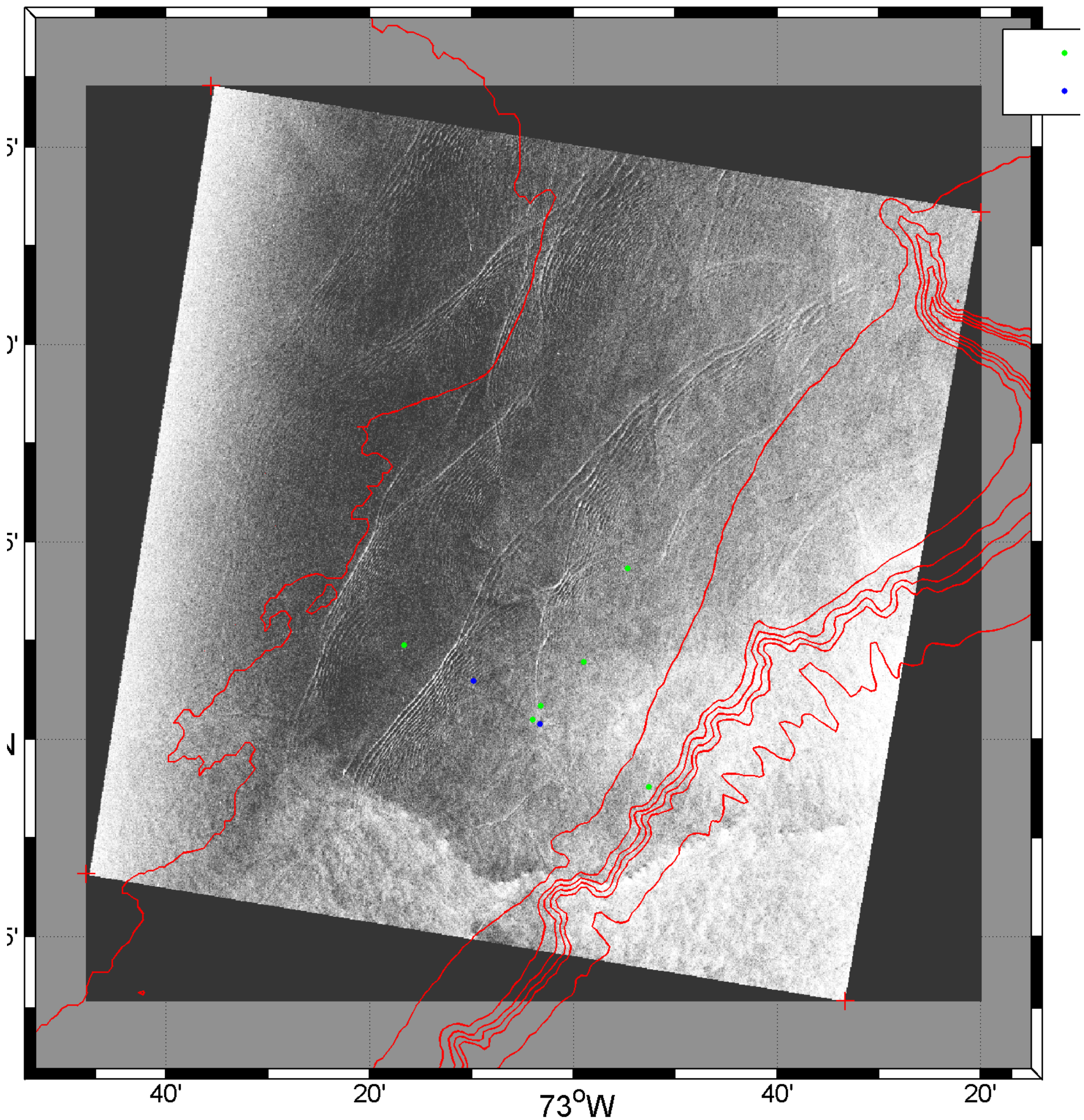
Space-time variations (fluctuations) of the sound field initiated by moving nonlinear internal waves in shelf zone of the ocean (shallow water) are considered. These fluctuations should be observed during rather long time (several hours and more).

Oceanographic features of nonlinear IW:

