

Progress Report

from the Woods Hole Oceanographic Inst.

Dec. 10, 1941

---

1) A second group of 10 Ensigns has received training at Woods Hole and they are now awaiting orders. Their equipment is ready to be shipped. Several officers from the Inshore Patrol of the First Naval District have also been sent to Woods Hole for shorter periods of instruction.

2) About 40 special slide rules have been made up and these greatly facilitate the calculation of the path of the limiting sound ray. In most situations the range at any depth can be known within 5 or 6 minutes after the temperature record is available. A considerable number of these slide rules have been distributed to officers who are interested in the refraction of sound in sea water.

3) A set of 5 charts summarizing the available bathythermograph observations in the Western North Atlantic has been prepared. It is possible that a general distribution of these results should be made independently of the revised report on "Transmission of Sound in Sea Water", which is now ready for final typing.

4) Reports received from various officers who are now using bathythermographs and the auxiliary equipment indicate some changes which will improve the performance of the instrument under operating conditions. Nevertheless, useful observations are now being made in a routine way from destroyers and, considering the difficulties involved, the results of the

present design are most satisfactory. Besides improving the existing model we have begun to build a more rugged, hand operated instrument for use by small patrol craft. It may also be useful for showing up thermal stability near the surface and the resulting poor echo ranging conditions.

5) A temperature-depth recorder for submarines has been built and given preliminary tests.

6) A study of the advantages to be gained by a towed projector (or listening device) as well as a brief analysis of the importance of refraction from the submarine's point of view, both emphasize the possible significance of sound channels. It is believed that sound transmission in such layers should be carefully investigated, if this has not already been done.

7) A bathythermograph has been installed on a ferry which crosses Lake Michigan daily. The Meteorological Department of the University of Chicago is using these observations to study the exchange of heat, watervapor, and momentum. Prof. Rossby and Dr. Church are revising the theory of wind currents in the light of the newer ideas concerning the turbulence in the surface layer and with their most valuable cooperation it is hoped that before long our understanding of the thermal structure in the surface layer will be placed on a sound theoretical basis.

8) Of recent weeks a considerable part of our effort has gone into underwater photography at the request of representatives of the Bureau of Ordnance. However, this work also has an anti-submarine slant in that recent tests show that a light can easily be seen at depths of around 100 feet, and possibly

submarine carry a light, she could be seen or photographed from the surface ship through an underwater port.

9) It should perhaps be added that the problem of how best to use the "Atlantis" in the future is a serious one. Moreover, it is not going to be easy to hold together her crew or, for that matter, to keep many of our laboratory technicians and assistants from also enlisting in the Army or Navy.

~~CONFIDENTIAL~~ NON-REGISTERED PUBLICATION  
*Declassified*

CALCULATION OF SOUND RAY PATHS  
USING THE  
REFRACTION SLIDE RULE

May 1943

BUREAU OF SHIPS

NAVY DEPARTMENT

WASHINGTON, D.C.

