Observations and modeling of angular compression and spatial coherence in sea surface forward scattering

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Spatial coherence in forward scattering from single (time resolved) interaction with sea surface from Shallow Water 06

Environment: Wind speed $\sim 6$ m/s, Waveheight $\sim 0.15$ m, stationary $> 6$ h

Comparative influence of sea surface C(Z) [thermocline]

Research sponsored by U.S. Office of Naval Research
Experimental site: off the New Jersey Continental Shelf, Water Depth 80 m

Shallow Water 06 (SW06) August 2006
August 10 2006 measurements:

R/V *Knorr* holds station at four source locations each at range 200 m from the receiver and separated in bearing angle by 90°.

Time: 0830-1500 UTC

Two, time resolved surface bounce paths studied.
R/V Knorr CTD cast 1107 UTC
Derived from WHOI Shark Temperature mooring 15 min avg. 0830 UTC
Derived from WHOI Shark Temperature mooring 15 min avg. 1330 UTC
R/V Knorr CTD cast 1107 UTC
upper receiver eigenrays and corresponding arrival structure

complex envelope for $i$th ping $x_i$

20-ping avg

RELATIVE LEVEL (dB)

RELATIVE TIME (ms)

RELATIVE TIME (ms)

RELATIVE LEVEL (dB)

0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2

0 5 10 15 20 25 30 35 40 45 50 55 60

RELATIVE LEVEL (dB)

0 10 20 30 40 50 60

RANGE (km)

0 20 40 60

DEPTH (m)

0 10 20 30 40 50 60

SB S D B BS

Direct Surface Bottom Bott-Surf Surf-Bott

20-ping avg

upper receiver eigenrays and corresponding arrival structure
Spatial coherence between (d) vertically-separated channels based on 20 ping avg

\[
\Gamma_{xy} = \frac{\langle xy^* \rangle}{\sqrt{\langle xx^* \rangle \langle yy^* \rangle}}
\]

4 receiver pairs and frequency (k)
6 combinations of kd
Average air-sea conditions for 0830-1500 UTC. Wind speed 6 m/s +/- 1 m/s

- **WAVE DIRECTION FROM (deg)**
  - 160°
  - 220°

- **FREQUENCY (Hz)**
  - 0.12 Hz
  - 0.34 Hz

**Graphs:**
- **WAVE DIRECTION FROM (deg)**
- **FREQUENCY (Hz)**
- **SPECTRAL DENSITY m²/Hz**

**Locations:**
- APL-UW wave buoy
- APL-UW wave buoy (loan from ARL-PSU)
- U. Miami ASIS buoy
~0.34 Hz wind waves from 220°

~0.12 Hz swell from 160°
Absolute value of vertical coherence vs normalized separation (kd) at 16 kHz
Absolute value of vertical coherence vs normalized separation (kd) at 16 kHz

Bearing 210
Bearing 120
Bearing 030
Bearing 300
Modeling of coherence will proceed with directional-averaged sea surface wavenumber spectrum $F(K)$

Plant model
6 m/s, 20000 m fetch
(Plant 2002)

Combination used in bistatic scattering computation
PDF for vertical arrival angle

sea surface bistatic cross section
via small slope approximation & wave number spectrum $F(K)$
(Dahl, 1999)

region producing same $\theta_A$
iso-speed analysis

mean vertical arrival angle close to specular angle $\sim 18^\circ$

Variance = 0.0078 rad$^2$
Analysis using measured $c(z)$ with thermocline

- Mean: $18^\circ \rightarrow 21^\circ$
- Variance: $0.0078\text{ rad}^2 \rightarrow 0.0042\text{ rad}^2$
The PDF for vertical arrival angle is readily converted to spatial coherence $\Gamma(kd)$

Alternatively, the van Cittert-Zernike Theorem can be utilized to estimate $\Gamma(kd)$ (Dahl 2002, 2004)

- $kd^*$ for $c(z) \sim 21$
- $kd^*$ for $c_0 \sim 14$

$$|\Gamma| = e^{-1/2}$$
Range of magnitude coherence for modeled spectrum: 4 – 10 m/s
Refraction conditions of SW06

Range of magnitude coherence for modeled spectrum: 4 – 10 m/s
Iso-speed conditions

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4 m/s → 10 m/s
\[
\theta_A = \cos^{-1}(n \cos \theta_S)
\]

\( n = \frac{1485}{1530} \)

\[
\frac{\partial f}{\partial \theta_S} \bigg|_{\bar{\theta}_S}
\]

\[
\text{var}(\theta_A) \approx \left( \frac{\partial f}{\partial \theta_S} \bigg|_{\bar{\theta}_S} \right)^2 \text{var}(\theta_S)
\]

Vertical angular compression factor

\[
- \frac{n \sin(\bar{\theta}_S)}{\sqrt{1 - n^2 \cos^2 \bar{\theta}_S}}
\]
Vertical Angular Compression

Large change in $kd^*$ predicted by the angular compression factor

Compression does not $\Rightarrow$ intensity

SW06 geometry:
TL *increased* by 1.5 dB (confirmed by ray and PE analysis)

$$\sigma_{\theta_A} = \frac{n \sin(\bar{\theta}_S)}{\sqrt{1 - n^2 \cos^2 \bar{\theta}_S}}$$

$0.72$

$kd^* \sim 1 / \sigma_{\theta_A}$

$\therefore kd^*$ should $\Rightarrow$ by $\sim 4/3$
Model comparison with data (14-16-18-20 kHz) plotted versus $kd$.
Summary

• Spatial coherence in sea surface forward scattering with strong thermocline

• Vertical angular compression: dominate effect greater than that linked to sea surface roughness and slope

• Vertical angles compressed while TL increased over spherical spreading (angle expansion in upward paths not balanced by downward paths)

• Mild refraction effects (influencing phase of $\Gamma$) observed in ASIAEX data (Dahl 2004) SWO6: strong refraction effects influencing both magnitude and phase

• Predictive model based on Snell’s mapping of angular variances