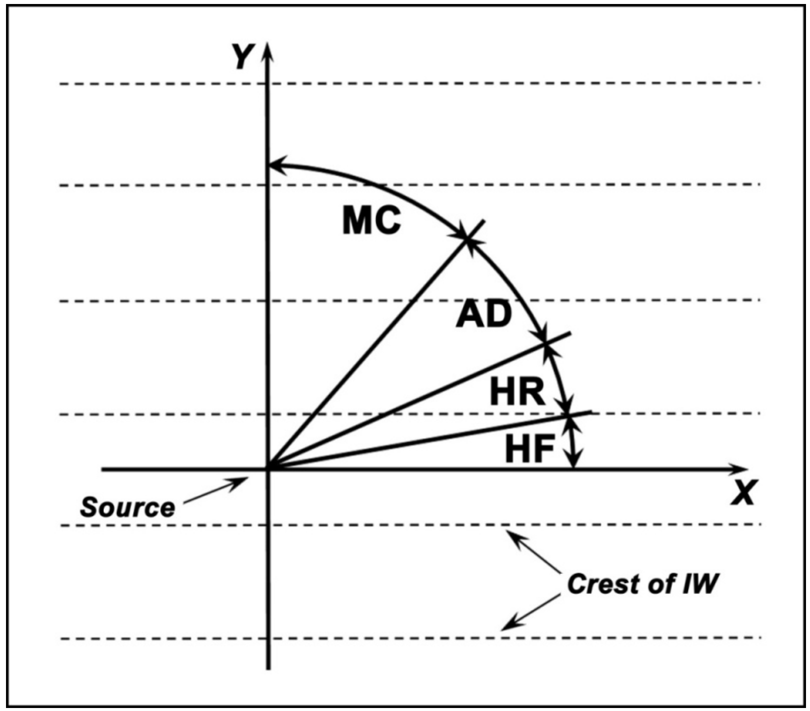
- shape (envelope) of IW is rather complex, but in the simplest case have KdV form. Wave lengthis ~300- 500 m, amplitude ~ 10-15 m, train can contain up to10-12 separate solitons;
- trains of IW arise in area of shelf break, live up to 10h, move at velocity ~ 0.5-1 m/s;
- direction of propagation is almost perpendicular to coastal line, (in SW06 we got angle diagram of width ~15 degrees for more than 50 moving trains during 3 weeks)
- wave front is long and almost planar, radius of curvature ~20-30 km
- perturbation of water media is concentrated in comparatively thin layer of thermocline (in SW06 thermocline ~ 10-15 m, water depth ~ 80 m)

Typical satellite image from SW06

Radarsat August 13, 2006 10:47 GMT



Mechanisms of fluctuations in dependence on direction of the sound propagation relative direction of propagation of internal waves



MC - modes coupling, AD - adiabatic, HR - horizontal refraction, HF - horizontal focusing

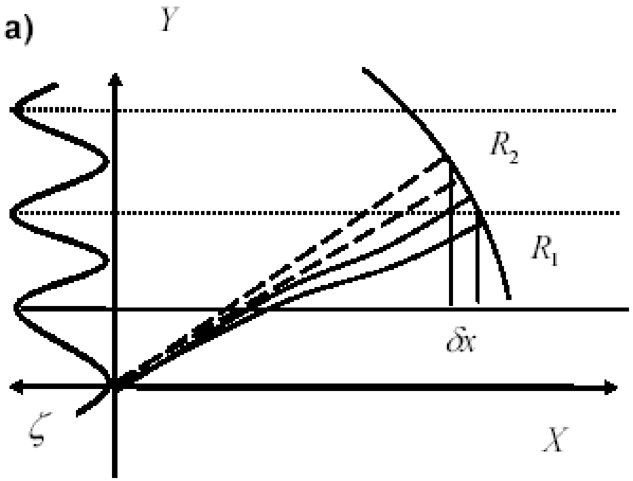
SI for intensity fluctuations

Total intensity as a function of depth $I(z,T)$, spectral $I_\omega(z,T)$,modal $I_l(z,T)$ intensities, and $I_{l\omega}(z,T)$ as a function of frequency and mode number. We analyze scintillation index for all types of intensity (averaging:

$$SI_{l\omega}^2 = \left\langle \delta I_{l\omega}^2 / \left\langle I_{l\omega} \right\rangle^2 \right\rangle, \delta I_{l\omega}^2 = \left\langle I_{l\omega}{}^2 \right\rangle - \left\langle I_{l\omega} \right\rangle^2$$

Horizontal refraction (HR)

a)



Vertical modes and horizontal rays (HR)

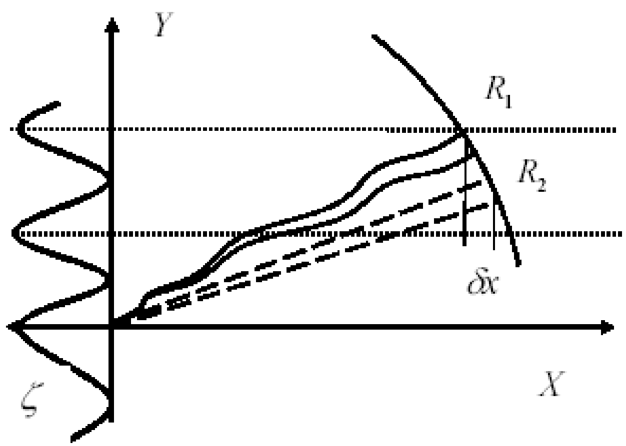
$$P_\omega(r,z,t)=\sum_l A_l(r,r)_s \psi_l(r,z)\exp[i(q_l^0\theta_l(r)-\omega t)]$$

Eikonal equation for HR

$$\left(\frac{\partial\theta_l}{\partial x}\right)^2+\left(\frac{\partial\theta_l}{\partial y}\right)^2=1+\mu_l(\vec{r}),$$

Correction to refraction index in perturbation theory

b)