~~SECRET~~

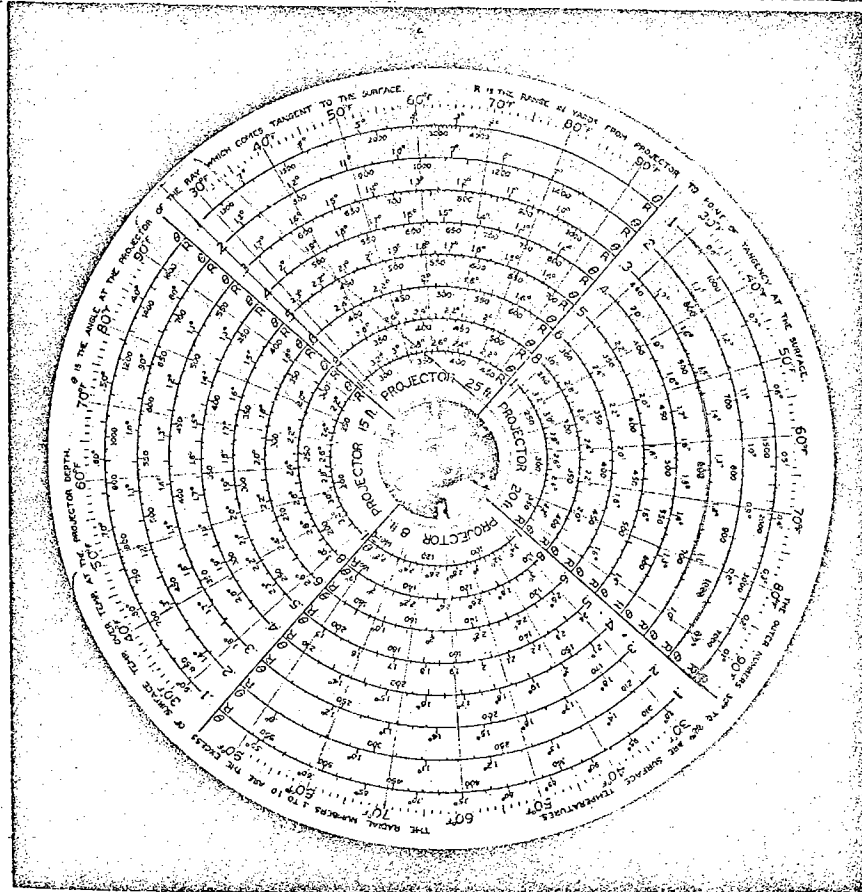
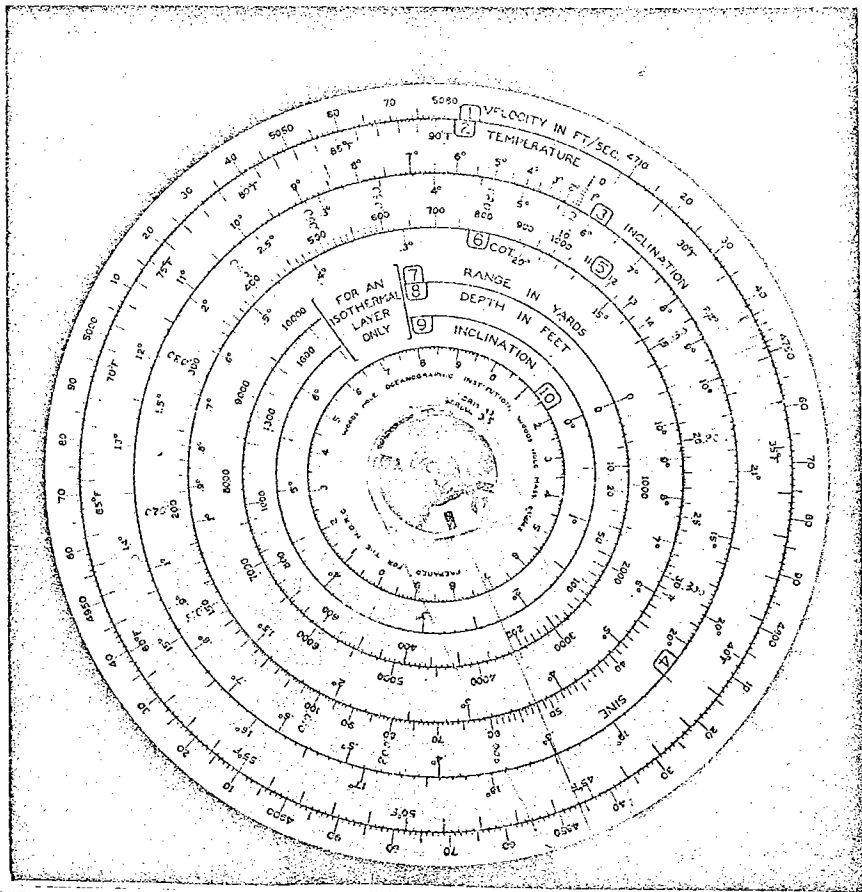
CALCULATION OF SOUND RAY PATHS  
USING THE  
REFRACTION SLIDE RULE

COLLABORATORS:

Woods Hole Oceanographic Institution.  
Division of War Research, University  
of California.  
National Defense Research Committee.  
Bureau of Ships, Navy Department.

## TABLE OF CONTENTS

	Page
I INTRODUCTION	
A. General Statement	1
B. Principles of Refraction Calculations	3
C. Principles of Intensity Calculations	6
D. Description of the Slide Rule	7
II RAY DIAGRAM CALCULATIONS	
A. Examples on Isothermal Layers	14
B. Examples on Downward Refraction	17
C. Examples on Upward Refraction	22
D. Examples on the Use of the Velocity Scale	26
E. Method of Proportions	27
III INTENSITY CALCULATIONS	
A. Introduction	32
B. Graphical Determination	32
IV PREDICTION OF MAXIMUM ECHO RANGE	
A. Types of Procedure	37
B. Glance at Bathythermograph Slide	37
C. Determine "Assured Range".	37
D. Draw Limiting Rays	37
E. Construct Ray Diagram	39
F. Construct Intensity Diagram	39
G. Exact Prediction of Maximum Range	40



# CALCULATION OF SOUND RAY PATHS USING THE REFRACTION SLIDE RULE

## 1. INTRODUCTION

### A. General Statement

The maximum ranges at which a target can be detected at sea with supersonic echo-ranging equipment are very variable and in most instances are not limited by the sensitivity of the equipment itself. Recently, the importance of refraction, or bending of the supersonic beam has been emphasized as a controlling factor in determining maximum echo ranges. Predictions based on the calculated bending of the sound beam have been found extremely valuable in establishing the effective maximum ranges which exist in any area at a particular time.

If the velocity of sound in sea water were everywhere the same, there would be no bending. A beam projected horizontally from a standard projector could be represented by a cone of circular cross section, with a fan shaped section in a vertical plane through the axis as shown in Figure 1. It should be noted that the edge of the shadow zone is not as clear cut as the figure indicates. In practice, a shadow zone is a region from which no useful echo can be obtained.

