The effects of non-linear internal wave curvature on acoustic propagation

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Lots of work in past 20 years on acoustics and IW’s

- BSPF (1992)
- SWARM (1995)
- PRIMER (1996-97)
- ASIAEX (2000-01)
- NEST (2007-08)
- SW06 (~2006)
What are the big issues??

- TL and its fluctuations
- Fully 3-D acoustics - not just slices of 3-D ocean – and direction of arrivals
- Array coherence
- Inversion for bottom in presence of fluctuating ocean
- Others…
Started with SWARM - Cross Shelf
IW Induced Coupled Propagation Gain/Loss Cases

PRIMER Noise Case – Net Amplification

Zhou Yellow Sea Case – Net Attenuation
Evolution - Boris’ IW Master Plan!
Fig. 2. Ray patterns in the horizontal plane for the case of a plane front of soliton packets. The dependence of the thermocline displacement on the transverse coordinate is shown on the left.
Temporal dependence of the sound energy for different depths in intensity gradation (relative units)
Mean intensity increase due to ducting (no spreading vs. cylindrical spreading)

For IW duct, have geometry

\[ R = \text{Ratio of areas} = \frac{r\theta_c}{w} \]

IW duct ( \( r_i = 20 \text{ km}, w = 1 \text{ km}, \theta_c = 7.5^\circ \))

\[ 10 \log R = 7.18 \text{ dB} \]

7-8 dB is a lot for sonar systems!! (And is observed)
IW Trek – The Next Generation

New IW Features to Include…. funkier!

Curved IW’s
Terminating IW’s
Field of IW’s with horizontal decorrelation
Horizontal Lloyd’s Mirror Crossing Wave Trains
So much for infinite plane waves...
First port of call - curvature

- First theory – Katznelson
- Computer models – Duda, Lin, et al
- Ridiculously simplified theory ("For Utter Dummies" version) – Lynch
- Shadowing by circular wavetrains - Lynch
Theory

• Theory for straight and curved IW acoustic ducts first published by Katznelson et al.
• Predicted the ducting by “straight” internal waves seen by Badiey, Lynch in SWARM (1995)
• Uses Weinberg and Burridge “Vertical Modes and Horizontal Rays” 3-D theory
• Formally shows curvature effects in curvilinear coordinates… not easily transparent
Fully 3-Dimensional Curved IW Propagation Simulations

Curved pairs of nonlinear internal waves. Waves scaled to fit SW06 area observations. 200-Hz. Modes separate and beams form.
Effects of nonlinear gravity internal waves on 3D sound propagation

— Acoustic modal focusing in an curved internal wave duct

- Radius of curvature=135km, frequency = 100Hz
- Mode 1 penetrates through internal wave duct, but mode 2 focuses in the duct.

CW source

internal wave troughs

SW06 SAR image, 23 July

Depth-Integrated Sound Energy
Higher modes trap better—hmmm!

- **Case 2**: curvature=135km, frequency = 200Hz

- Modes 1 and 2 penetrate through internal wave duct, but modes 3 and 4 focus in the duct.
Some Simple Theory

- Previous work clearly displays frequency and mode dispersion effects, light piping (and leakage from curved pipe), etc.
- But doesn’t have simple physical insight into how parameters of problem (frequency, mode number, IW strength, background waveguide structure, etc.) affect trapping and leakage of modes.
- Maybe looking to a simple theory picture is useful?!
- Lynch an excellent choice for very simple stuff
Start with Weinberg/Burridge 3-D horizontal ray/vertical mode theory

- Have a local horizontal index of refraction for each mode at a given frequency

\[ n_n(x, y, \omega) = \frac{k_n(r)}{k_n(0)}. \]

- After get the index of refraction field by computing the modes at all x,y, then trace rays in the horizontal
- Product of the ray (horizontal) and mode (vertical) gives acoustic field !!!
Trivializing wave trains 101
(and can you spot the missing line)
Simple modal waveguide model

\[ \gamma_n H = (m + 1/2)\pi \]

\[ k_n = (k^2 - \gamma_n^2)^{1/2} \]

\[ Z_n(z) = \frac{2}{H} \sin(\gamma_n z) \]

\[ \Delta k_n = \frac{1}{2k_n} \int_0^p \frac{\Delta q Z_n^2(z) dz}{\rho(z)} \]

\[ \Delta q = -2\Delta c(z) \frac{\omega^2}{c_0^3(z)} \]

\[ \Delta k_n = \frac{-(D, H) \Delta c \omega^2}{c_0^3} \]

- Simple model – rigid bottom background waveguide plus ML, IW perturbations
- The background eigenvalue plus the appropriate perturbation is what we want -> eigenvalue at each point in (x,y)
- The mode function vertical dependence put in via 2OPT. Shows why higher modes trap!!
See that energy will preferentially leak out the exterior and that apparent angle of source has $W$ curvature effects in it – can calculate with rays
Mode Ducting… Critical angle most relevant

Mode Scattering… Fresnel scale $R_F$ as well as critical angle

$S - R$ Distance $L$ ; Wavelength $\lambda$ ; $R_F = (\lambda L)^{1/2}$
## Approx Fresnel Scales (m)

<table>
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<th>$L$ (km)</th>
<th>Freq (Hz)</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
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<td>273</td>
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<td>15 km</td>
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<td></td>
<td>612</td>
<td>433</td>
<td>354</td>
<td>306</td>
<td>274</td>
<td>250</td>
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</table>
Shadowing

- IW has critical angle of ~5 degrees (max)
- Can give an x-y plane shadow behind a linear IW, but only if source is within 1-2 km of IW front
- Curved IW’s allow one to see shadowing effect for source considerably further away
- Might be an observable effect in data..???
Poorly drawn example of how “falling away horizon” gives critical angle before the tangent point, and thus a shadow region behind the IW.
Anti-Shadowing

• Having source on interior of IW curves produces an opposite effect – makes shadowing unlikely

• Horizon curves toward the rays, making effective angle larger
Where are we now?

- Have analytical forms for the eigenvalues, the ray paths for the geometries considered, where shadow zones are, etc.
- Are doing some examples to compare to detailed numerical models (i.e. see if we can simply explain some features in models)
- Want to look at data to see if we have some situations where the shadowing and dispersive light piping/leakage effects might be seen. Curvature of IW’s in 3-D thermistor string array region and thus near WHOI VLA looks attractive!
Some new work – if time !!

By same gang of thieves…
Now note horizontal decorrelation/termination of waves
Horizontal dispersion – Galtons Box in the Ocean
Binomial-> Gaussian
Physics of terminating duct (Lin)
Lin’s open ended duct work continued (Galton’s box input gets more complicated!!)

(a) Internal square wave model

(b) Vertical mode comparisons

(c) Sound radiation from the termination

Radiation beam pattern of a vertical propagating mode
(Total radiation field = sum of all vertical mode radiations)

Horizontal modes $\phi_{nm}$ associated with $m^{th}$ vertical mode
Radiation Beam Pattern
decomposed to individual vertical modes
Other pieces of the menagerie to add

- Crossing internal waves (Reilly-Raska)
- Horizontal Lloyds mirror (Reilly-Raska)
- Mid-frequency (Katznelson)
- ???
Quo Vadis?

- What does one see with all of this stuff present? (for sure it’s Funky!)
- Are there distinct signals/signatures from each “process” we’ve examined, [time, angle, frequency, intensity,..] or do they produce similar, “additive” effects?
- How to describe this mess - with random medium approach…?!?
- What does this mean for naval and other ocean acoustics applications?
Questions??

"This is the part I always hate."