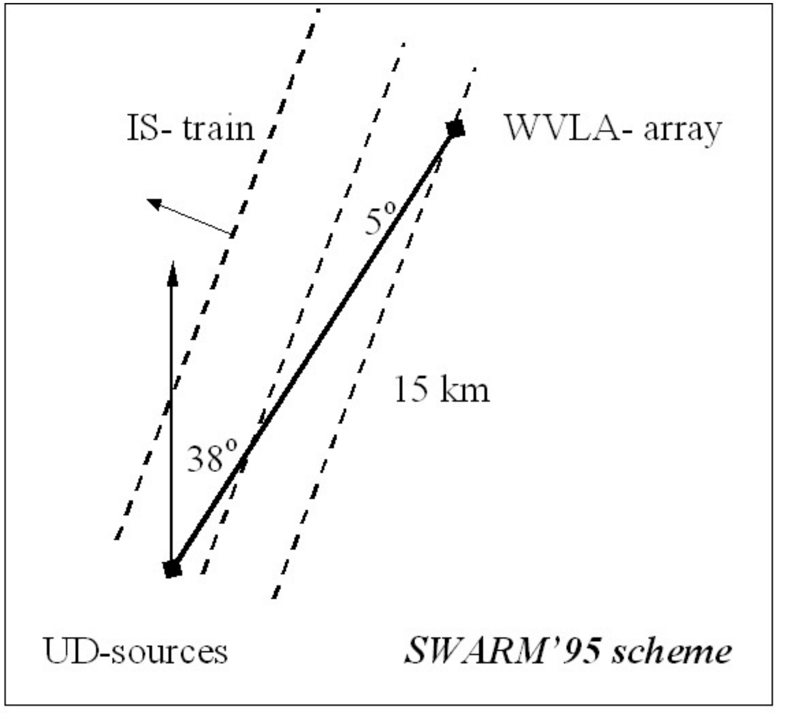


$$\mu_l=-\frac{2Qk^2\zeta(r)}{(q_l^0)^2}\int_0^H\left[\psi_l^0(z)\right]^2 N^2(z)\Phi(z)dz$$

Scintillation index in ray approximation

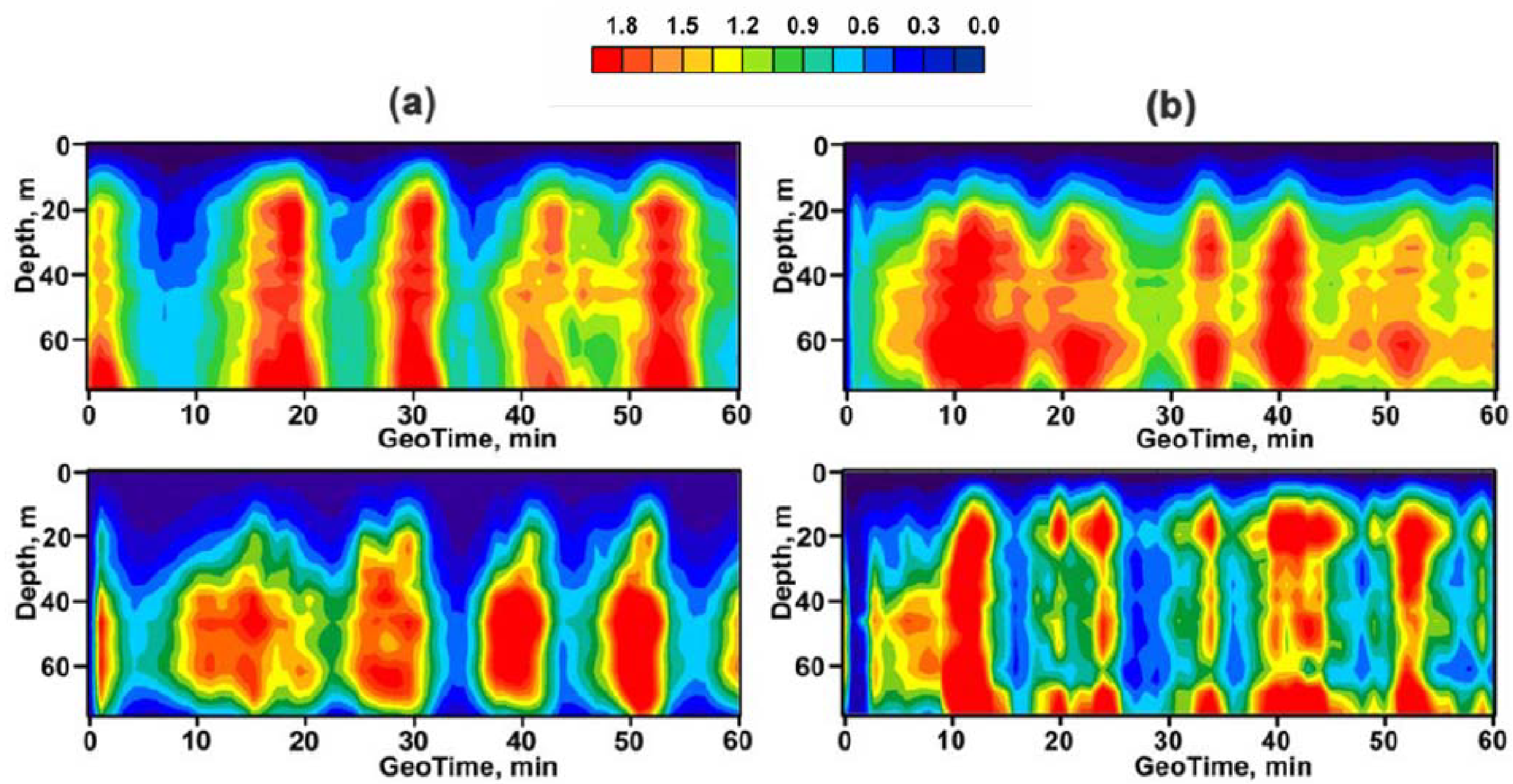
$$SI_{l\omega}^2=\frac{\left|\mu_l^0(\omega)\right|}{2\sin^2\chi_s}$$