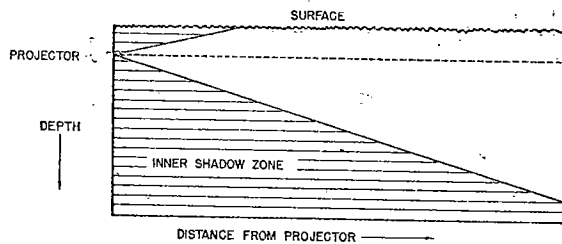


Figure 1

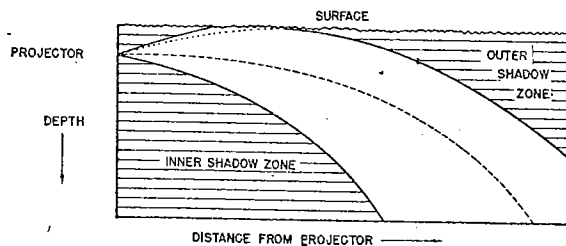


Figure 2



Under most conditions, the sound beam does not travel in a straight line but is bent upward or downward as in Figure 2, split and distorted, owing to changes in sound velocity with depth. The velocity of sound in sea water increases with temperature, with pressure, and with salinity. Ordinarily, salinity differences between the surface and a depth of 600 feet are small and may be neglected because variations of sound velocity with salinity are also small. The increase of sound velocity due to pressure amounts to eleven feet per second at a depth of 600 feet. That is, the portion of the wave front at 600 feet travels eleven feet per second faster than the portion of the wave front at the surface, if the temperature of the water is the same at both depths. A temperature difference of only three degrees Fahrenheit will affect the velocity of sound as much as a pressure difference equal to 600 feet of water. As temperature differences of ten or more degrees in a 600 foot water column are common, and as these differences vary from place to place while the pressure difference is the same all over the world, it is clear that knowledge of the distribution of temperature with depth is essential for the calculation of echo ranges. This information can be obtained at any time from a moving vessel by the use of the bathythermograph, an instrument which gives, on a small glass slide, a record of temperature against depth.

For a quick, but approximate, determination of sound ranging conditions, simple tables have been drawn up. These tables are shown and their use described in "Instruction Manual for Bathythermograph Observers, - Part II, Prediction of Sound Ranges from Bathythermograph Observations", published by the Bureau of Ships.

For some purposes, however, an exact knowledge of the refraction pattern is desirable. When the temperature distribution is complicated, or when information is wanted concerning the variation of maximum echo range with target depth, the simple methods no longer suffice. To simplify the more extended calculations the Refraction Slide rule, the use of which is described in the present Manual, has been developed.

The refraction slide rule can be used to calculate the maximum echo range obtainable from a target at any depth, in any type of water, provided the variation of temperature with depth is known from a bathythermograph observation. Frequently it is sufficient to calculate the path of a single limiting ray which bounds the shadow zone and is horizontal at the depth of maximum velocity. In some cases however, a ray diagram must be constructed, showing the deflections of various rays in the sonic beam. These diagrams can be produced in 2 to 15 minutes, depending on the complexity of the water and the skill of the operator.

If the target is in the beam, the intensity of sonic energy reaching it will frequently depend primarily on the range, regardless of the type of refraction which the beam undergoes in transit. However, in some cases the intensity in part of the beam is reduced by refraction far below the value appropriate to the distance, and this decrease must be taken into account. The slide rule can be used, both to calculate the shape of the ray diagram and to calculate the relative intensity in any part of the ray diagram. With even a little experience in working with ray diagrams, it is quite easy to know in advance which cases are worth the additional time required for intensity calculations.

Thus, the slide rule can be used first to calculate ray paths for the construction of a ray diagram or the determination of the boundaries of the shadow zone, and second, for calculation of relative intensities along any ray in cases where there are abnormal variations of intensity with range. On the basis of these calculations the maximum echo range can be predicted with reasonable accuracy for any type of water conditions.

In depths less than 100 fathoms, where the bottom is smooth and hard and the refraction is downward, extension of the range by reflections from bottom must sometimes be taken into account.

## B. Principles of Refraction Calculations

The refraction of sound rays depends on the variation of the velocity of sound from point to point. The treatment of refraction in the present report is based on the following assumptions about sea water, justified in the light of experience.

1. The velocity of sound in the open ocean depends only on temperature and pressure, changes in salinity being negligible.
2. The water is stratified horizontally, which means that a bathythermograph lowering defines the temperature-depth relation at all points within echo-ranging distance of the point of lowering.
3. The temperature-depth curve on the bathythermograph slide may be approximated by a series of straight lines, which means that the water column may be considered made up of a series of layers, each having a constant thermal gradient.

Any ray passing through a horizontally-stratified medium remains in a single vertical plane. The inclination to the horizontal for any point on the ray is given by Snell's Law. Let  $V_0, V_1, V_2$ , etc. be the velocity of sound at points along the ray, and let  $\theta_0, \theta_1, \theta_2$ , etc. be the inclination of the ray at the corresponding points. Then Snell's Law, which is the entire basis of ray diagram construction, states that

$$\cos \theta_0 / V_0 = \cos \theta_1 / V_1 = \cos \theta_2 / V_2 = \dots \quad (1)$$

The following deductions can be made from Snell's Law:

1. Suppose that the velocity of sound increases steadily with depth, as shown by the solid line on the right hand side of Figure 3A, where the velocity (horizontal scale) is plotted against depth (vertical scale). To the left in Figure 3A is shown a typical ray in such a case, where the depth scale is the same as on the right-hand side, but the horizontal scale now represents distance from the projector. This combination of two graphs together is a very useful way of indicating the effect of a particular velocity distribution on the ray diagram. In Figure 3A,  $V_0$  and  $\theta_0$  represent the velocity and inclination of a ray at the projector. Let  $V_1$  and  $\theta_1$  relate to any other point on the same ray. Equation 1 shows that as  $V_1$  increases,  $\theta_1$  becomes smaller, becoming zero when  $V_1 = V_0 / \cos \theta_0$ , and this particular ray can never reach a level where the velocity is greater than this value. A ray leaving the projector with a greater initial inclination can obviously reach a level of greater velocity. When the velocity decreases with depth, the corresponding path of a ray through its maximum level is shown in Figure 3B.