Layout of the SWARM'95 experiment



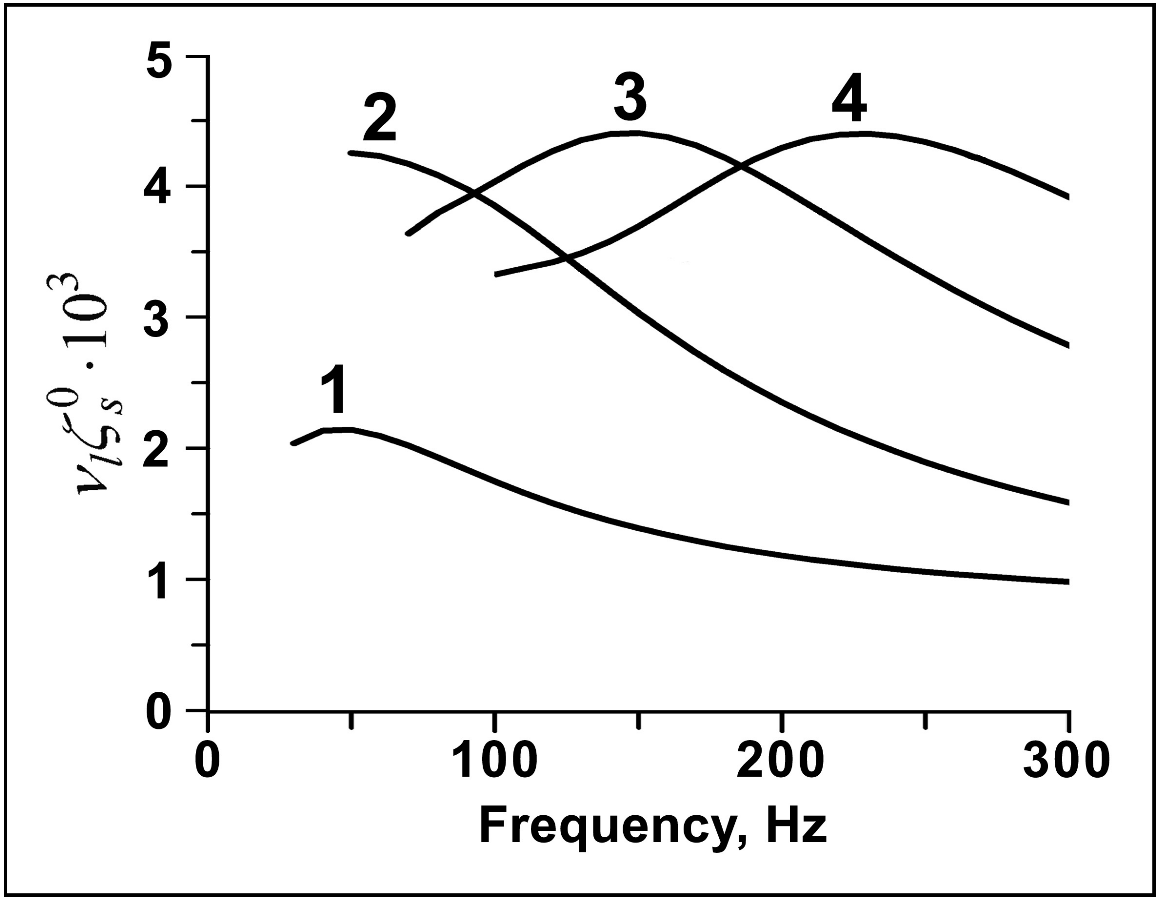
Signals were radiated every minute during a few hours from airgun and received by vertical array. Intensity of received signals fluctuates with period about 12-14 min

Depth distribution of intensity $I(z,T)$.(a) and (b) correspond to different time periods and depths of the sources. Top panels –airgun, low panels LFM source (SWARM'95)

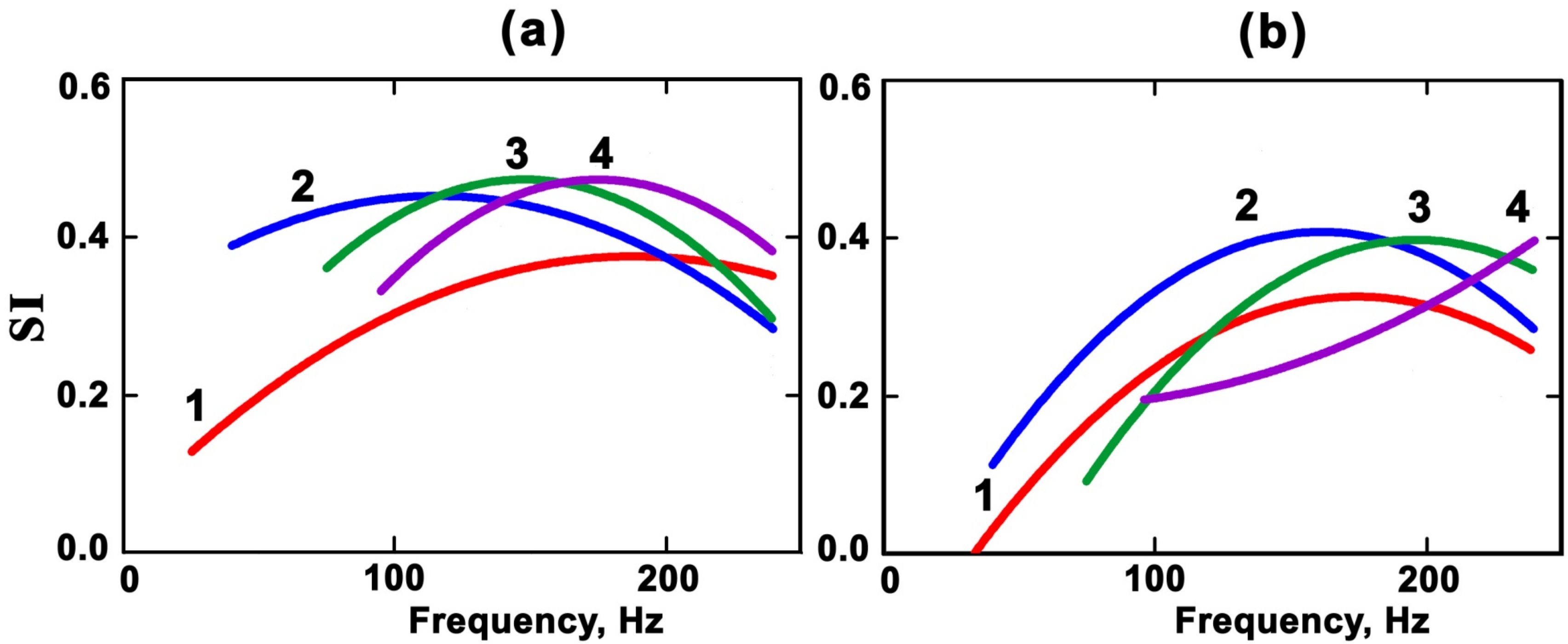


Synchronicity in depth and frequency of fluctuations (Buoyancy frequency) can be explained by HR mechanism of fluctuations

Frequency dependence of modal refraction index μ_l in horizontal plane for he SWARM'95 conditions



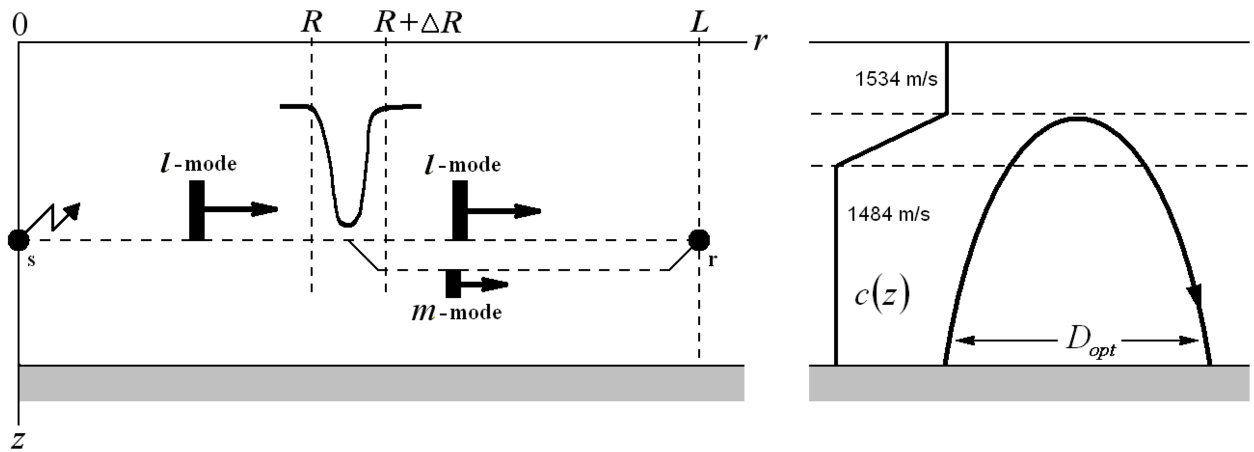
SI as a function of frequency and mode number for the SWARM'95 experiment



(a) and (b) correspond to different depths of the source and different time periods

We see correspondence with theoretical frequency dependence of refraction index for individual modes.