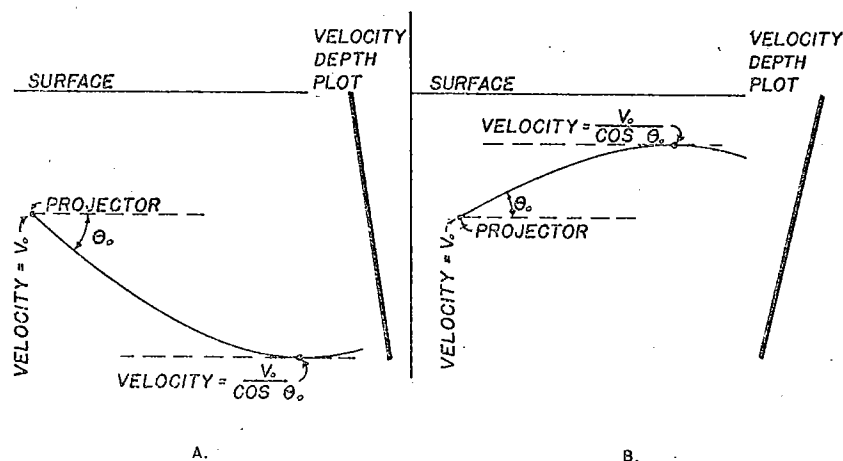


Figure 3

2. Suppose that with increasing depth in the water the velocity increases to a maximum  $V_m$  and then decreases, as shown in Figure 4. All rays for which the initial angle  $\theta_0$  for the ray at the projector is so small that  $V_0/\cos\theta_0$  is less than  $V_m$  will recurve and return to projector level.

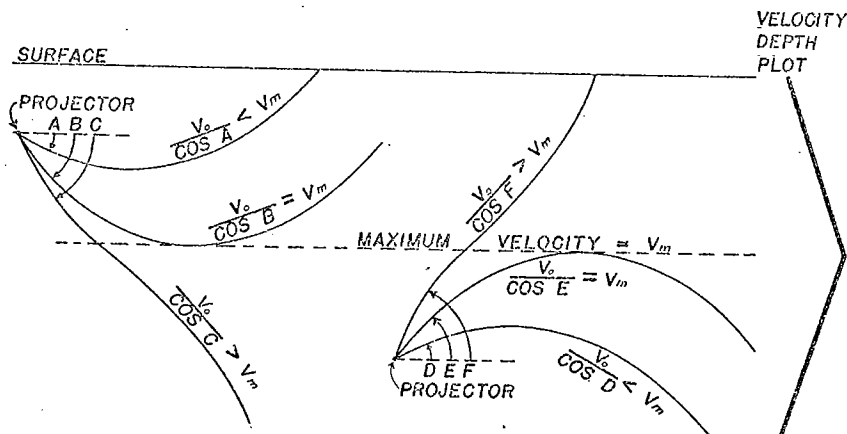


Figure 4

On the other hand, all rays with initial angles,  $\theta_0$ , sufficiently large to make  $V_0/\cos\theta_0$  greater than  $V_m$ , will penetrate beyond the level of maximum velocity and will not return to projector level.

3. In a horizontal layer in which the temperature-depth graph is a straight line, all rays are circular arcs having centers on a single horizontal plane, as shown in Figure 5. If  $V_s$  is the velocity of sound at the surface of the layer, and if the velocity changes with depth at a rate  $g$ , the velocity at any depth in the layer is  $V = V_s + gh$ . The distance from the surface of the layer to the plane of centers is  $V_s/g$ . It may assist the memory to note that the plane of centers is the plane at which the velocity would be zero if the layer extended that far and maintained a constant rate of change of velocity.

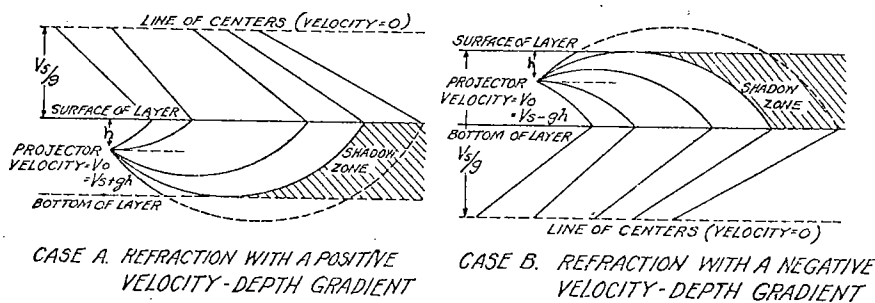


Figure 5

4. If a ray recurves and thus passes a given depth twice, Equation 1 shows that its inclination on the two passages will be equal in magnitude. For example, if one of the rays in Figure 5 leaves the projector level, with an inclination of  $+3^\circ$ , and returns to projector level, its inclination will then be  $-3^\circ$ .