Fluctuations due to modes coupling



Pulses radiated from the source, corresponding to a sum of normal modes,
create additional modal pulses in the area of perturbation, each propagating with their own group velocities, and these in turn change the sound field at the receiver. For other positions of IS, there will be other composition of modes created and other sound field at the receiver. We understand this variability as temporal fluctuations. Typical frequencies of fluctuations are about ~1-10 cph.

Theory

After acoustic interaction with the soliton, we have another modal decomposition for the sound field. We will describe this decomposition using S-matrix formalism

$$P_\omega(r,z;R) = iS(\omega)\sum_{m,l}\frac{\psi_l(z_s)\psi_m(z)}{\sqrt{8\pi i q_m r}}S_{ml}(R+\Delta R,\omega)\exp\left[i\left(q_m(r-R)+q_l R\right)\right]$$

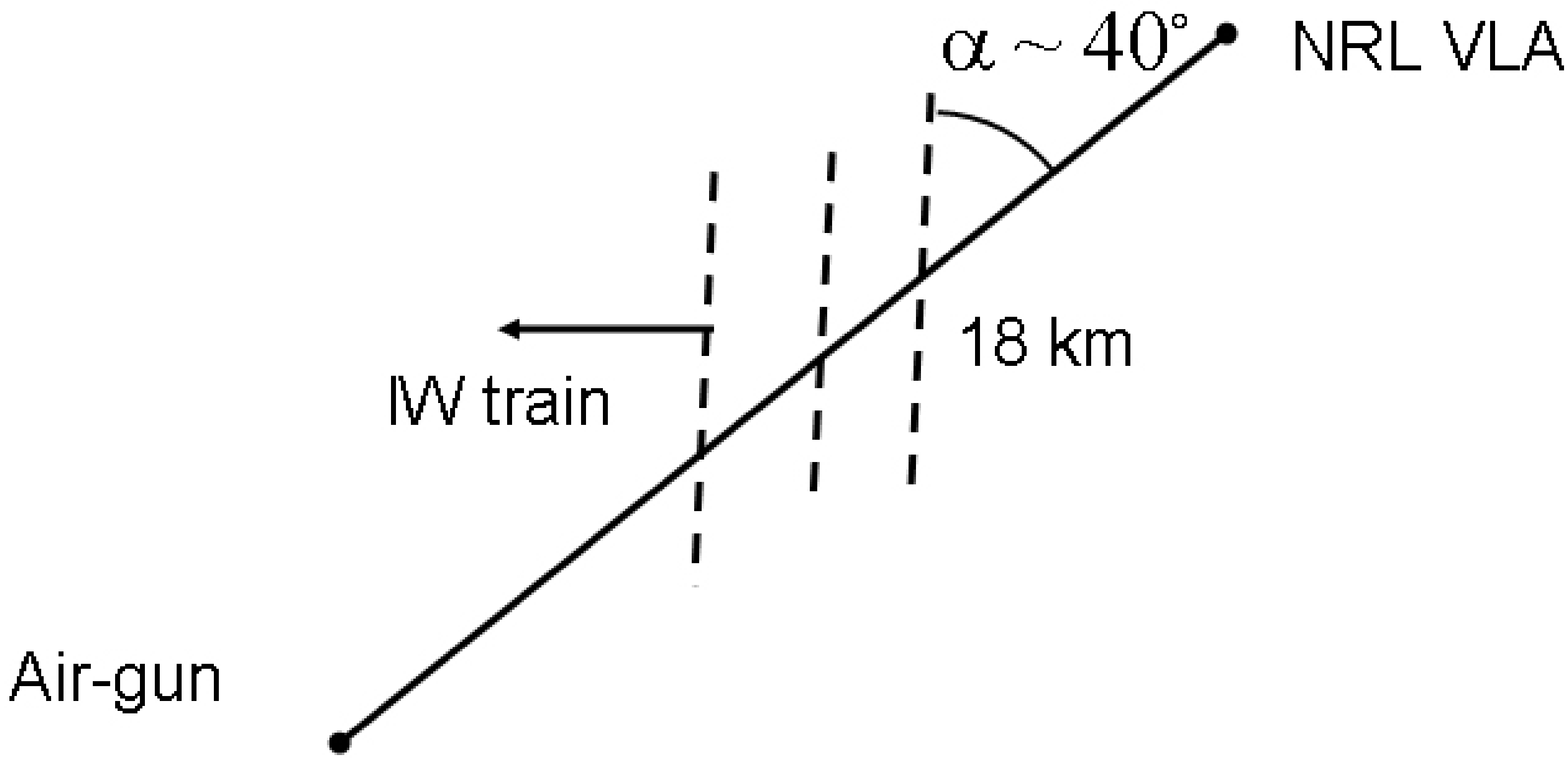
$$\frac{d\mathbf{S}}{dr} = \mathbf{WS} \qquad \mathbf{S}\left(R\right) = \mathbf{I}$$

$$W_{ml}\left(r\right) = i\frac{k^2\exp[i(q_l-q_m)r]}{\sqrt{q_m q_l}}\int_0^H \frac{\delta c\left(r,z\right)}{c}\psi_m\left(z\right)\psi_l\left(z\right)dz$$

$$R = vT$$

Is position of moving perturbation, so we can get temporal dependence of the sound intensity