5. If the variation of velocity with depth is such that a given ray encounters the same velocity at two different depths, it is seen from Equation 1 that the ray will have the same inclination at both depths.

### C. Principles of Intensity Calculations

The method of calculating intensity given in this report assumes that no sonic energy is lost by scattering or by absorption in the water. The loss in intensity is considered to be due simply to the geometrical spreading of the beam. With this assumption, the total energy between any two rays is the same at all distances from the projector; since the intensity is equal to the energy crossing an area one foot square placed perpendicular to the beam, this quantity decreases as the distance between any two rays increases.

When the velocity is constant throughout the water, all rays are straight and the perpendicular distance between any two rays is directly proportional to the distance. If account is taken of the lateral spreading of the beam in addition to the spreading in the vertical plane shown in the simple ray diagram, the familiar inverse square law results; that is, the intensity at any point is inversely proportional to the square of the distance from the projector. Where variations in velocity bend the rays strongly, it is evident from inspection of the ray diagram that there are places where the effect of refraction is to spread the rays apart, decreasing the intensity below the value which would otherwise exist at the same distance from the projector. This is particularly true when a ray reverses its direction of curvature at a point where the inclination is slight. On the other hand, in some cases the rays converge at some points, leading to large increases in the intensity.

Figure 6 contains ray diagrams for two types of refraction. In each ray diagram the angles between successive rays at the projector are all equal, so we may consider that equal amounts of energy go into each of the spaces between rays. It is obvious that at point A the effect of refraction has been to spread the energy over a wider area, decreasing the intensity, while at point B the effect is the opposite.

All intensities calculated in this report are relative intensities, the intensity at any point being given in decibels (see Page 13) below the intensity at a distance of one yard from the projector, considering the projector to act as a point source of sound.

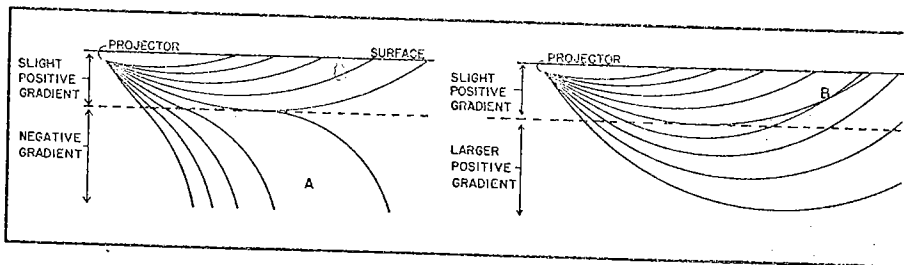


Figure 6

#### D. Description of the Slide Rule

The front side of the rule (see Frontispiece) contains ten circular scales and two index arms. The longer arm contains a long index mark and a scale of depth extending from zero to 600 feet. If the bearings of the rule are in proper adjustment, rotation of the longer arm causes the shorter one to move with it so that the angle between the two remains constant. Rotation of the shorter arm does not affect the position of the longer one.

The scales of the rule numbered from the outside to the center, are as follows:

Scale 1. Velocity of sound in feet per second for sea water. The graduations on this scale are spaced so that the angle included between any two velocity settings,  $V_0$  and  $V_1$ , is proportional to  $\log V_0 - \log V_1$ .

Scale 2. Temperature of the sea water in degrees Fahrenheit. On Scale 2, opposite each velocity on Scale 1, is the temperature at which, in sea water of standard salinity (35 parts in 1000) at atmospheric pressure, sound will have that velocity. To find the velocity at any temperature and depth the auxiliary depth scale of the long arm is used. Set this arm so the given depth coincides with the given temperature on Scale 2, and read the velocity on Scale 1 under the main index line. Illustrations of typical settings appear in Figure 7.

Scale 3. The inclination of a sound ray relative to the horizontal. The graduations of this scale are spaced so the angle included between two settings  $\theta_0$  and  $\theta_1$ , is proportional to  $\log \cos \theta_0 - \log \cos \theta_1$ , the constant of proportionality being the same as that used for Scale 1. It is used in conjunction with Scales 1 or 2 to solve Snell's Law (Equation 1) in the logarithmic form:

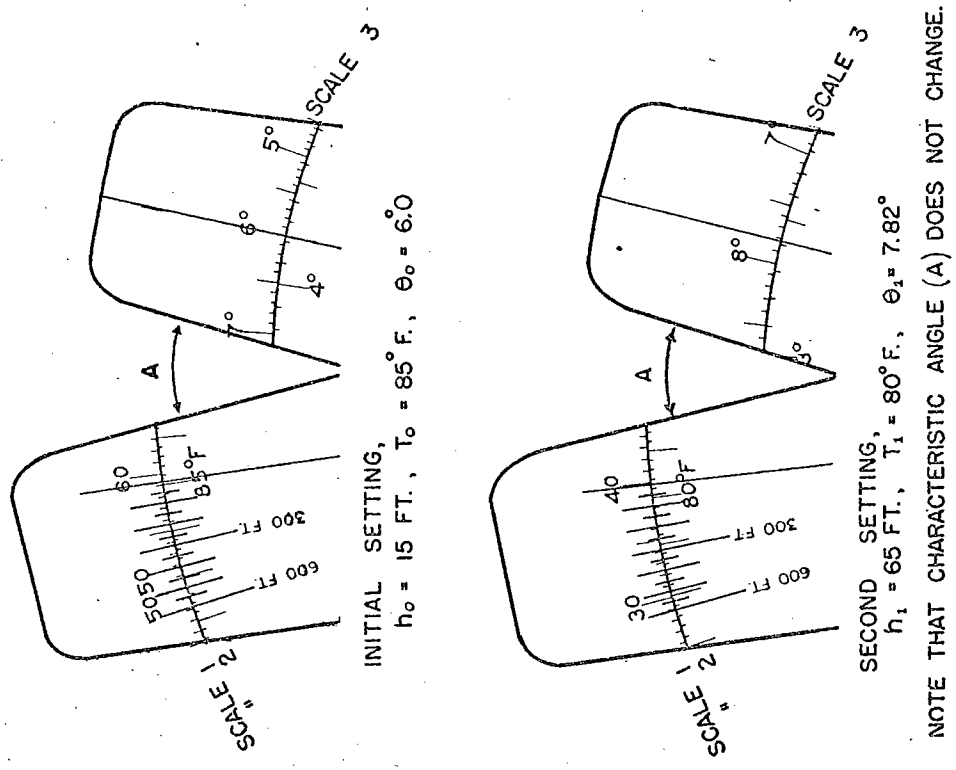


Figure 8

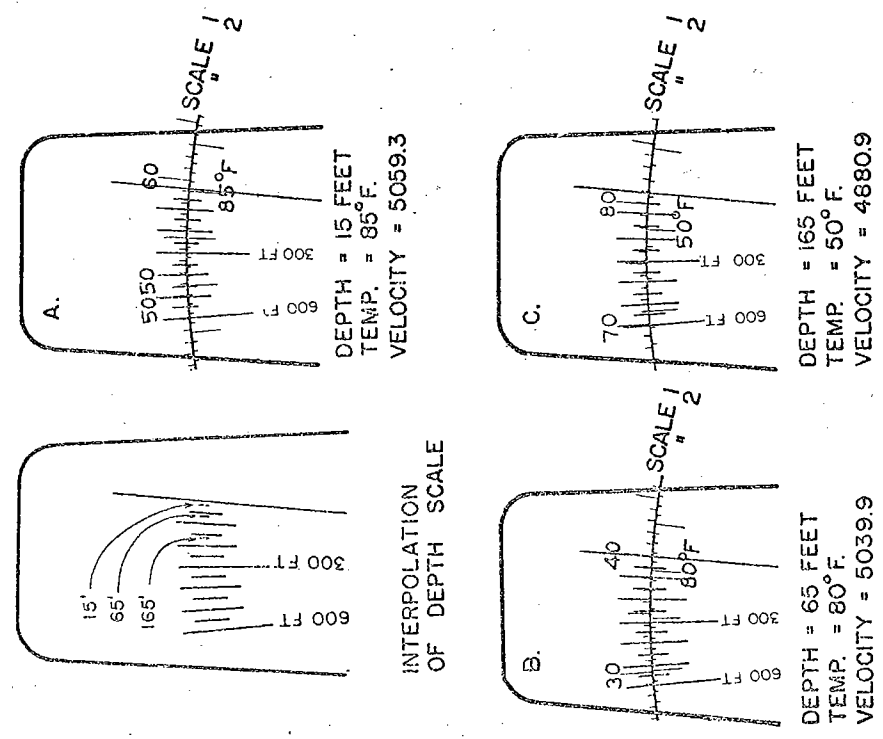


Figure 7

$$\log \cos \theta_1 = (\log \cos \theta_0 - \log V_0) + \log V_1$$

To trace a ray which has the inclination  $\theta_0$  at a depth  $h_0$  and a temperature  $T_0$ , set  $h_0$  on the long arm to coincide with  $T_0$  on Scale 2 and set the index of the short arm on  $\theta_0$  on Scale 3. The angle between the two arms now represents  $\log \cos \theta_0 - \log V_0$ . This angle between the arms is characteristic for the given ray and should remain unchanged until all inclinations on that particular ray have been determined. Thus, the initial setting in calculations for a given ray consists of arranging the scales to show (1) a depth, (2) a temperature (or a velocity which replaces both depth and temperature), (3) an inclination. To find the inclination of this ray at any other point where the temperature and depth are  $T_1$  and  $h_1$ , set  $h_1$  on the long arm in coincidence with  $T_1$  on Scale 2 and read  $\theta_1$  under the short index on Scale 3. Figure 8 illustrates this calculation.

One feature of Scale 3 which may require explanation is the method of graduation for inclinations between zero and  $2^\circ$ . By reference to Figure 10B, it is seen that the curved line connecting zero with  $2^\circ$  is divided into ten parts, corresponding to intervals of  $0.2^\circ$ , by a set of parallel lines. The settings of the index are made on the intersections of these lines with the curve.

Scale 4. The sine scale is graduated so that the angle between any two settings,  $\theta_0$  and  $\theta_1$ , is proportional to  $\log \sin \theta_0 - \log \sin \theta_1$ . It is related to Scale 5 in such a way that opposite each angle on Scale 4 is the sine of that angle (except for decimal place) on Scale 5. If the user has difficulty keeping the decimal point straight, he may write in the space between these two scales, as seen in Figure 20, the sines of a few representative angles in black lacquer. The sine scale is used only in calculations of intensity. From Figure 10A, it may be seen that  $\sin 5.02^\circ = .0875$ .