Layout of the SWARM'95 experiment

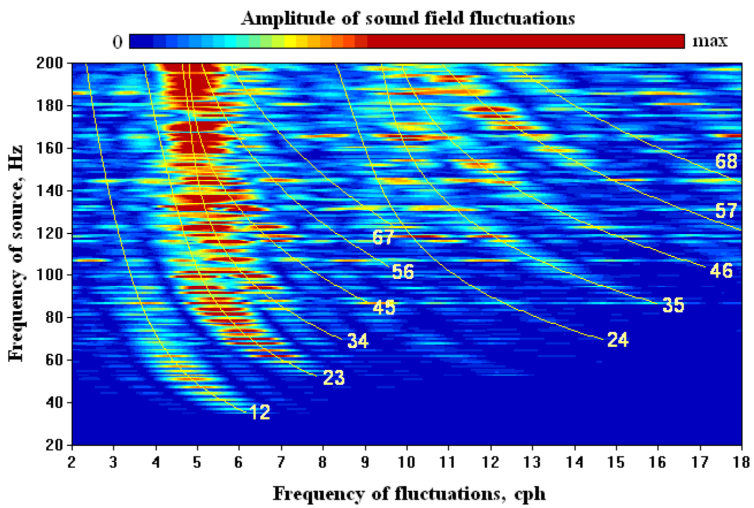


We consider sequence of pulses radiated by airgun and received by VLA during two hours

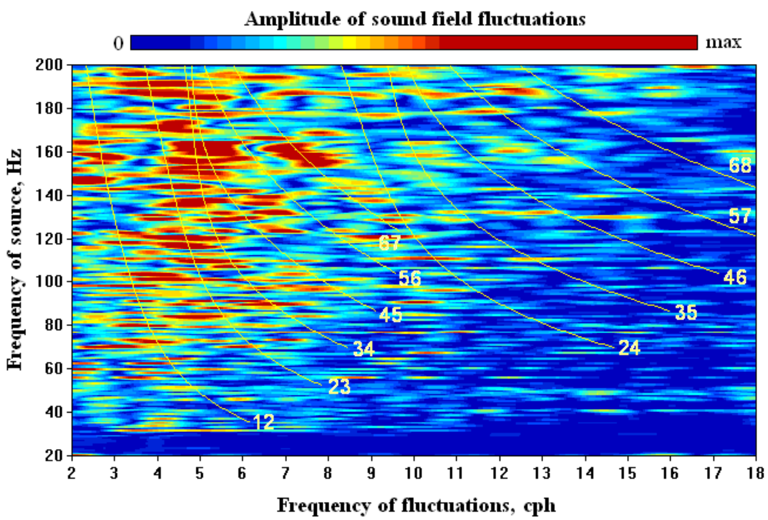
Spectrum of temporal fluctuations

$$I_\omega(T) = \frac{1}{2\rho c}\left|P_\omega(T)\right|^2 \qquad G(\omega,\Omega) = \int_0^{\Delta T} \delta I_\omega(T) e^{i\Omega T} dT \qquad \Delta T \text{ ~ a few hours}$$

theory



experiment



Predominating frequency $\Omega_{opt} \sim 2\pi v / D_{opt}$

v Is velocity of IS

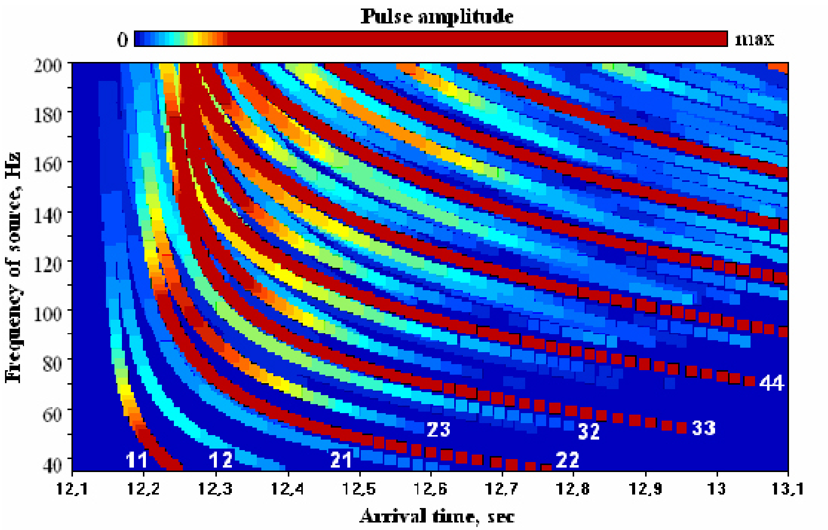
D_{opt} Is “optimal” ray cycle

Arrival time fluctuations (time frequency diagram)