Scale 5. The logarithm scale is a standard two-cycle logarithm scale graduated from 10 to 1000. It is used for all operations of multiplication and division and may be used in conjunction with Scales 4 and 6. In general, the decimal place cannot be determined from the rule.

To obtain the product  $ab$ , (1) set the long index on  $a$  and the short one on 10, (2) move the long arm until the short index is at  $b$ , and (3) read the product under the long index, as in Figure 9, which illustrates the multiplication of 219 by 0.86.

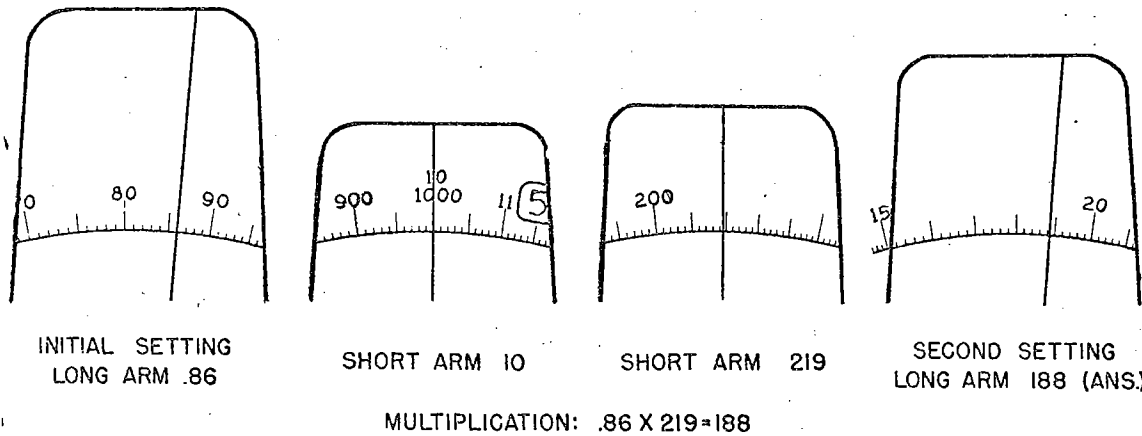


Figure 9

Figure 10 illustrates the multiplication  $60 \times \sin 5.02^\circ = 5.26$ .

To obtain the quotient  $a/b$ , (1) set the long index on  $a$  and the short one on  $b$ , (2) move the long arm until the short index is at 10, and (3) read the quotient under the long index, as in Figure 11, which illustrates the division of 5.88 by 6.82.

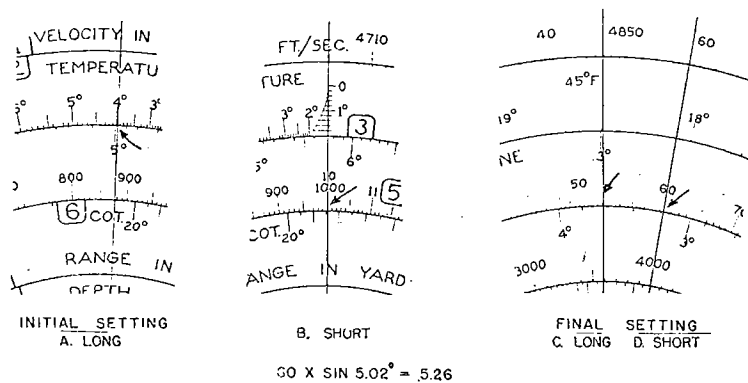
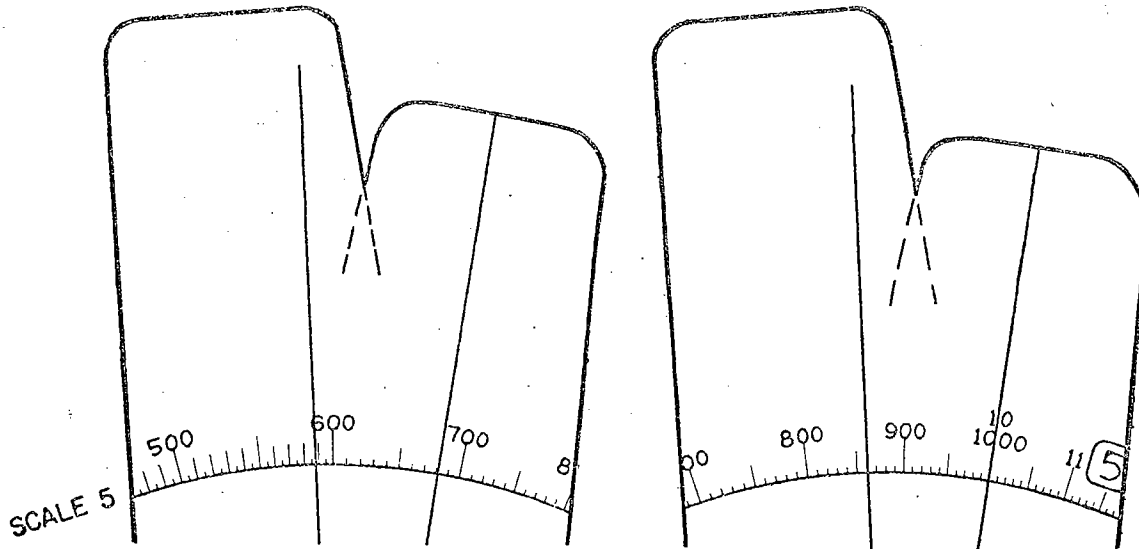