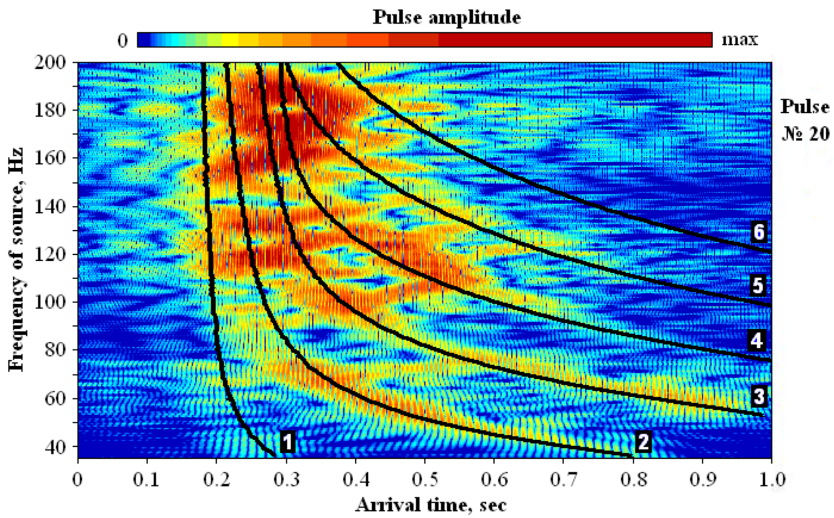
$$t_l = \frac{L}{v_l^{gr}}$$ Arrival time for the l-th mode without coupling (*L* is distance)

$$t_{lm} = \frac{R}{v_l^{gr}} + \frac{L-R}{v_m^{gr}}$$ Arrival time for mode *l*, coupling with mode *m*

theory



experiment

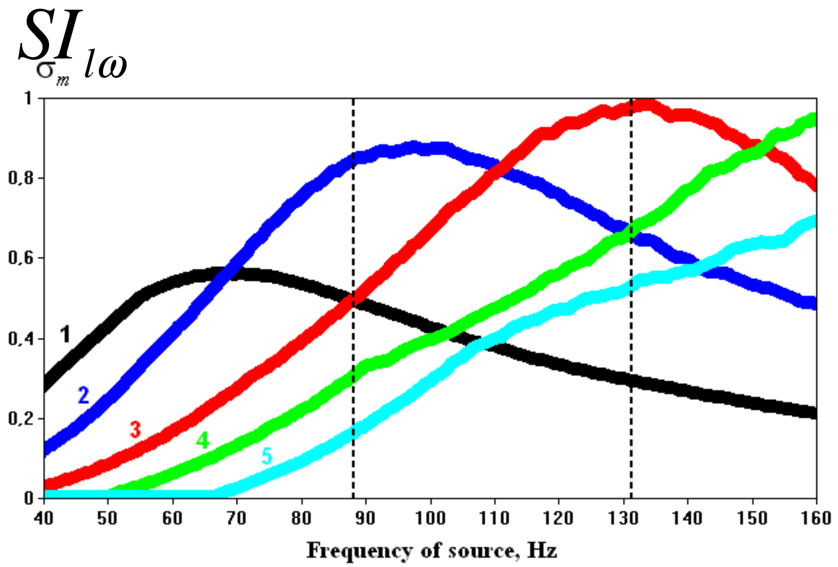


Arrival times of additional (created) modes are concentrated in area $t_{opt} \sim L / v_{opt}^{gr}$

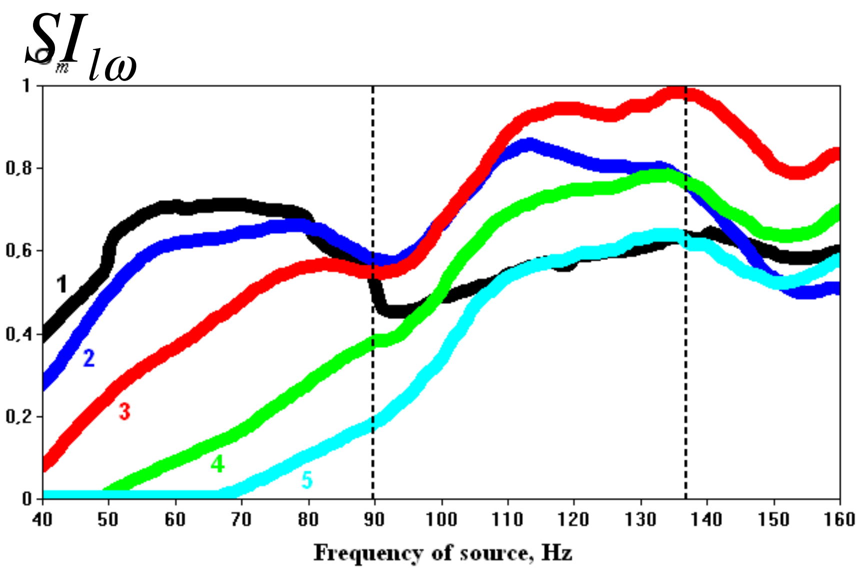
Frequency dependence of modal fluctuations

Maximums correspond to frequencies where adjacent modes have the most significant coupling. These pair of modes have turning point in thermocline area

theory



experiment



Conclusion

- Moving internal waves (trains of solitons) initiate fluctuations of the sound intensity
- Physical mechanisms of fluctuations depend on direction of propagation of the sound signals
- *SI* and some another characteristics of fluctuations have “invariant” parameters (predominating frequency, correlation time, arrival time etc) depending only on properties of unperturbed waveguide