Figure 10

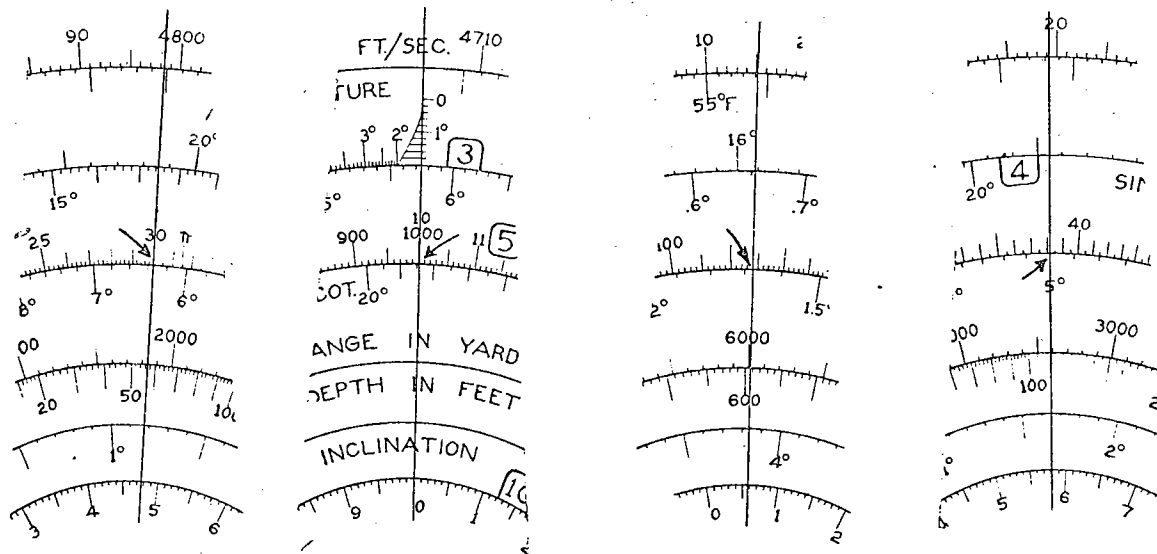


INITIAL SETTING  
 SHORT ARM 6.82  
 LONG ARM 5.88

SECOND SETTING  
 SHORT ARM 10  
 LONG ARM .86 (ANS.)

DIVISION:  $5.88 / 6.82 = 0.86$

Figure 11



A. LONG  
 FIRST SETTING

B. SHORT

C. LONG  
 SECOND SETTING

D. SHORT

$\Delta h = 30 \text{ FT.}, \bar{\theta} = 5^\circ, \Delta X = 114 \text{ YDS.}$

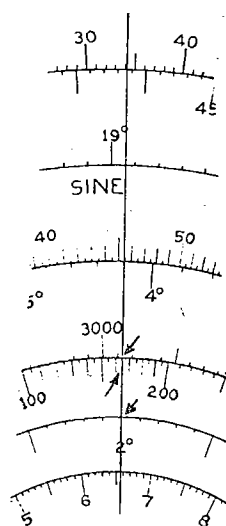
Figure 12

Scale 6. The cotangent scale is used to compute the horizontal travel  $X$  of a ray in a layer when the thickness of the layer,  $h$ , and the angles of entrance and emergence,  $\theta_0$  and  $\theta_1$  are known.

The equation involved is

$$X = h \cot \left[ (\theta_0 + \theta_1) / 2 \right] = h \cot \bar{\theta} \dots (2)$$

In the actual use of this scale: (1), on Scale 5, set the long index on  $h$  in feet and the short index on 10; (2), move the long arm until the short index coincides with the given mean angle,  $\bar{\theta}$ , on Scale 6; and (3), read the range  $X$  in yards under the long index on Scale 5. This scale has been graduated according to  $(\log \cot \theta) / 3$  in order that the range will come out in yards if the depth is expressed in feet. For example, for  $h = 30$  feet,  $\theta_0 = 6^\circ$ ,  $\theta_1 = 4^\circ$ ,  $(\theta_0 + \theta_1) / 2 = \bar{\theta} = 5^\circ$  one obtains  $X = 114$  yards, as seen from Figure 12.



ISOTHERMAL LAYER  
 $\theta = 2^\circ$ ,  $h = 165$  FL.,  $X = 3120$  YDS.

Figure 13

Scales 7, 8 and 9. These scales are used for isothermal layers only. A single setting of an index line across these three scales gives values of  $X$ ,  $h$ , and  $\theta$ .  $X$  and  $h$  are the horizontal and vertical dimensions of a segment of a ray having one end horizontal, and  $\theta$  is the inclination of the other end of the segment. For instance, a ray with an initial inclination of  $2^\circ$  will travel 3120 yards laterally and 165 feet vertically in isothermal water before becoming horizontal, as seen from Figure 13.

